RIGHT THINKING

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A STUDY OF ITS PRINCIPLES AND METHODS

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PREFACE

Seventeen years ago, when I was insufficiently aware of the extent of my ignorance, I published a book entitled *Principles and Problems of Right Thinking.* Recently this book has, deservedly, gone out of print. Were it not for two considerations I should have shed no tears over its demise and should long ago have become absorbed in other enterprises.

But the passing of the years has convinced me more than ever, first, that there is great need for a book covering essentially the ground which that book ambitiously endeavored to cover, even though the product of one's hardest endeavor might contain many defects. I determined, therefore, to attempt the task again, in the hope that this time it might realize more of its aim and, at least, mislead the reader less than before; I was encouraged in this determination by the fact that, even with the weaknesses of its earlier form, it had been found useful by not a few college teachers of logic and kindred subjects.

Second, I became especially convinced that there is serious need for the inclusion in such a volume of a systematic treatment of reasoning as evaluation, however sadly the treatment might fall short of what its readers have a right to expect. If there is any distinction between wise and unwise evaluation, particularly in choosing ends of conduct, the student needs to be shown that there is, and on what rational ground that distinction rests. And in this connection I should be distressed to appear to leave the theme on the note of sceptical relativism which dominated its treatment in the earlier book; like many others at that time, I found myself forced in honesty to such a relativism and saw no clear way of getting beyond it. But I believe now that it is possible to get beyond it, without forsaking either open-mindedness or intellectual in-

subject of statistical methods; and Professor H. D. Laube on the topic of the nature of legal concepts. Professor Ernest Nagel, of Columbia University, gave me the benefit of his criticism of a tentative draft of the chapter on probability. I wish here to express my thanks for all these aids. The reader will kindly remember, however, that none of the friends mentioned is responsible for the way in which these matters are finally presented.

Constructive criticisms of any part of the book will be deeply appreciated, and of course I should be glad to be apprised of any outright errors that have slipped through, or any dubious statements proffered with unjustified assurance.

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Ithaca, N.Y.
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SUGGESTIONS TO TEACHERS

This book has been written with the aim of providing material sufficient for a two-semester course. Since many teachers who will use it as a text have, however, only one semester at their disposal, the following suggestions regarding the selection of chapters may be helpful.

1. For an Orientation Study of Man as Thinker
   - Part I—Entire
   - Part II—chaps. 7, 8, 9 to p. 165
   - Part III—chaps. 15–19, 23
   - Part IV—chaps. 25, 26, 28, 29
2. For a Study Emphasizing Scientific Method
   - Part I—chaps. 1, 2, 5, 6
   - Part II—chaps. 7–11
   - Part III—Entire
3. For a Study Emphasizing Formal Logic
   - Part I—chaps. 1, 2
   - Part II—Entire
   - Part III—chaps. 15–17, 20–22, 24
4. For a Study Emphasizing Rational Evaluation
   - Part I—Entire
   - Part II—chap. 7
   - Part III—chaps. 15, 19, 23, 24
   - Part IV—Entire

PART I

THE ADVENTURE OF THINKING

CHAPTER 1

THINKING—AND RIGHT THINKING

Man as thinker

The famous Harvard philosopher, Josiah Royce, is reported to have remarked in an argument with his equally famous colleague, William James, "The trouble with you, James, is that you don't know how to think." What a devastating criticism!—that is, were it meant without humorous exaggeration, and with no presupposition of genial friendship. Not only are philosophers supposed to know how to think—man in general has been considered different from and superior to the lower creatures precisely in virtue of this capacity. He is *homo sapiens*, man the thinker. Not to know how to think, then, is to fail at a crucial point; it is not quite to be a man at all.

Most of the things that men prize in the intricate system of living called civilization are achievements of man as thinker. The accomplishments of some of the lower animals are great, but they have not created human culture with its impressively distinctive institutions. To realize this is more important than ever in a day when the result of man's labor is being tested in a crucible of fire, so that it will be clearly shown what parts of this vast structure can endure and what parts will crumble in irretrievable ruin. Science would be impossible if man did not think—both pure science in its quest for an understanding of nature, and applied science with the astounding inventions that testify to its practical power. Literature, art, religion, philosophy, and the patterns of political and economic life are likewise products of man's fertile and disciplined imagination; and any exercise of disciplined imagination is a form of thinking. Moreover, none of these enterprises can be sustained and developed from age to age

by tradition and habit merely. Their effective preservation demands continued thinking, as a few illustrations quickly show. Appreciation of a work of art cannot occur unless to some extent one thinks the artist's thoughts after him; and it is no longer an artistic creation when it ceases to be appreciated as such. Continued use of such a marvelous invention as the radio or the automobile requires that some in each generation comprehend anew the laws of nature that are utilized in its construction and acquire facility in practically applying this comprehension. The world we live in is a vastly different and far greater world because man thinks.

But, as Royce's remark suggests, observation of the reflective meanderings of our neighbors gives us frequent occasion to note that knowing how to think is not such a universal or fully developed capacity as might be desired. Consider the numerology column in one of our large city papers.

Q. My name is ——. Born in Kentucky, worked hard all my life but can't make things go. What's wrong with my name? T. E.

A. My boy, I wish fate had contrived to have me born into this world under the same name you bear. Your name, dear Kentuckian, is exactly the name a certain world-renowned man wore before fame knocked at his door and he was exalted to the seat of the mightiest of the mighty. I refer to Thomas A. Edison. He sometimes signed himself T. A. Edison and sometimes T. Edison up to his thirty-second year. Up to then he had hard and heart-breaking struggles. He was a genius long before the world knew it. And at last he began to sign his name Thomas A. Edison, a name perfect in its vibrations.

Before the change to the signature, Thomas A. Edison, his destiny number was 7, which is neutral and in melody with the words chance, complex, absent, empty, frequent, hunger, left, oppose, pale, trials, and woe. And so he resumed the A. as the middle initial which was his birthright and his signature. As you will find if you work it out yourself, this tunes him in with these highly positive words: achieve, axiom, brain, complete, exact, enlarge, fellowship, faith, goal, God, hope, happiness, intellectual, independence, magnetic, success, superman, unity, and vast, all of these words having the atomic weight value of 8, which is Thomas A. Edison's destiny number.

So, as you can see, there is something in a name, as it is written and as it vibrates. Many have changed their names so as to be more harmoniously attuned to life.

What is it
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Is this thinking? One wonders. And in moods of humility, at least, one discovers that his own previous mental meanderings have not always led to the conclusions now clearly seen to be right. Apparently, a distinction must be drawn. All of us who bear the human form think, but not all think *straight* in a high percentage of our thinking.

Consider a case where the authors are far more competent men than the writer of this numerology column. In February, 1945, two groups of college presidents gave public pronouncements, one group favoring conscription in the United States after the war, the other opposing it, so far, at least, as any immediate commitment was concerned. One important premise in the reasoning of the two groups was the same, namely, that when peace returns there is bound to be a change in people's ways of thinking about this problem. But one group proceeded from this premise as follows: In peacetime we shall be less conscious of the danger of war than we are now, hence we ought to take advantage of the present war psychology to commit ourselves to a policy of adequate national protection. The other group proceeded from it as follows: After the war we shall be less under the influence of blind emotion than we are now and can reason more calmly and clearly, hence we ought to postpone till then any decision on such an important issue as the issue of peacetime conscription. Now both of these groups of eminent scholarly gentlemen could hardly have been reasoning correctly, since, in part from the same premises, they reached contradictory conclusions.

Such instances as these, when fairly faced, are very challenging. They force us, if we are sufficiently inquisitive as well as sobered, to confront seriously the questions: What then does it mean to think straight? What principles do we need to grasp and embody in habit if we are to assess confidently the thinking of others, and if our own thinking in the future is to be wiser than it has been in the past?

These questions furnish the guiding theme of the pages that follow. We propose, in other words, to engage in a study of man as thinker. But our study will not be that of a psycholo-

gist—describing thinking just as he finds it going on—except where such a description provides us a necessary foundation. We shall examine *man as seeking to think aright,* and every part of the enterprise will be affected by this consideration.

The reader should not, however, expect any magical transformation in his own reflective habits from participating in this quest. To look for magic anywhere is to court disappointment. There should, however, be a difference between one who has considered systematically what it means to think aright and has made a serious effort to profit by it, and one who has not. When the latter has painfully corrected one mistake or outgrown one superstition he will not know how to protect himself against falling into another, if the two are not very similar. Whereas the former will be on guard because he will know the main types of erroneous thinking into which people are likely to fall, and will understand why they are erroneous. All education assumes that fuller knowledge brings fuller self-control, in matters to which the knowledge pertains. We shall assume this too, but shall not expect any particular degree of rational self-mastery to result at once. A foundation will be laid, on which we can continue to build as long as we will.

First it is necessary to discover what kind of affair thinking is, and under what conditions it can be right or wrong. This preliminary inquiry may be opened by asking: When do we think? Whenever we are awake, or only sometimes?

It might seem that the latter answer should be given. All are familiar with the old rustic who, on being questioned as to how he spent his time, replied, "Sometimes I set and think, and sometimes I just set." And an unimaginative damsel is reported to have answered her lover's jealous appeal, "Darling, do you always think of me?" with a reply no doubt sufficiently appeasing: "Yes, dear, whenever I am thinking at all." If, too, we hesitate to admit that our numerologist friend was thinking, we evidently have in mind a limited conception of what "thinking" includes. But we will understand right thinking better if we commence with the most generous conception

that might be made plausible. Now during all our waking hours something, however vague and profitless, is flitting through our minds. In some sense—a very broad one, to be sure—thinking is always going on, and the amount and variety of it, as revealed to inward analysis, is rather startling. As J. H. Robinson remarks, "Our thought moves with such incredible rapidity that it is almost impossible to arrest any specimen of it long enough to have a look at it. When we are offered a penny for our thoughts we always find that we have recently had so many things in mind that we can easily make a selection which will not compromise us too nakedly."[1]

What common forms does thinking take?

But this varied and fitful conscious activity is obviously not all of the same sort. Let us examine a typical stream of it in order to distinguish a few of the common forms that it takes.

One summer morning I took my family for a ride in a little motorboat off the shore of Lake Michigan. During part of the trip my thinking was so vague and effortless and so pliable to the mood of the moment that it is exceedingly difficult to recover it with confidence. There were, of course, the sensations of relaxed bodily comfort in the genial warmth of the sun and the gentle play of the breeze; there were the feelings of spontaneous muscular activity as I changed position in the boat from time to time, or found my attention attracted by this or that object around me. These experiences, taken by themselves, would perhaps not be regarded as thinking; but interspersed with and accompanying them were the free constructions of imagination, built by the passing mood in response to cues offered by the environment, each quickly waning and fading away as some new color, shape, or sound forced itself upon my attention. The steady purring of the motor reminded me of an earlier difficulty in adjusting the flow of gas to the carburetor, and suggested how much more fun it would be to have an engine geared to secure far greater speed than this one could achieve at best. The succession of sand bars extending irregularly into the lake from

[1] *Mind in the Making*, p. 37.

the shore stirred my imagination to picture playfully a war between different schools of fishes, who used these bars as natural ramparts for protection and cover in attack. Such thinking went on whenever nothing more incisive controlled my mind in any more fruitful fashion.

At times this stream of vague sensations and loose fancies passed into appreciative enjoyment of some feature of the scene through which we were passing. The blue of the lake, as it shifted its pattern under the puffy clouds screening it here and there from the sun; that lone clump of birches giving the knoll on which it grew a white crown above the green mantle of the pines; these will exemplify, from the trip mentioned, the many common experiences which suddenly surprise us with beauty and hold our thoughts eagerly captive.

But a large concrete slab set up on the shore ahead attracts our attention. We draw near it and stop. On it are inscribed the words: "On this spot was made the first settlement of ——, by ——, in the month of ——, ——. Erected —— by the Historical Society of ——." We had hardly absorbed this news when one of us noticed a strange dark object floating in the lake some distance farther. As we wondered what it might be we saw several people gathered on the shore opposite it. I started the motor and moved toward them. Soon a man emerged from the group and ran down the beach to meet us. When we were near enough he dashed into the water, clad, as we now saw to our surprise, only in his underclothes. In response to his frantic waving we drew up. "A boy is being carried out to sea in a barrel!" he shouted. "He's going fast!" "We'll get him," I called in reply, and turned the boat lakeward. At once my thinking became a quite different sort of thing from any of the above types. The challenge of the situation controlled it. First thought—shall I land the children, so that there will be no danger of their causing anxiety? For the drifting boy was already more than half a mile from shore, and was swept on by a stiff offshore breeze. And it is not impossible for a small boat to be upset. No, for every moment is precious. The barrel must be harder to balance as it moves farther out in the

waves stirred up by the offshore wind; moreover, our boat is pretty steady for its size, and if the youngsters sit down in the bottom they will both be safer and make an upset less likely. So I steer directly for the drifting barrel. As we approach, two other important questions need an answer. Is the boy self-possessed, or is he so excited that he might jump for the boat too soon and thus endanger us as well as his own chance of rescue? Can I bring the boat near to the barrel without tipping it over before we quite reach it? I slow down at a distance of a few rods and scan closely the boy and the large half cask which forms his conveyance. He seems quiet and is maintaining his balance as well as the ripples permit; his strange craft, too, seems fairly steady. So I greet him in a matter-of-fact tone, and stop the engine just far enough away for the boat to glide up beside him. He scrambles in and soon we have him ashore.

Daydreaming or reverie

What different types of thinking are exemplified in this illustration? Well, let us note them in order. First, much of the time was occupied by a free play of imaginative activity merely responsive to the mood of the moment. This is what is called *reverie* or *daydreaming*. Though casual, loose-jointed, and usually worthless for anything beyond itself, it is, as Robinson affirms, "our spontaneous and favorite kind of thinking." By this he means that most of our conscious life, and vastly more than we should like to admit, is occupied with precisely this lazy and undirected play of fancy.

Esthetic appreciation and acquisition of information

In the second place, there was *esthetic appreciation*. This form is sufficiently identified in the illustration. Enjoyment of the beauties that surround us in nature and art is universal enough for no one to have difficulty distinguishing it from mere reverie, and it would as surely be violating common usage to deny that it is a kind of thinking.

A third type is revealed in our *securing information* from the tablet. Here we were adding to our previous store of knowledge by absorbing a set of ideas expressed in words. Since the relation of these words to the corresponding facts has long been familiar, such appropriation of knowledge may

involve no conscious effort. Yet it requires concentration during the reading, and presumably no one would refuse to regard it as a form of thinking specifically different from those already enumerated.

The solution of a problem

A fourth kind of thinking is also apparent in the illustration, quite different from any of these three. In it, a challenge to our powers has thrust itself upon us. Thinking gains at once in purposefulness and zest. All our conscious resources are mobilized in an effort to meet the situation successfully. The occurrence of the challenge sets a goal which we desire to reach—in this particular case, the rescue of the imperiled boy. Let us broaden somewhat the ordinary meaning of the word "problem," so as to refer by it to any such challenge, of whatever sort, that commands our attention and controls the mental activity at once embarked upon. Now it is precisely this ensuing process that exemplifies the type of thinking at present to be distinguished. The desired goal guides our thinking, and the outcome of the thinking guides whatever subsequent action is needed. Thinking of this fourth type is thus essentially a method by which to meet the demands and resolve the perplexities that we from time to time face. It is what we ordinarily mean by the word "*reasoning*."[2]

Let us examine these four illustrative types of thinking in the light of our concern to find out how to tell right thinking from wrong thinking.

Which forms may be right or wrong?

What would it mean to know how to daydream aright? The reader probably feels this query to be somewhat ridiculous, and indeed it is. Any daydream is appropriate from the standpoint of the one it has captured if it is interesting, and from the standpoint of others if it is the imaginative expression of a sociable character. Important it doubtless is to improve our daydreaming, but beyond widening our outlook so that we may have a larger variety of ideas at command, such improvement is a moral rather than an intellectual mat-

[2] This word is sometimes used in a narrower sense, as equivalent to "inference." See below, chap. 7, pp. 107 f. In the present book it will have the wider meaning here defined.

ter. In short, while a daydream may be morally good or bad, it may not exactly be either correct or incorrect, right or wrong. It remains the wandering play of fancy.

A decisive contrast appears if we pass over the second and third types for a moment and ask the same question of the fourth: What would it mean to reason aright? And examination of this contrast shows that it depends essentially on the following consideration. Daydreams are concerned with no *judgments which claim to agree with the realities that we confront.* Now such judgments—together with the verbal *assertions* in which they find expression, and the readiness to make them on appropriate occasion, which we call *belief*—obviously may be right or wrong, correct or incorrect, true or false, depending on whether they do or do not agree with the realities to which they refer. And thinking is pertinently characterized by these same adjectives according as it proceeds or does not proceed in the manner appropriate to attaining right judgments. But reverie is not occupied with assertions about real things, nor the beliefs expressed in them. So, while a reverie may be entertaining or insipid, dramatic or dull, pretty or ugly, wild or sober, it can hardly be right or wrong. When I pictured the battle of the fishes between the submerged bars of the lake, I did not believe that such battles actually took place. To have done so would have given sufficient warrant for my admission to an insane asylum, since the commonest forms of insanity consist just in inability to distinguish imaginative creations from real events. It was simply a fancy momentarily entertaining, to which the bars, as perceived, merely offered the suggestive cue. To be sane and normal is to be able to hold fast the distinction between such fancies and genuine beliefs about the objects in question, such as, for example, that the fourth bar from shore is about six feet below the surface of the water. To make judgments of this sort, upon which one is ready to stake action when occasion arises, is clearly to think in a way which may be right or wrong, and one very different from daydreaming.

Discussion need not be expanded, however, to show that

the fourth type of thinking *is* concerned with judgments about real things, and gains its essential value and vindication from its ability to yield correct ones. For the purpose fixed by the nature of the problem, and controlling the course of reasoning, can only be attained by our finding out how, in that situation, true assertions can be made instead of false ones. Thus, in the rescue of the boy from his barrel, a happy solution of the problem depended on our reaching correct judgments, first as to the advisability of landing the children before going after him, second as to his calm or excited condition, and third as to the steadiness of his craft. By beliefs about these things the action fulfilling our purpose was guided, and accordingly thinking was solely occupied in bringing to light pertinent suggestions on just these points and weighing the evidence for their truth or falsity.

It will be well to distinguish at once the two main kinds of problem with which a thinker may be confronted. Many, probably most, of our problems are essentially *practical*—that is, in them we are trying to decide what to do in some situation which is distressing, or at least not fully satisfying, as it stands. The illustration just used is an instance of this kind. Some problems, however, are essentially *theoretical*—which means that in them we are trying to understand or explain some puzzling occurrence. If, for example, after having rescued the boy, I had sought to find out why he happened to be on the lake in such a queer craft, the problem then would have been a theoretical one. And in puzzles of this sort the controlling motive is curiosity, which is never the dominant incentive in a practical problem.

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Later we shall have to make further distinctions within each of these two kinds. In all cases, however, the thinking which endeavors to solve the problem is intimately concerned with judgments about reality, and it is this circumstance that makes it possible for it to be right or wrong thinking. If it follows a method calculated normally to yield correct judgments it is right thinking; if it fails to do so it is wrong thinking. It must be remembered, of course, that sometimes factors upset our

calculations which could not be foreseen in advance, and that often the time available for reflection is insufficient to permit the best solution that would otherwise be possible. In the light of these considerations, right thinking may be provisionally defined as *reasoning which proceeds in the manner most likely under the circumstances to reach correctness in the judgments with which it is concerned.*[3]

A brief return to the second and third types of thinking, which have been temporarily neglected, is now in order. Under what conditions is such thinking susceptible of correctness or incorrectness?

In what sense is esthetic judgment right or wrong?

To take type two first: If we mean by the appreciative activity there described mere personal absorption in the attractiveness of the scene, such as could be expressed in the words, "I like this," or "This looks pretty to me," the adjectives "right" and "wrong" are just as inappropriate as in the case of daydreaming. Here is simply an immediate feeling, not an assertion about a real object. But if, in the experience, we are endeavoring to apply an objective standard, such as would be expressed by the words, "This is beautiful," meaning that others should find it so as well as we, then clearly the assertion falls in the domain of truth or falsity and our thinking may correspondingly be right or wrong. For to mean "beautiful" is to mean something real and definite about the object of our thought, even though it is a more difficult quality to verify than such qualities as colors or shapes.

In what, acceptance of information?

As for the third type, there is no ambiguity of this sort about it; it is clearly susceptible of being right or wrong. For the process issues in a belief; I accepted the statement on the tablet as true, and trustfully added it to the rest of my historical knowledge. What essentially distinguishes it from the fourth kind of thinking is that while one is absorbing the information his attitude is a predominantly passive one. He may not raise any question about the accuracy of the statement

[3] In Part IV, but not earlier, we shall need to broaden our base in a way which can no longer be adequately stated in terms of the adjectives "true" and "false," "correct" and "incorrect." See below, pp. 657 ff., 667.

at all, and if he does so, it is presumably not until after he has finished apprehending its meaning. Whereas, in problem-solving thought, one is active at each stage of the process, alert to all the factors that experience tells him might be pertinent. He is not just accepting an idea externally supplied; he is energetically establishing one for himself which will be, he hopes, trustworthy.

These considerations strengthen our persuasion that when we talk about knowing how to think aright, the thinking to which we must refer is occupied in some manner with the formation of judgments, and with the assertions in which they find expression.

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But my ready acceptance of the information on the tablet reminds us again of the fact that beliefs adopted may be wrong instead of right—that we fall into error as well as reach truth. Perhaps I should not have just passively absorbed this information; a critical question might appropriately have been raised about its trustworthiness. Suppose that I had raised such a question; what would have happened to my thinking? Evidently, I should at once have passed from the third kind of thinking to the fourth. I should be facing a problem whose desired solution would control the ensuing mental process, including the suggestion of ideas and the search for evidence which would lead, if successful, to a more justified belief. And the same thing happens, of course, when doubt arises about matters with which we are concerned in other modes of thinking, such as the second. Suppose that, while one is enjoying a charming prospect, the question comes to mind whether his feeling of pleasure is merely personal, or points to an objective quality of the scene. He would immediately pass into the fourth kind of thinking, since he would now be endeavoring to solve the problem posed by these two incompatible alternatives. Obviously, in either case, this shift will be futile unless we know how to engage in that fourth sort of thinking aright—that is, with reasonable hope of attaining the best degree of success that is possible. Otherwise, not only are mistakes likely whenever we directly attempt to reach

justified assertions, but there is also no reason to expect that we shall be able to correct erroneous beliefs which in other situations we have been led to accept.

It is time to bring together these various considerations and see what they indicate about our proposed study of how to think straight. They appear to show that if anyone entertains unfortunate beliefs this is either because he has failed to reason when it would have been wise to do so, or because, attempting to reason, he has failed to use the most appropriate methods. Certain questions at once arise which, it would seem, we shall need to investigate.

Outline of subsequent chapters

(1) What sort of affair, in fuller detail, is this problem-solving procedure which appears to be our recourse when we try systematically to reach true judgments about the realities with which we are concerned? (2) Is it our only recourse in such a situation, and if not, what are its advantages and disadvantages in comparison with other procedures that are available? (3) Why do we allow beliefs to gain entrance into our minds without some assurance that they are correct? (4) What makes us hold on to them once they are adopted, and how may we best avoid serious consequences from the forces thus operating? (5) If reasoning provides the best way of solving problems and establishing beliefs, how should it be guided so that the judgments reached by its aid will be as correct as possible—that is, what specific conditions must be respected if mistakes in its use are to be avoided?

These questions will now be attacked in some detail, and in the above order. First, a more careful analysis of a typical case of problem-solving will be undertaken, since the results of that analysis promise to be very helpful in answering the other questions.

EXERCISES

1. After reading Columbia Associates in Philosophy, *An Introduction to Reflective Thinking*, Chapter 1, describe the nature of reasoning in relation to habit and impulse.
2. Jot down six of your beliefs at random. Describe briefly in each case an instance of thinking in which it played a part.

3. In just what ways are true judgments essential to good reasoning? Illustrate from both practical and theoretical problems.
4. Write a brief essay on the values and dangers of daydreaming.
5. Outline the chapter briefly but clearly.

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CHAPTER 2

ANALYSIS OF A TYPICAL CASE OF REASONING

Everywhere in science and in life, what is hard to understand when viewed as a whole becomes easier to master when dissected into parts. Now a typical instance of reasoning may be dissected in several ways, each revealing important aspects of its structure. Most helpful for our present enterprise, however, is a temporal analysis; that is, one which traces the natural order of sequence which any case of reasoning follows and notes the distinguishable steps into which that process may be broken up. And it appears that one can distinguish five steps, related to each other in a definite temporal order.[1] The first of these might perhaps be omitted without loss, since it merely registers the fact that the person concerned has stopped doing something else and begun to reason, but on the whole it seems best to include it, both for the sake of completeness and to indicate the relation of any case of problem-solving to its context in the larger stream of one's conscious life.

How shall we analyze reasoning?

With this need in mind we may describe the initial step as the *occurrence of a perplexity*. As noted in the first chapter, all of us much of the time, and some of us almost all of our waking time, are engaged in other types of thinking than that which aims at the establishment of true judgments. We are pursuing a congenial reverie, or absorbed in gloom or enjoyment, or passively acquiring information, or doing something else which demands no active intellectual effort under

Step one—occurrence of a perplexity

[1] This analysis is derived mainly from John Dewey's *How We Think*, 1910 edition, chap. 6. His later *Logic*, chap. 6, gives a more technical formulation.

the guidance of a definite aim. Some event, accordingly, must occur to recall us from such casual activities and confront us with the necessity of winning a correct judgment about certain real things—either things on which we have hitherto had no definite opinion, or ones as to which we have now found reason to distrust our previous opinion. This happening is usually external to the play of ideas in which we have been absorbed. A certain habitual action, for example, meets an obstacle which checks it until a way of completing it in spite of the handicap is thought out. A housewife used to cooking by gas in the city, when faced with the task of preparing dinner over an oil stove in a summer cottage, will meet such checks frequently until new habits have been formed. Or a novel event will excite our curiosity and challenge us for an explanation. But the event need not be external. Especially with those in whom intellectual curiosity is active, some occurrence in the free play of ideas itself may suggest an interesting problem, or the mind may (after a period of relaxation) be attracted again to perplexities previously baffling the effort to reach a satisfying answer.

Step two—clarification of the perplexity

The second step consists in *clarification of the perplexity*—that is, in an examination, designed to reveal precisely what the perplexity is. It localizes the problem at some particular point in space or in some specific object or part of an object, or (if it is not the kind of problem to which such localization pertains) the aim of the examination is to describe more definitely its nature. A word may be helpfully borrowed here from a special field of problems, namely, medicine. Expressed in its language, step two consists in a "diagnosis" of the problem, a determination of just what it is that is unsatisfying about the situation which the reasoner confronts. In practice this step usually comes to focus in the formulation of a question, indicating what sort of answer will be sought and regarded as appropriate. Here is revealed the characteristic difference between the second step and the first, where the problem is merely felt as a challenge to thought, but in which the interest which is baffled and the kind of solution which

would satisfy it are not yet clearly realized. To exemplify this in a very simple case—suppose that I am walking along a road not far from the lakeside, and notice a path branching away from it which had not previously caught my attention. Thought is quick—so quick that it is often very difficult to separate clearly the distinct steps it reveals—yet one can distinguish the mere feeling of a challenging novelty on seeing the path from the question which soon springs to mind: "Where does it lead?" With this question I am in the second step; it clarifies the perplexity and makes precise just what I want to find out. Behind it lie the rapid observations: that here is something new, that it is a path, and that it leads to some place which people wish to reach. Such are the simple facts which, together with a bit of curiosity in my nature, explain why the experience is a problem and why the problem takes this specific form.

Step three—appearance of suggested solutions

The third step is the *coming to mind of suggested solutions of the perplexity.* To continue with the above illustration, the thoughts quickly occur: "Perhaps it leads to a spring"; or "It may be a shortcut to another part of the road"; or "It may lead over to the lake." What is the source of these suggestions? It is my previous knowledge about paths, roads, and the general nature of the environment in which I am strolling. One can entertain no suggestion whose elements have not been derived from his previous experience, although these elements may be combined in new ways.

Step four—deducing implications of the suggested solutions

The fourth step consists in *deducing some implications of the suggestions* thus entertained, and *evaluating the suggestions by their aid.* In simple cases, the drawing of these implications is performed so rapidly that practice is required to discover its presence. When expressed in words it frequently takes the form of a conditional sentence whose first clause, introduced by the word "if," supposes the truth of the suggestion under consideration, the second clause being introduced by the corresponding particle "then." Let us pursue the same illustration. "If the path leads to a spring," I say to myself, "then there will be indications of a stream of water

near, and of the use of the path to carry water; if it is a short cut to another part of the road, then there will be a wide turn in the road soon; if it leads to the lake, then it will have the appropriate direction and there will be signs that bathers have come and gone by it." Examination of these bits of deduction discloses that any suggestions we are able to entertain fall into an ordered pattern, such that each part of it has systematic connections with the others. Assuming any part, we can therefore affirm something about the others on its warrant. Thus, the suggestion of the path as a short cut to another stretch of the road implies a wide turn in the road; it is part of a geometrical structure requiring such a turn in the road as another part. The idea that the path leads to the lake demands that it have a certain direction and renders probable some indication of its use by bathers.

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What next? Well, if what is implied by a suggestion can be tested quickly and without much effort, we pass at once to step five as soon as the implication has been drawn. This would very likely be the case with the implication of the second suggestion above—that the path is a short cut to another part of the road. By merely lifting my eyes I can ordinarily tell whether the wide turn in the road a short distance ahead, which is implied by this suggestion, is a reality or not. If it cannot be thus readily tested, we usually consider the implications of other suggestions first, compare their plausibility in the light of past experience, and proceed to test them in the order of greatest promise or economy of effort. Sometimes more than one may be tested by the same procedure.

In any case, the thinker is in the fifth step as soon as this process of testing has begun. For that step consists in *verification of the suggestions, i.e.*, in some action or observation, engaged in for the purpose of determining which, if any, of the suggestions as implicatively developed, offers an adequate solution of the problem. The third and fourth steps constitute the peculiarly mental part of the process—that is, they could be performed, as far as their essential character is concerned,

with one's eyes shut or even when removed from the physical setting of the problem. With the fifth step, the mental and the physical become united once more as they were in the first and the second. In the case of practical problems this verification consists in carrying out directly in action the suggestion selected as most promising. This is what happened in the problem of deciding how to rescue the endangered boy. In the case of theoretical problems, concerned with the explanation of some puzzling fact, one ascertains, either by observation or experiment or both, whether the circumstances implied by the selected suggestion are really present or not. The illustration that has been pursued is of this sort; clearly, the idea that the path leads to water cannot be regarded as proved unless what it implies is discovered to be actually the case.

Let us complete this illustration. I reach the entrance to the path, my observation guided by the deductions drawn from the suggestion. No sign of a stream of water yet, and there is no turn in the road as far ahead as I can see. I follow the path to the top of a little knoll. Still no sign of a spring, but I see that the path advances in the general direction of the lake. I scan the path more closely. In the firmer soil a few feet ahead of me is the print of a bare foot. A child's? No, it is too large. Now a conclusion to the entire course of reasoning thrusts itself irresistibly upon me. The path leads to the lake, and is used by the cottagers across the road when they go bathing. That is, the implications drawn from the idea that the path leads to the lake I now find observably present, whereas the implications of the other suggestions are not present. I assert, therefore, that the path does lead to the lake and regard the assertion as proved, or verified—it is, I believe, true, or right, or correct. Of course this does not mean that I might not, by continued investigation, find the belief thus expressed mistaken. Though favorable as far as it goes, the evidence is meager and is possibly compatible with some other destination of the path. But I do not propose to carry my investigation farther, because it is not important

enough at present to reach a more certain judgment about the matter. In other words, by saying that my idea has been verified and that the belief reached is true, I often mean simply that it is correct enough to serve as a basis for whatever subsequent thought or action might be occasioned in me by the existence of the path. Were I a scientist engaged in establishing conclusions as thoroughly dependable for any purpose as the present tools of investigation make possible—in this instance, let us say, a surveyor charged with making a careful map of the region—I should not allow myself to be satisfied with such a rough-and-ready result. Then it would be my responsibility to survey the course of the path fully and to chart its relations with surrounding points, so that everybody who used my results would find in them accurate guidance.

In some cases of reasoning there may follow what is temporally a sixth step, consisting in a résumé of the preceding thinking to uncover any inadequacies that might be corrected. When time permits and the problem is at all important this is certainly a desirable addition, but it reveals no special conditions that a consideration of the five steps as first performed would omit.

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We return now to the fifth step for a further important consideration. Different problems exhibit essential differences with respect to the presence of this step. And the fundamental difference is that some theoretical problems are concerned entirely with implicative relations, so that no observation or action subsequent to the fourth step is needed. These are the problems of formal science, of which mathematics is the most familiar field. It would be unnecessary to appeal to external observation to determine whether five plus five is equal to ten. We can show that the assertion must hold from the very nature of these numbers, whose relations are exhibited by a deductive process solely. Other theoretical problems are all concerned with the explanation of some fact that is more than an implication, and in their case a fifth step is required to complete the course of reasoning.[2]

[2] In Part IV we shall discover another difference, affecting practical problems. It would be needlessly confusing to introduce it here. See pp. 586 ff.

It will be well to illustrate mathematical thinking briefly. Our aim is simply to discover certain implicative relations between defined concepts, such as numbers or geometrical forms, without any reference to the physical objects to which these concepts might be applied. In the solution of such problems the fourth step is often accompanied by making visible marks—drawing figures, jotting down equations, etc.—but this activity is merely a way of facilitating the deductive process; it is not analogous to the kind of observation or experiment that constitutes verification of a suggestion. Let us imagine ourselves as students beginning the study of geometry.

Having mastered the simpler properties of parallel lines in relation to an intersecting straight line, we are asked to prove that the three angles of a triangle are equal to two right angles. The problem being thus defined, we consider various suggestions of ways to solve it. These will take the form of constructions to be added to the triangle, whose implications, it is hoped, will prove the theorem. Doubtless several such are imagined and are followed up in vain before the sug-

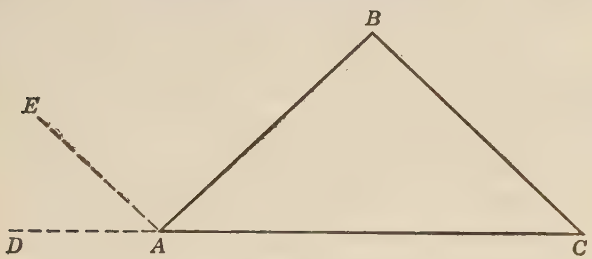

Fig. 1.

gestion occurs to extend the base of the triangle *AC* to *D* and then see whether the external angle *DAB* cannot be shown to be equal to the two internal angles *ABC* and *ACB*. For, since *DAC* is a straight line, angles *DAB* and *BAC* are together equal to two right angles. Remembering now our theorems about parallel lines, we construct *AE* parallel to *CB*. Further implications at once appear. Angle *DAE* equals angle *ACB*, being the exterior interior angles produced by the

intersection of parallel lines with a straight line. Also angle *EAB* equals angle *CBA*, being the opposite interior angles. Therefore, we proceed, angle *DAB* equals the sum of angles *ABC* and *ACB*, and the straight angle *DAC* equals the sum of angles *ABC*, *ACB*, and *BAC*, the three angles of the triangle. But, as a straight angle is equal to two right angles, this is what we had to prove.

Now every part of this demonstration, subsequent to the occurrence of the fruitful suggestion, belongs to the fourth step of the piece of reasoning. It is simply a series of implications about the external angle *DAB*, forming a chain of deductive inference.

Were we mathematical geniuses discovering this theorem for the first time, the situation is not altered in these fundamental respects. The range of suggestions and implications will be wider, but again such physical activity as is undertaken will be an aid to the imagination merely and will not constitute an additional verifying step.

Pertinence of this five-step analysis in other problems

The illustrations of the five-step analysis chosen in the preceding paragraphs are taken from simple practical and scientific problems. Had the selections been drawn from reasoning in art, religion, or philosophy, precisely the same steps would be revealed, but they would interpenetrate in a more complex manner, and the thinking would usually not ride on to any such decisive and early end as is frequently possible in simpler cases of reasoning. But analysis of any stage in such a course of thinking would be able so far as essentials are concerned to locate and describe it in terms of this five-step sequence, just as can be done with a simpler problem.

With a little practice the student will find it easy to take any unitary instance of reasoning from his own experience and subject it to a similar analysis. The following are the chief difficulties which, in the case of some, obstruct ready mastery of this procedure.

Limitations and significance of the analysis

1. In the first place, it is important not to misinterpret its scope. On the one hand, the analysis applies only to thinking as actively concerned in the solution of a problem. There are

other kinds of thinking, and also we often traverse briefer mental processes whose results can be used in later reasoning, but in which not all these five steps are present. Our casual observations illustrate such processes. Watching the lake in the morning sunshine, I may simply notice, for example, that its color is a deeper blue than usual, without having previously been in any perplexity to which this observation affords an answer. On the other hand, whenever we are trying to solve a problem the analysis does apply, even though at first sight it may be hard to see that such is always the case. Any given instance of reasoning may greatly emphasize one of these five steps and subordinate the others to it so fully that, except to very careful attention, they may hardly seem present at all. Thus in mathematical thinking the fourth step always looms especially large; in many problems of natural science the verifying experiments seem the only significant factor, on account of their difficulty and the ease with which the preliminary processes are performed; in the work of a doctor the other steps usually follow readily once step two has been adequately completed; while in various problems in all fields everything else may be readily performed except conceiving the right suggestion, with the result that the appearance of the needed suggestion, when it comes, may bulk so large in value as to dwarf in significance the rest of the process.

We come now to more specific difficulties.

Other questions about the analysis answered

2. The difference between steps one and two is primarily a difference between the occurrence of the perplexity and the clear statement, usually in a question, of just what the perplexity is. Psychologically, this corresponds to the difference between vaguely feeling that one is puzzled, and noting the definite conditions which must be met if the puzzle is to be solved. Sometimes, of course, the two steps merge completely. This is the case when the problem is given us at once in its precise formulation, as when we are assigned an original theorem in geometry or commissioned a specific errand. In other cases a distinction can be noted.

3. There is a temptation sometimes to regard steps two and

three as merging, where the problem is such that only two or three alternative solutions seem possible. Shall I go to class this morning, or cut? seems like a precise statement of the problem as well as of the suggested solutions. But the processes are very different and they must not be confused, however closely connected. In this case the problem, as clarified in step two, probably was: Shall I go to class this morning? arising because of some event which makes the accustomed routine distasteful. Step three will accordingly consist in facing the two alternative possibilities: (a) I shall not go; (b) I shall go; and the implications of each will be drawn in step four.

If, after a definite question has been formulated and before possible solutions are considered, any further facts are observed and kept in mind throughout the rest of the reasoning as guides to the proper solution, their observation should be included in the second step. In the illustration given early in the chapter, I might have noted, before suggested solutions occurred, that the path was quite narrow and apparently little travelled. Such an observation would have a bearing on my attempt to decide which of the suggestions entertained are more plausible than others.

4. Those whose thinking is closely linked with action sometimes pass over steps three and four so rapidly that they find it difficult to isolate and identify them accurately, particularly step four. It will help if we fix firmly in mind just what these steps accomplish. Step three is the coming to mind of possible solutions of the problem, step four a process of inference in which the relevant implications bound up with these suggested solutions are traced. The latter can always be expressed in the "if . . . then . . ." form. The dividing line between steps four and five is the line between *deducing what ought to be* the case if such and such a suggestion is the right one, and *looking to see* or *testing by appropriate action* whether it is the case or not.

5. Finally, a misunderstanding of the temporal relations of these steps must be avoided. They always succeed one another

in the order named, in the sense that suggested solutions do not occur till the problem is sufficiently clarified to make them possible, nor can the implications of a suggestion be deduced till the suggestion itself has appeared, etc. But one should be careful not to suppose that a given step is necessarily finished when the succeeding one begins. The cases where this is so are the exception rather than the rule—they are the momentary puzzles where the first suggestion seems to afford at once an adequate solution. In most cases (and the examples given earlier in the chapter are of this sort) the process is more complicated. As soon as the problem is sufficiently well defined for a suggestion to emerge, we welcome it and trace its promising implications. If these seem favorable and are such that observation or action can readily test them, we pass at once to the fifth step. If the suggestion is there rejected we return to the third step and are open to another suggestion, or if this seems futile without further clarification of the problem we revert to the second step. Then the same process is repeated. Often, however, the observation or experiment required in the fifth step takes more time. While we proceed to carry it out, further suggestions occur and their implications are pursued. If one of these seems more promising, or can be quickly tested, we follow it through to the fifth step, postponing the earlier ones. Sometimes a continuous series of observations can be made, or experiments devised, which test a number of different suggestions. The most interesting experiments in scientific history have been of this sort. Oftentimes, moreover, steps two and five will merge in the sense that an observation which disproves a certain suggestion may also clarify the whole problem in such a way that an entirely different group of suggestions arise. Thus the attempt to mend a piece of broken machinery with wire may, while failing, indicate that the break was somewhat different than had been supposed. Thought may thus retrace its steps time and again while occupied with a given problem, until some suggestion is hit upon which is found to furnish a solution or the attempt to solve it is given up as a bad job. Thus we may meet baffle-

ment at any one of these steps and return to any preceding step, expanding its contribution in the hope of overcoming the obstacle. Otherwise each part of the process occurs in the definite place in the temporal order above indicated.

Three technical terms

This analysis of a typical unit of reasoning gives the basis for explaining two words and a phrase which will be frequently employed in subsequent chapters.

One of the words is "relevance." In listening to an argument or participating in a discussion, we often hear the criticism raised that such and such a statement is "irrelevant." What does it mean for a word or sentence to be relevant, and how does one distinguish in any given case what is relevant from what is not? The answer is: Relevance is determined by the nature of the problem whose solution is being sought. Any consideration is relevant if it helps to clarify that problem, provide a suggested answer to it, constitute an implication of such a suggestion,[3] or bring to attention evidence which verification must take into account. Otherwise it is irrelevant. In brief, the guiding principle is that anything is relevant if it offers constructive value at any step of the process of solving the problem.

The phrase is "realm of discourse" or "universe of discourse." What it signifies is that the meaning of the words employed in any investigation or discussion is determined by the range of facts and implications that are relevant to the problem faced. Thus, in the illustrative problem employed in the present chapter about the path and its direction, the universe of discourse would be my geographical environment, with all its component facts and relations; anything outside that environment would be excluded, although it might enter the realm of discourse determined by another problem. It is in this way that ambiguities in the meaning of words are resolved. For example, the suggestion came to me, among others, that the puzzling path might lead to a "spring." Could this word mean a "piece of elastic metal," or a "sudden leap," or "the season preceding summer"? Obviously not, in this

[3] But see below, chap. 7, pp. 123 ff.

universe of discourse, although in another it might mean any one of these things. Here, the nature of the problem indicates that the word must mean the kind of spring to which a path in wooded hills might lead, *i.e.*, a "natural flow of water from the earth." Two or more problems may, however, have the same universe of discourse if the range of facts and implications relevant to both of them is the same.

The other word is "meaning" itself. What does one mean by "meaning," within any given universe of discourse? Well, we have just said that the meaning of any word employed in an investigation is determined by the range of facts and implications that are relevant to the kind of problem faced. Now when such a statement is carefully considered in the light of the above analysis of reasoning, it appears that the meaning of any word has three aspects (or "dimensions," as we shall often call them). These need to be clearly distinguished, and for each there is a generally accepted technical term. Suppose, for example, that a person who noted my use of the word "spring" in the context of the problem above discussed, were to ask: "What does that word mean, or signify?" He might intend by this question any one of three things.

He might intend, first: What does the word imply that is equivalent to it, *i.e.*, what other word or phrase would be a good definition of it? And an appropriate answer, if this is what is intended, would be: It means "water flowing by natural causes from the ground." For, obviously, from any statement using the word "spring" in this context there could be deduced a statement using this equivalent phrase. This would instance the *syntactic* meaning of the word. He might mean, second: To what fact or facts does the word refer, *i.e.*, for what sort of fact would one look if he were verifying some suggestion about a spring? And the answer in this case would be: He would look for some location in which water flowing from the ground can be observed. Any such fact would instance the *semantic* meaning of the word. Finally, he might mean (especially if he put his question in the form, "What is the significance of the word?"): How is it related to the pur-

pose of the one who is using it, *i.e.*, what help does it promise to give him in reaching an answer to the question he has raised? And if this is the meaning the reply might be: It means a possible destination of the path whose course and direction have puzzled him. Here we should be dealing with the *pragmatic* meaning of the word.

These three dimensions of meaning will occupy us more in detail later.[4]

EXERCISES

1. Analyze in terms of the steps of a complete course of reasoning a recent attempt in your experience to solve:
   a. A practical problem.
   b. A theoretical problem involving no fifth step.
   c. A theoretical problem involving verification.
2. Solve the following cryptogram, and then analyze the reasoning by which you solved it in terms of the five steps:
   XLMW MW IEWC XS WSPZI.
3. Analyze in these steps the way in which you solved the problem: What courses shall I take this semester?
4. Specifying some appropriate universe of discourse, explain the syntactic, semantic, and pragmatic meanings of the following words:
   a. Cloud
   b. Fascist
   c. Swing (as verb)
   d. Black
   e. False

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[4] Cf. Part II, pp. 148 ff.; Part III, chap. 24; and Part IV, pp. 596–602.

CHAPTER 3

REASONING AND ITS ALTERNATIVES

With the temporal structure of reasoning now in mind, we may turn to the second of the topics mentioned at the end of Chapter 1. Does one always proceed to reason when facing a problem, and if not, is reasoning always the best way of meeting it?

Is reasoning our only way of meeting a perplexity?

The illustrations thus far used have encouraged the assumption that whenever one is involved in perplexity he does engage in a course of reasoning—that the only way in which human beings are equipped to meet such a situation is to proceed in the manner above analyzed. But this assumption can hardly be allowed to pass without examination. In case it proves false we confront a further challenge which can hardly be ignored. If there are alternative methods to the method of reasoning, it is our responsibility to see what they are, and to compare them sufficiently with the five-step process just dissected so that we can tell whether there are circumstances under which one of these alternatives is more likely to lead to a happy outcome than reasoning is.

Well, such alternatives certainly exist; indeed, one surveying human conduct might easily reach the conclusion that reasoning is the least popular way of meeting problems. Just what it accomplishes will be more clearly revealed by examining these other methods. In studying them, the main clue to hold in mind is that a problem is always a *novel* situation to the one who faces it, and that accordingly one of the essential characteristics of reasoning is its capacity to guide us in accommodating ourselves to the novel as contrasted with the customary in our experience. When a situation is entirely fa-

Reasoning as adjustment to novel situations

miliar—hardly more than a repetition of some occurrence that has already been frequently met—there is no need of reasoning or of any of the alternatives now to be considered; we act at once in some instinctive fashion, or bring into play some acquired habit. Much of our day's routine, such as arising, performing our toilet, going to breakfast, etc., is carried on by spontaneous impulse and habit without conscious attention; it is when we face a difference between today's demands and those of previous days, such as new lessons assigned, new engagements to be met, and the like, that some distinctive line of conduct is needed, taking the form that it does because of these novel demands.

Alternatives to reasoning —authority and intuition

Now a survey of human practice will disclose two broadly different ways of gaining adjustment to novel situations besides the use of reasoning. One is the way of following somebody else's *authority*. The other is the way of *intuition*, under which term I mean to include everything in the nature of guess, hunch, inspiration, and the like—in short, any direct, spontaneous, or impulsive acceptance of an idea.[1] As to the former, a brief reference only is necessary, for it is quickly shown not to be an independent or ultimate method. It is obvious that following authority is justifiable only when the person whose authority is taken as our guide knows more about the problem than we, and has himself used the best method in meeting it. In case he in turn followed authority, the question to which we are forced is: Who was the original authority, and what method did he use in reaching the solution that we now propose to adopt? This brings us back to the other alternative, for any primary source of authority will prove to have used either the way of reasoning or some form of intuition. How, then, does the method of intuition differ from that of reasoning, and does it, in some guise or under some circumstances, lead to better results?

Nonrational ways of meeting novelty

A clear and adequate answer to this question is implied by the analysis of reasoning given in the preceding chapter, but

[1] The word is thus used here in the popular rather than the technical philosophical sense.

it will be profitable to approach it from the somewhat wider perspective suggested by the fact that both reasoning and intuition are methods by which to meet the occurrence of some challenging novelty in our experience. Mother Nature has been experimenting in a variety of interesting ways in connection with the circumstance that both routine and novelty occur in her world and that all her creatures must be accommodated to the latter as well as to the former. In the case of inorganic objects this has been accomplished by their being quite indifferent to what happens as a result of the impact upon them of new surroundings. Put a piece of iron where dampness is constant, and it will gradually adapt itself by turning into a pile of rust. It is obviously willing (if we use a little animistic phraseology) to become something else than iron if the nature of the new situation requires; as between remaining iron and becoming rust it shows no preference. The adjustment of an organic creature, on the other hand, is different; it exhibits a determined effort to maintain, in the presence of a novel situation, the characteristic pattern of activity which makes it the kind of creature it is. The adaptation is active rather than passive; there is an insistence that, if possible, the novelty shall be met in such a way that the life of the animal or plant will be preserved and furthered.

But, within the organic world itself, Nature has not been confined to a single technique for attaining this result; she has proceeded on at least two broadly different assumptions. In the case of some living things, the assumption apparently is that novelties are so infrequent or unimportant that they may safely be neglected, at least if certain compensations are provided. Some species of insects are the creatures which best exemplify this procedure; they are a rather notable success from the standpoint of the question whether this assumption can be justified or not. They are equipped with very complex reflexes which enable them to take advantage of quite intricate situations as long as such situations continue to repeat themselves in essentially the same way. The ammophila wasp, for example, provides its larvae with food by paralyzing a cater-

pillar in as many as nine different nerve centers and laying its eggs beside the victim. This is a complicated procedure; a human being could only master it at considerable pains, and, with rare exceptions, other animals would probably be unable to do so at all.

Reasoning as a form of learning

In the case of most other forms of organic life Nature's assumption has been that novelty cannot be safely neglected.[2] She has accordingly provided them with a more or less definite capacity for *learning*—this word being a general term applicable to any method by which living creatures adjust themselves to the presence of unfamiliar events, and acquire the lesson which it will be appropriate to use if any given novel event occurs again. And man, in virtue of his physiological structure and evolutionary connections, belongs among this group of organisms; viewed from this standpoint reasoning is therefore one of the ways of learning that Nature has devised in the course of her varied experiments. But it is different from the ways followed by other species of this group, although analogous in many interesting respects to the method of animal learning about which in recent decades psychologists have gained the fullest information. For this reason the characteristic virtues of reasoning (and of intuition as its main human alternative) may be strikingly revealed by a brief study of learning among the lower animals, so developed as to permit detailed comparison with the methods of reason and of intuition. The accepted phrase for describing the form of animal adjustment to novelty which psychologists have investigated most painstakingly is "trial and error" learning.

Main characteristics of trial and error learning

The essential characteristics of trial and error learning, at a rather complex stage of development, are revealed in the mastery of puzzle boxes by such animals as rats, cats, dogs, raccoons, and monkeys. E. L. Thorndike's work on cats and dogs is an early example of such experiments. The animals were placed in boxes, with food on the outside, the construc-

[2] I do not mean to imply that insects are quite unable to adapt themselves to new conditions. But the distinctive virtue of the best known insects does not lie in this direction.

tion being so arranged that they could get out by pulling a wire, clawing a button, drawing a string, raising a thumb latch, or in some similar fashion. At first they responded by random clawings and bitings at different parts of the box; the successful movement was hit upon only by accident. When the animals were replaced in the box the useless movements became fewer in number. There was a tendency to concentrate on the appropriate part of the box and the time wasted was much shortened. As the experiment continued to be repeated, more of the useless movements were omitted, until finally the right one was performed unerringly as soon as the creature was fastened in. In the case of cats, in general, about fifteen to twenty trials were required for this stage of prompt and assured response to be reached. Other experimenters have found that monkeys are able to master, by the same process, very complicated combinations of fastenings which have to be dealt with in a certain order, running sometimes up to nine or more in number. In problems of this sort, raccoons show ability intermediate between that of cats and monkeys.

Generalizing from these results, one may say that the animal fumbles around with its various possible responses until one of them succeeds in extricating it from its difficulty or else it abandons the attempt. In the former case, the appropriate act is gradually established and the useless movements are gradually eliminated. The various animals differ from one another greatly in the number and complexity of these possible responses, the rapidity with which they can be brought into use in a perplexity, and the degree of permanence with which acquired adaptations can be maintained.

Weakness of the trial and error method

Now in a broad sense, as we shall see, this method of trial and error learning is quite all-inclusive and universal, the most brilliant example of human intelligence offering merely an illustration of it at its highest level of efficiency. If we take it in the narrower sense, however, there is a very significant difference between the two sorts of learning. Learning by the trial and error method as above described is a process in which a good deal of blundering activity seems to be essential. Put the

animal in a perplexing situation, and instead of surveying his fix, clarifying it by analysis, deriving various suggestions and comparing them in the light of their implications, he either at once begins to do something or else passively accepts his state until the motive to get out is aroused. And when he acts he does so impulsively rather than thoughtfully. This is the case even in repetitions of the same situation. Recent experiments indicate that he may be guided by some perception of the structure of the puzzling facts, especially perception of the relation between proven means and desired goal. But in general this perception apprehends what he confronts as a rather vague whole; it does not systematically analyze the situation in detail and give him the guidance that such analysis could provide.

Contrasting characteristics of reasoning

How about the method of reasoning? In just what characteristics is it different from the above process? Obviously, it too involves an element of fumbling, but the fumbling is mainly of another sort; it is trial and error by ideas in one's mind, eliminating much of the need of physical action. Let us describe these differences in the light of the five-step analysis outlined in the preceding chapter, and with the aid of an illustrative situation as closely similar to the puzzle box of Thorndike's cats as would be plausible. Suppose that a human being of ordinary ability and range of experience wakes up in a strange room. The strangeness and confinement being unpleasant, he wants to get out just as the cats did. But instead of blundering at once into action, or lying torpid till his want is more imperious, he embarks on the second step of a piece of reasoning; he explores the situation, noticing its various parts and concentrating on those which promise assistance. He observes that the walls are of stone—no use doing anything with them. A window—iron bars around it fastened in the stone—no use doing anything there, either. A door in the wall—and now attention becomes more alert; since doors have commonly provided a way out of rooms, the expectation is peculiarly vivid that something done to the door will be most likely to succeed. The man approaches the door, scanning it

more closely as he does so. No latch, knob, or lock in the usual position. Further observation. Ah! something similar to a bolt at the top of the door. Now comes the first definite suggestion of a solution to his problem, followed by its deduced implication. Unfasten the bolt, and the door will probably open. This idea is sufficiently promising to lead to verifying action. A pull at the bolt. It fails to give. Another, harder pull —and it slowly gives, but springs back when pressure is relaxed. This clarifies his problem further and leads to another idea as to how it might be solved. Obviously, some way must be found to pull at the door and hold the bolt down at the same time, despite the absence of any knob. Second suggestion—hold the bolt down with one hand while pushing at the wall beside the door with the other. This, too, is promising enough to be tested in action without waiting for further ideas. The man follows this suggestion, and with a vigorous push the door opens.

The differences examined

What are the essential features of such a typical exhibition of reasoning that are not found in trial and error learning as displayed in the puzzle-box experiment? It is evident that they appear mainly in the second and fourth steps. If the animal's performance is described in terms of what happens in a case of reasoning, we should say that when he falls into a perplexity he follows whatever suggestion comes to him promptly in action, without any clarifying analysis of the situation to insure that the suggestions will be promising, and without any critical evaluation of suggestions in the light of their deduced implications. If the first act is unsuccessful he continues in the same spontaneous and uncritical fashion with others, until either his problem is solved or his repertoire of possible doings is exhausted. The reasoner, on the other hand, prepares for the birth of suggestions by systematic analytic observation, discriminating the various parts of the situation instead of merely taking it as a baffling whole, and selecting for concentrated attention the parts which, in the light of their similarity to objects familiar from past experience, enable him to forecast what would be likely to result from this or that action

upon them. Thus, when suggestions are elicited they are more than mere random possibilities; they have definite relevance to the detailed nature of the perplexing situation. And the fourth step is equally distinctive. Instead of acting at once on a suggestion, the thinker anticipates what else would be the case if he does so, compares it with the implications of other suggestions, and selects as a guide to action the one which seems most promising in the light of such comparison.[3] If no single suggestion appears adequate to the situation, he combines (as in this illustration) two or more ideas into a complex suggestion, never tried before in this particular form. Thus when he finally engages in action there is a maximum of justified expectation that it will be appropriate to the unique character of the problem. Fumbling is not absent, but a large share of it is transferred from the field of muscular performance to that of anticipatory reflection; action is postponed until a suggestion has been elicited or constructed offering the most promise, in the light of past experience, of meeting the challenge successfully.

Distinctive advantages of reasoning

The essential virtue of systematic clarification thus arises from the circumstance that if the situation as a whole were similar to one with which a thinker is already familiar, it would not be perplexing—he would act at once in the appropriate way. But since it is different from anything previously faced, he finds it helpful to discover parts of it that are not different, in order that his previous experience of their ways of behaving may be utilized for present guidance. The essential virtue of anticipative deduction is that, as a result of it, the fumbling required becomes mainly a matter of memory and controlled use of imagination, dependence upon overt action being reduced to a minimum.

Now the practical advantage of being able to learn in this way hardly needs elaboration. It saves time and energy, and it

[3] In this comparison of reasoning with trial and error learning, of course, reasoning in practical problems only is being considered. In these, the fourth step always takes this form: If such and such is done, then such and such results may be expected. The tracing of implications is here equivalent to anticipating the consequences of a certain act.

does not stake the individual's life or welfare on each suggestion in succession as impulsive action would often do. It saves time, because memory and anticipation are far quicker than physical activity. They can resurrect a prior experience with this or that object and follow it through to its promised outcome in a minute fraction of the time which the actual performance would require. It is for this reason that the possessor of such ability may forecast future events and prepare for them while they are taking their own time to arrive. It avoids much danger, because oftentimes serious consequences are staked on the overt performance of an act, and if it is possible to anticipate such consequences and select for action suggestions in the order in which they are most likely to be successful, much dangerous testing of other possibilities is avoided. A child lost near dusk in a deep wood, for example, will not be able to commit himself to action in many different ways before he will be in great danger. Ability to analyze, to bring forth relevant suggestions and foresee their consequences, is, in such a plight, of the highest value. Learning by reasoning thus appears to be by far the more efficient procedure of the two. It includes all the possibilities of the method of trial and error, and this highly advantageous power of analytic observation and constructive imagination in addition; hence its possessor has an excellent opportunity to outrun all competing species in securing whatever ends he desires to attain. Moreover, any development which enables this process to be performed more effectively will, other things being equal, increase its value.

Man is the only animal able to learn by this method of reasoning in any systematic or extensive way. But he too must fall back upon the method of physical trial and error whenever he meets a situation which he cannot adequately analyze in terms of familiar factors. His rational knowledge and adaptation are, therefore, never complete. The chemist experimenting with elements he has never known combined; the boy facing a mechanical puzzle, the relations of whose parts he cannot entirely disentangle in imagination; and all of us

when struggling to master complex performances which involve coordinating various parts of the body, such as swimming, bicycling, dancing, etc., illustrate instances of learning where a great deal of overt doing is required. Human beings, too, often learn by blundering ahead and seeing which bit of blundering succeeds. It is only through the gradual advance of knowledge and experience that the element of fumble can be more and more transferred from muscular action to the quicker and less risky play of rational analysis.

If we ask for an explanation of the comparative superiority of man as reasoner, a large part of the answer is doubtless to be found in his systematic development of a remarkable intellectual tool—language.

To analyze a puzzling situation so as to bring out its various elements clearly, hold fast the way in which they are related and combined, and be prepared to recognize them in new problems, must be a very difficult process until language is also developed to the point where definite symbols are available by which to refer to them. By the aid of language, a baffling area is not permitted to remain a vague unit, taken merely in bulk, so to speak, and responded to as a whole, with perhaps some recognition within it of means-goal patterns and a few spatial or mechanical differences. It is forced to lay bare its wealth of distinctions which are attended to separately and named. It is analyzed into definite objects, in their specific relations and with their specific qualities, each capable of recognition when it appears in some new situation. It thus gains a richness of detailed organization—any feature of which is usable in subsequent experiences—that lies quite beyond the capacity of creatures without a well-developed language.

Comparative values of reason and intuition

What light, now, do these considerations throw on the nature of intuition, and on its value as a method of solving problems in comparison with that of reasoning?

The main and striking consideration is surely this—that most cases of intuition are essentially like the method of blundering trial and error in one very important respect; that is, they involve commitment to an impulsively suggested

action without the discriminating analysis and critical comparison with possible alternatives that are the distinctive marks of reasoning. At least this is true of such intuitions as may be grouped under the terms "guess" or "feeling," and likewise with the "hunches" and "inspirations" whose foundation turns out to be merely their emotional appeal. In terms of our five-step analysis, intuition is partial or telescoped reasoning—it is the occurrence and acceptance of a suggested solution without the controlling checks which analytic clarification and deductive elaboration might provide.

In many perplexities, of course, one has no alternative than to follow intuition, in lack of past experiences similar enough to offer helpful material for reflective comparison to seize upon, or when insufficient time is available to profit by them. But to do so when full use of reason is possible is to fail to take advantage of an invaluable human capacity. Doubtless, indeed, there are cases covered by the loose word "intuition" that really instance the employment of reasoning, in such degree as the cases permit. Certainly we sometimes find ourselves in problems where the only comparison possible is performed so rapidly, or the analogies with past experience are so vague, that we cannot definitely note any critical evaluation, and seem to be trusting the first plausible suggestion that wafts its way into our minds. Such is perhaps the situation in the judgments that we make about new personal acquaintances. Here at least, woman's intuition has often given ground for a favorable comparison with man's use of reason. But in any event the answer to our present question is decisive. If the only cases in which intuition is justified are those in which either nothing else is possible or in which the intuition is really the best approximation to reasoning that is available, it is difficult to see how a defensible case can be made for intuition as an alternative method to reasoning in the solution of our difficulties. When we cannot quite reason, we will naturally fall back on intuition, and if the first intuition fails, fumble like the lower animals. When we reason, as the above comparison of human with animal learning reveals, we are

doing all that the way of intuition makes possible and more—before an intuition arises we insure by detailed observation that it will be relevant to the problem, and before committing ourselves to acceptance of and action on it we secure warrant, by comparative deduction of implications, that the intuition finally selected will be the most promising of those available, so far at least as the time and pertinent past experience at our disposal permit. It can surely never be sensible to follow the less when the more complete method including it is at hand.

There is an interesting way in which the reader may test the soundness of this conclusion.[4] Let him make predictions, guided merely by his spontaneous intuitive hunch, about ten events which should occur within the following three months. Then let him make predictions about ten others, in this case being guided by the most careful and complete course of reasoning that the time available permits. The two groups of events should be as similar in other respects as possible, especially in the degree of difficulty in the way of accurate anticipation, and the predictions should be definite enough so that there will be no question whether they have been fulfilled or not. At the end of the three months, the success achieved by the first group of predictions may be compared with the success achieved by the second group.

It is true that some persons who have formed the habit of careful reasoning become overcautious; they are more hesitant to act in emergencies than those accustomed to following vivid intuitions. But unwillingness to act when action is appropriate does not seem to be essential to the process of reasoning, and the conclusion to draw is, therefore, that thinkers must remember there is always a time to stop reasoning and plunge into action, which in some problems may come very soon. If this principle is not forgotten one may affirm confidently that the soundest basis for dependable judgments, and the best way of dealing with the challenging novelties

[4] It would be still better if this test were carried out by a group of readers such as a college class using the present book as a text. The combined result, in the case of all members of the class, ought to be quite clear and convincing.

which seem to be inevitable in animal experience, is that supplied by reasoning. Let us summarize the main considerations which support this conclusion.

The virtues of reasoning as a method

As compared with impulsive commitment to the first idea which dawns, that is, with intuitive action, reasoning is patient, exploratory of other possibilities, and deliberative. The good reasoner can and will, however, follow the way of intuition whenever he has insufficient opportunity for reflective criticism of ideas that are vividly suggested. He will use intuition but make sure that it is relevant and carefully evaluate it whenever possible.

As compared with the passive acceptance characteristic of trust in authority, reasoning is active, critical, and independent. The reasoner is not satisfied with judgments and beliefs which merely accord with the experience of someone else; they must accord with his own experience so far as it provides relevant material. Thus he can follow the testimony of another where he has no time or opportunity to investigate the matter for himself and where reason tells him that the testimony is apt to be trustworthy, but not elsewhere. He will use the reasoning of others, while always guarding against the danger of being imposed upon by undependable testimony.

As compared with the mechanical fixity of habit—our way of accommodating ourselves to the repetition of those phases of experience which we have already learned how to meet successfully—reasoning is alert, flexible, and reconstructive. It is ready to discover something new in what appears at first sight a mere repetition of something old, and to meet whatever challenge it may bring. The reasoner is thus free from the bonds of habit whenever significant novelty so requires, while able to fall back upon it whenever a habitual response is appropriate and sufficient for his needs. He will use habit but never be imprisoned by it.

EXERCISES

1. In what respects is reasoning a trial and error process?
2. Just what is meant by the assertion that reason includes intuition

and more? Is there any kind of intuition of which this is not true?

3. After reading Köhler, *The Mentality of Apes,* write an essay, "The Chimpanzee as Thinker: His Capacities and Limitations."
4. Discuss the problem, under what circumstances the acceptance of authority is justified.

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CHAPTER 4

HOW WRONG THINKING OCCURS

But if reasoning expresses a natural capacity of man, and is clearly superior to intuition, how is it that this lesson has not been universally learned? How does it happen that people find satisfaction in appealing intuitions, and continue under many circumstances to place confidence in them? If the considerations dwelt upon in the preceding chapter are sound, it is evident that intuition is much less likely to issue in correct judgments than is the full use of reason, since it lacks, or performs very hastily, processes which are quite important in contributing toward whatever success reasoning achieves. Following intuition rather than the complete course of reason, we are more apt as a result to fall into error instead of attaining truth. Why haven't we all come to see this clearly? Moreover, since trusting intuition may not be the only source of our mistaken judgments, this question leads to a broader and more fundamental one, the third among those listed at the close of the first chapter. This question is: Why do we ever adopt mistaken ideas? It would seem a proper part of our task to investigate the course of a typical piece of problem-solving once more, in order to lay bare the forces that lead to erroneous judgments, and to discover how most effectively to guard ourselves against them. **How does error arise?**

All of us wish to avoid adopting false beliefs and making incorrect judgments, for two reasons. In the first place, we want to know the truth; we prefer to believe what is true rather than to be deceived even when it might be quite comforting or to our interest in some other way to hold a mistaken idea. In the second place, many practical problems in which

we become caught are pretty urgent; on their solution our lives and welfare, or those of others, depend; and when this is the case nothing else seems so important as to gain correct judgments about the facts with which we must deal.

The human cost of erroneous beliefs

Our repugnance at being deceived in the beliefs we adopt becomes especially strong when we glance at the history of certain notions which have enjoyed wide social currency in the past. Let us pick some earlier century and see how universally prevalent were beliefs that are now, in all educated circles at least, relegated to the limbo of superstition. Not many generations ago, in the Western world, it was a practically universal belief that the earth is the astronomical center of the universe, that heavy bodies fall with greater acceleration than light ones, that night air is apt to cause malaria, that people who doubt orthodox religious ideas are immoral, that old women who act queerly are probably trafficking with the devil. Few raised any question about the truth of such notions. People kept on believing them for no more rational reasons than those which decided the style of their clothes and the language they spoke. Indeed, it was dangerous even to question these doctrines, for one who questioned was himself regarded with suspicion. Now it would hardly be maintained that such ideas were not fraught with sorry results for those whom they affected. To believe in the earth as the center of the universe—still more to believe that it is flat, as was for many people an accompanying superstition—was to support a set of doctrines that now seem childish, acceptance of which tended to forestall both the discovery of the laws of modern dynamics which underlie much of modern applied science, and the explorations which developed modern commerce and trade into a world-wide enterprise. To suppose that heavy bodies fall faster than light ones was to accept a set of notions about physical objects that proved, among other things, inconsistent with the establishment of quantitatively verifiable knowledge, and with the invention of labor-saving machines which make practical use of the laws of modern physics. The idea that night air causes malaria was one of a set of medical

beliefs which, as long as they prevailed, stood in the way of any serious attempt to control the plagues by which the world was periodically scourged.[1] To hold that religious conformity should be enforced by law was to accept the principle of the inquisition, and to suppose that old women could traffic with the devil meant to establish and continue one of the most horrible persecutions of innocent and helpless folk that the world has ever known.

What a blessing for humanity if these false notions could have been discovered and rejected as such earlier! Truth is clearly very important—both for its own sake and for its bearing on man's practical well-being. Moreover, since there were, even in the days when these superstitions were generally accepted, scattered individuals who realized their falsehood and were struggling to establish truer ideas in their place, what a blessing if such individuals could have been singled out as objects of confidence, support, and honor, rather than forced to labor not merely in penury, but under the weight of suspicion, hatred, and at times active persecution from those whose welfare was being furthered by their work! Is it not a sobering thought that many of the men whom we at a later time pronounce unusually deserving of sympathetic understanding and far-reaching influence were little better able in their own day to secure these boons than the common criminal? The question becomes pressing: What are the causes of this tragic blindness? What are the reasons, in human nature and in man's relation to the world around him, that account for such pathetic behavior? How do erroneous ideas get believed in the first place, and why is it so hard for people to abandon them once they have been accepted?

Where is the clearest present exemplification of right thinking?

Without the aid secured by a systematic endeavor to answer these questions, there would seem to be no reason why we might not today fall prey to a similar blindness. For as life and change go on, there must be hosts of beliefs which we naturally tend to accept without question, but which will

[1] We cannot get along without night air, hence as long as such a notion is accepted there seems no way to bring malaria under control.

prove in time to have been just as superstitious and mischievous as the discarded dogmas which we now look back upon with wonder that people should ever have been silly enough to entertain them. This is especially likely to be the case with ideas about man himself and his social relationships. In recent times the entire surface of the globe has become a compact neighborhood, binding human beings everywhere into most complex and intricate patterns of interdependence; yet our moral and political ideas, together with the policies of action to which, under their guidance, we blithely and fanatically commit ourselves, may still reflect assumptions appropriate (if at all) only to days when communities were small, close-knit, and isolated from each other except in minor and very limited ways. Total war, destroying or maiming twenty-five million or more young soldiers and civilians in every generation, is perhaps only the most obviously sickening price that we pay for continued failure to overhaul these traditional beliefs. How can we play our part in locating such wrong notions, and contribute as fully and rapidly as possible to their replacement by better ideas? If reasoning is the way to truth and the most dependable key to progress, here is where it is most desperatelv needed. And since no one of us can do more than a small bit of this constructive work himself, where shall we turn for the most adequate judgments available on which to build? It is important to identify the men of our generation who in this field or that of human endeavor are furnishing us with the most dependable knowledge to date and the most promising anticipatory theories. It may well be that the gap between the notions of the man in the street today and those of the most enlightened group in our communities, on matters momentous to human well-being, is at least as great as that between the abandoned superstitions of earlier centuries and the beliefs of the average educated contemporary.

How do erroneous ideas—or any ideas—get believed?

To these questions we now address ourselves, confining the present chapter to the basic query: How do incorrect notions ever find entrance into people's minds and get themselves accepted as true?

Well, how do any notions, mistaken or appropriate, persuade us to adopt them? What are the causes which operate here? The fact that we sometimes think correctly and sometimes incorrectly indicates that the causes which determine the course of a bit of reasoning have no necessary connection with either the truth or the falsity of the beliefs to which the process leads. If we can discover what these causes are, how they work, and what kind of result each tends to produce, we can perhaps tell why some of the notions that we find ourselves entertaining are fanciful or superstitious while others are more soberly pertinent to the situation they purport to describe. The preceding chapter has, of course, offered some light on this matter; it seems plain that whatever causes lead us to be satisfied with an uncriticized intuition are likely to foster, except for happy accidents, inadequate judgments; whereas whatever forces encourage us to a critical evaluation of intuitions before accepting any of them tend, comparatively considered, in the direction of a more satisfactory conclusion. But it is clear that before one can criticize an intuition it must occur to him—it must be consciously considered as a possible solution of the problem with which he is confronted. What are the causes that account for the occurrence of these suggested solutions? And which of the causes that operate here produce suggestions possessing a good chance of proving correct; which, suggestions that are likely to be mistaken? It is important to concentrate now upon the third stage of the process of reasoning and see from this new perspective what happens in it, and how it happens. Perhaps the result of this study will also indicate why it is that we often follow the way of plausible intuition in preference to that of systematic reasoning.

What causes control the suggestion of ideas?

Our analysis of a unitary instance of reasoning into its five steps has revealed the fact that as we proceed from whatever clarifying observations we engage in at step two into the following step, our thinking becomes a succession of ideas, following a definite order. We are asking now what causes determine why this or that particular idea rather than another pops into our minds when it does—why, after one has been analyz-

ing a perplexing object or event, do the suggested solutions that in fact occur to him present themselves in the sequence that they do? This third step is a fairly mysterious affair at best. It seems to be at the mercy of many accidents, as is evident, for example, when we note on how many perplexing occasions the suggestion which later, in a more leisurely moment of retrospect, bobs up as the only sensible way out of the difficulty, fails to do so till too late to be of help. "If I had only thought of that in time!" we say to ourselves in sorrow or disgust. Evidently there are irrational barriers between the relevant deposits of past experience and one's present perplexities, which make it impossible to be sure that the best solution one knows (in the sense of having at some time learned it) will be suggested at the time needed. The man who found a way to break down such barriers would be fortunate indeed; his thinking would gain an assurance that the rest of us quite lack.

Laws of connection between ideas—similarity and contiguity

The psychologists have devoted considerable study to this question, and the results of their work are available in what may be called in nontechnical language *laws of connection* between ideas. There appear to be two general laws of this sort and a number of more specific ones, which it will be worth our while to examine briefly. The two general laws are those of *similarity* and *contiguity*. By the former of these is meant the easily verifiable fact that attention to anything naturally leads us to think of something like it. Observing a photograph on my desk, I readily think of the friend of whom it is a likeness. That this law of similarity is a real factor in determining what ideas appear in a course of reasoning is evident from the illustrations with which we are now acquainted; a suggested solution of a problem is some way which has been found helpful in meeting past situations similar in this, that, or the other respect to the present perplexity. The hero of the preceding chapter found his way out of the unfamiliar room because he could note and put to practical use the likeness between its door or latch and others with which he had previously been acquainted. It is because of this circumstance

that reasoning by analogy is inevitable in any sort of human inquiry; "analogy" simply means similarity between two or more objects in certain of their properties or relations, when used as a guide in thinking. In fact, the main virtue of systematic observation of a puzzling situation is that it permits such connections through similarity to get under way in our minds. The situation as a whole is not exactly like any to which we are accustomed, and that is why it puzzles us; but by breaking it up analytically into parts, some of which do show a close resemblance to familiar things, we can use that likeness to elicit promising ideas as to what might be done.

By the law of contiguity is meant the equally verifiable fact that from any experience of an object our minds are readily carried to something that in some previous situation has been adjacent to it in space or time. Noticing a door leads us to look for knobs and latches which, as we have found, have an intimate spatial relation with doors; or to think of opening it, which is the act often following in close temporal sequence upon observing it.

A particular form of association by contiguity is especially important. I refer to the bond between means and end, *i.e.*, the contiguity exhibited between a desired goal and the mode of action found to lead successfully to it. The kind of association by contiguity established through such experience of successful achievement appears to be present in all stages of animal and human learning. Any dependable means-end pattern becomes closely knit in memory once it has been discovered; as applied specifically to reasoning, this means that whatever has proved itself successful in leading to the desired solution of a past problem acquires unusual vigor for future recollection in the case of similar problems.

But any object or idea is similar, in some respects at least, to a host of others, and has been contiguous, on some occasion, with many different things and events. What determines which of these similarities or contiguities will be most apt, in any given situation, to seduce our attention as providing a possible clue to the solution of a problem? Why does observa-

tion or memory select one rather than another as its guide in the sequential order which is followed? A door is like a wall, a gate, a table top, and many other objects. What decides to which of these things perception of the door will carry our thoughts?

Factors of advantage—frequency, recency, and vividness

Here the more specific laws of connection offer help. They are sometimes called *factors of advantage* in the connection of ideas. Apparently there are many such factors, and psychologists differ considerably in the selection and treatment of them. Three, however, that seem clearly supported by experience, are those of frequency, recency, and vividness. By the law of *frequency* is meant that the mind tends to be carried from any experience to the idea of whatever has been most commonly connected with that sort of experience in the past; by the law of *recency*, that it is likely to think of something recently connected with such an experience, so that memory of it is fresh. The law of *vividness* expresses the fact that our thought is easily led to something which possesses peculiar intensity—for example, on account of some strong appeal to our emotions; it will be more apt in any context to slip over the threshold of present consciousness than an idea which lacks this quality. We are, as all know, creatures moved by desire for various satisfactions, and powerful emotions are bound up with these desires; ideas that harmonize with our emotional demands are constantly ready to seize our attention just for this reason.

Bearing of these factors on truth and falsity in belief

Now which of these factors are likely to lead our thinking astray when they control the sequence of ideas, and which are more apt to guide in the direction of true judgments? The answer is tolerably clear and obvious. When the train of thought is determined by contiguity in the form of a means-end sequence, especially when the factor of frequency co-operates, our memories of past connections are resurrected in such a way as normally yields promising suggestions for meeting a present problem. For experience teaches us that Nature's habits are sufficiently uniform so that what has been repeatedly successful in meeting a perplexing situation in the past

is more likely than any alternative to meet successfully similar situations in the present.[2] Suppose that fire breaks out in some part of my house. If I recall at once the usual consequences of unchecked fire, and the means most frequently successful in putting it out, my sequence of ideas is following the course adapted to secure right judgments and the consequent happy solution of the problem faced. I am thinking in terms of the appropriate universe of discourse. Were the play of suggestion led merely in the direction of some recent experience of fire, or one with especially vivid interest for me, it is obvious that only by accident would ideas thus begotten give the guidance I require. For the most part they would be quite irrelevant.

Indeed, in situations like this, where it is essential to the preservation of life and the satisfaction of basic human needs for correct judgments to be reached, associations by means-end contiguity and frequency ordinarily are the ones that control the course of our thinking; we have formed the habit of resisting the seductions which mere recency or vividness can bring into play. Within the field of such vitally necessary beliefs there operate what Dewey describes as physical and social "sanctions" of correct thinking,[3] *i.e.*, forces which tend to prevent inappropriate ideas from occurring to us, or from winning our uncritical acceptance if they do occur. The burnt child dreads the fire; he will not even think of getting too close to it for comfort, or if he does, he will quickly remember its painful effects and jump back. Similarly, the spanked child dreads the slipper; he will still be tempted to acts that would violate the demands of well-ordered family life, but before he engages in them he will also think of their likely consequences.

But in matters not thus vital to life and essential social needs, the factor of frequency may dominate thinking uncontrolled by means-end contiguity, or such erratic factors as recency and vividness may do so. The latter, especially, may

[2] This fact will be discussed at greater length in Part III. See below, pp. 304–309.

[3] *How We Think*, 1910 edition, chap. 2, sec. 3.

dominate the current of ideas and encourage the acceptance of beliefs quite lacking in adequate support.

How the factor of frequency may lead thinking astray

The first of these irrationalities may be briefly illustrated by referring to one of the results reached by Piaget in his studies in the logic of children. He reveals clearly how, in the explanations offered by children, connection by frequency is obvious, although it does not really answer the question asked. For example, a child of eight explains that "the moon does not fall down, because the sun is not shining, because it is very high up." The connection between the moon and such facts as these betrays the factor of frequent association but it is not the constant relation needed to explain why the moon does not fall. The same irrational functioning of the principle of frequency cooperates in the formation of many superstitious beliefs and magical practices. Students of primitive thinking have often noted cases of "sympathetic magic," of which a good illustration is the attempt to control some person or event by repeating some formula containing its name. Now nothing is more frequently connected with any fact in our experience than its name, hence a strong tendency of suggestion to follow the clue thus offered; the trouble is that experience does not attest this kind of constant relation as promising effective understanding or mastery of the thing named.

Factor of vividness the chief culprit

But the factor of vividness is probably most seriously and pervasively responsible for the birth of erroneous notions. Something happens to add striking emotional intensity to some experience. A beloved leader, present at a dinner party of thirteen, is suddenly overtaken by betrayal and death. The absorption of attention and interest in such a tragic event gives the relation between it and the earlier fact a peculiar vividness, which may secure that the number thirteen will be more apt to arouse the suggestion of dire tragedy, especially if recency cooperates, than ideas much more frequently connected with it, but commanding no such powerful emotional support. Thus a persistent belief may arise that thirteen is an unlucky number. The same principle may help to explain many other superstitions, such as that it is dangerous to start

a journey on Friday, that passing under a ladder brings bad luck, and that various articles of diet must be avoided. Many of the taboos so common in primitive life, which shackle it by a mass of quite irrational restraints, doubtless owe their origin to the vivid connection of some casual deed with a subsequent happening of dire consequence. In view of the forces inevitably affecting our thinking, it is as natural to prohibit acts that vividly suggest dangerous outcomes as those frequently observed to lead to them.

Virtue of reasoning as compared with intuition

Now if candidates for belief thrust themselves upon us in each of these various ways, further important reasons are evident why it is desirable to follow the complete course of reasoning instead of seizing upon the first plausible suggestion that comes to us. If we at once adopt such an appealing intuition without critical evaluation, how can we be sure that it has not arisen in our minds through some quite irrational factor such as emotional vividness or recency? These factors are there, ready to operate whenever they have a chance. Whereas, if we engage systematically in steps four and five of a reasoning process before definitely accepting a suggested idea, we will at least become aware that it has no rational promise if such is the case, and give ourselves a chance to consider other suggestions whose occurrence is due to different factors and which may be decidedly more valuable.

Why the intuitive method easily satisfies

In fact, here is the main explanation of the fact that creatures capable of reasoning often trust the intuitive method even in problems where some critical evaluation of suggestions would seem to be possible, and find satisfaction in doing so. We tend naturally to accept whatever directly satisfies us, and to avoid replacing it by something that is not so appealing. Well, an emotionally attractive idea is directly satisfying just on that account, and even a quite terrifying image may exercise a powerful fascination over us. Hence, unless one has already formed the habit of cautious reasoning about whatever subjects he confronts, such a suggestion, on account of its immediately appealing character, will appear to carry its own adequate warrant and to render critical testing in the

light of other experiences superfluous. It strongly grips him; why make it run the gantlet of systematic comparison with alternatives that do not? So its adoption without more ado is very easy; he allows emotional congruity to usurp the place of genuine relevance. In matters not vital to our welfare it may take a long course of frustration before we learn the lesson that some forces in our make-up which favor the acceptance of ideas are not dependable and need to be brought under effective check. Indeed, even after we have acquired the habit of wariness and criticism in dealing with certain types of problems we may find ourselves blithely trusting vivid intuitions in others, or placing implicit faith in solutions offered by admired heroes who had themselves followed the path of intuitive inspiration rather than that of careful reasoning.

Beliefs expand by the same forces that determine their acceptance

Moreover, it is not surprising that the same factors which affect our way of accepting suggestions likewise help to determine the manner in which they become woven into a larger fabric of ideas in our minds. One who has habituated himself to the systematic and critical use of reason, with especial care that the fourth and fifth steps are adequately performed before suggested conclusions are adopted, will enlarge his body of knowledge with wise caution; the interconnections which tie together its various parts will reflect his constant demand for rational consistency, and each item will be buttressed by appropriate evidence. But one who allows himself frequently to succumb to appealing intuitions, accepting them without careful consideration of their implications and without verification, will naturally allow his agglomeration of beliefs to expand in the same way. New suggestions which emotionally harmonize with notions already accepted will readily attach themselves to the pile; mere dramatic congruity becomes in effect the principle governing the enlargement of his supposed knowledge. Thus it was that with primitive man's acceptance of the plausible idea that objects around him were possessed of personal qualities like those he was gradually discovering in himself, beliefs in fairies, dryads, gnomes, and picturesque tales about them readily followed in his mind. Or believing,

perhaps as a result of some dream about a deceased parent, that dead ancestors were somewhere living the same kind of life they had lived here, men easily adopted the further judgment that they should be provided with homes, weapons, and food. Whole systems of ideas thus branch from the root of a single accepted idea, and if the latter is without adequate foundation a vast tissue of error may accumulate around it. Imagination constructs an exciting drama rather than waiting for critical reason to build the sober body of accurate knowledge. We see essentially the same process today in the way in which gossip, given an enticing belief on which to start (whether with or without good evidence), will run a course of magnification in successive tellings which is almost impossible to check, and whose final product it is hopeless to refute. The play of imagery, set at work in one person's mind by the story, will be so vividly in operation before he communicates it to another that some intriguing addition will be supplied, and the account in its final form will be spicy indeed. One entertaining error thus inevitably begets another, and a gradually swelling collection of superstitious notions becomes the heritage of each generation. Indeed, we may note in this connection the strong tendency of the popular mind to trust suggestions generated merely by interesting similarities even when the beliefs involved are of some practical concern. A farmer's wife, being challenged to justify her assertion that it is dangerous to pare corns during the new moon, replied, "Well, while the moon is growing big the corn will grow big too, won't it?" And the persistent belief in seaside communities that births tend to occur at full tide and deaths at its ebb is another illustration of the control a dramatic analogy can exert. The notion seems to be justified by no verifiable evidence.

What can be done to further the eliciting of relevant suggestions?

An understanding of the factors operative in the occurrence of suggestions to our minds enables us to see the reasons for certain methods which experience attests as helpful in furthering the mysterious work of eliciting promising ideas when we need them. In his book on *The Art of Thought* Graham

Wallas describes the methods which he believes have proved their value in this regard. In the case of problems subsidiary to others, or of secondary importance, he finds it desirable to give attention to other difficulties while waiting for promising suggestions regarding the one in hand. This economizes time and does not seriously hamper the process of subconscious incubation. On the larger and more basic perplexities, however, a period of complete intellectual relaxation is desirable. This may include physical relaxation also, or it may involve moderate bodily exercise, not carried to the point where conscious effort is needed to control the muscles used. Its main virtue is that intense concentration on a problem often gets our minds tangled up in a set of useless ideas, whose interconnections only become more strongly fixed (because of the factors of frequency and recency) when we consciously try to break free from them. Relaxation loosens their hold, and gives a chance for other associations to become effective. Thus it permits a subconscious sifting process to take place, in which relevant experiences can separate and combine in manifold ways, till an idea emerges which promises to organize them all in the precise way required by the perplexity. Also, certain habits of intellectual work can be formed which facilitate this process to some extent. Three of these may be mentioned. First, one should set apart a regular time each day or week for creative thought. This is not merely to avoid waste of time, but because constructive imagination, though a mysterious affair, can be led to subject itself in a measure to temporal routine. After persisting in the habit for some time, one will find his power to elicit fruitful connections gaining its maximum effectiveness at the time he has set apart, in a fairly dependable way. Second, the surroundings under which one works should be as free from external distractions as possible (especially sudden irruptions upon attention) and should be esthetically pleasing. This insures, on the one hand, opportunity for continuous concentration on a single train of thought, and on the other a gentle and tonic appeal to imaginative fertility. Third, it is very

important to have a few blank cards constantly on one's person or by one's side, and to jot down at once a suggestive idea before it is lost. The infant mortality of new ideas is exceedingly high. Many people find the moments of relaxation in bed before dropping off to sleep especially favorable to the occurrence of valuable suggestions—if cards and a pencil are not handy, the illuminating thought may refuse to reappear in the morning. Memory is especially untrustworthy in the case of such sporadic intimations. Wallas refers, in passing, to the pathetic case of a man who suddenly conceived a brilliant idea—the solution, it seemed, of the problems of a lifetime. Instead of recording it at once he went into his garden to thank God for it, whereupon he found, on rising from his knees, that he had forgotten it and could never afterward recall it. The cards should be filed in a set of folders labeled with the names of the main subjects to which one's mind recurs.

EXERCISES

Exercises covering this chapter will be found in Appendix A, pp. 743–748.

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CHAPTER 5

HINDRANCES AND AIDS IN THE CORRECTION OF WRONG THINKING

Once beliefs have been adopted by an individual or community, many forces operate to give them continued sway, even though they may be quite erroneous, and even though occurrences are frequent which might be expected to throw doubt upon them. Let us survey the most potent of these forces, taking up thus the fourth topic mentioned at the conclusion of Chapter 1.

Error perpetuated by our natural submissiveness

Foremost among them is man's natural submissiveness, in virtue of which traditions currently accepted take root in the minds of the young while they are thoroughly docile and uncritical. All human beings can assimilate ideas acquired in the experience of previous generations without needing to repeat the process by which they were first established. But it is clear that such assimilation, in itself, will be just as apt to rivet in youthful minds the superstitious notions of their elders as the more dependable ones, and thus to perpetuate error as easily as truth. This transmission of ideas is in part conscious, expressed in public educational practice; in part it is the still more effective because subtly insinuating permeation of thought-habits by beliefs present implicitly in all that is done and said. What is consciously taught may often be questioned by a clever youth; what is unconsciously imbibed simply because it is everywhere implicitly acted upon, meets no resistance at all. It is accepted with no more criticism than the practice of joining in the hunt or of eating when the family dines. Most of us will find that a large number of our fundamental notions were gained in no more ra-

tional manner than this unconscious absorption of the prevailing convictions by which all the thinking of our trusting youth was influenced. That it should be thus is quite inevitable, for we are not born sceptics. The habit of viewing beliefs critically, of taking what parental authority or group tradition inculcates "with a grain of salt," to say nothing of engaging in radical doubt, is a kind of wariness only gradually forced on one by the difficulties into which he is thrown through a too trustful acceptance of current notions. Otherwise there could be no adequate motive for doubt. Education easily takes advantage of this docility; it becomes, throughout by far the greater part of human history, an instrument for indoctrinating the young firmly in the accepted beliefs of the community and fostering by every art their native attitude of innocent submission and unquestioning trust. And by education is meant the work of all social agencies, including the family and church as well as the school, which function toward this end.

The power of an inherited language

One aspect of this unconscious permeating of thought-habits by traditional ideas strikingly reveals its inevitability and subtle power. Even when we face a problem and endeavor to solve it by our own reflection we find it necessary, initially at least, to describe the factors discriminated in it, to formulate the suggested solutions that come to our minds, and to trace the implications deduced from them, in the words picked up from current usage; we do not invent a new language, calculated to express as appropriately as might thus be possible the course of our thinking. But an inherited language is essentially the deposit of ways of thinking and bodies of ideas that have become so widely accepted over a long period of time that nobody in the community using it questions them. When the only word for "grain" which a youngster finds available implies a supernatural power concerned either to satisfy or to frustrate man's need for food (as is the case in many primitive languages), how can he be expected to take a scientific attitude toward the processes of agriculture? However critical we are capable of be-

coming, we take the current language for granted as the normal medium whereby to describe and preserve in memory what is significant in our experience. It limits any universe of discourse in terms of which our thinking can proceed.

When boys and girls thus emerge from childhood they find themselves possessed of a mass of notions taken quite passively and without question, but as much a part of themselves as their muscular skill and their second teeth—notions which furnish the intellectual eyes through which they view the world. What will be likely to upset these beliefs, however inadequately based, even then? Well, if one is adventurous and exploratory by nature, some few of them will land him in practical trouble; this will perhaps prod him to try other ideas inconsistent with those previously accepted, and he may find them more successful. Thus he may be led to the revision of some of his beliefs in the direction of more adequate opinions. But by far the greater number of his ideas are not thus vital to practical welfare, while on the other hand two very important factors strongly support continued loyalty to the tradition he has come to share.

Painfulness of doubt or suspense

In the first place, the period of doubt or suspended judgment which necessarily intervenes between the time when one begins to distrust an accepted idea and that of its replacement by something better, is naturally quite painful. A belief once adopted is not a merely contemplative affair; it is an established habit, providing ready guidance for action on the objects with which the belief is concerned. And smoothly channeled habits are essential to effective action. If a decisive act is needed while one habit has been given up and before another has taken its place, it is not apt to be forthcoming. Whatever step is taken will be hesitant and weak. The individual in a state of suspended judgment thus feels himself lost in a practical helplessness which is one source of the pain attending the revision of habit. Only gradually can the greater satisfaction that he finds in the superior succeeding conviction acquire sufficient attachment to the preceding sense of doubt so that it counteracts the natural

unpleasantness of the doubt and renders it tolerable. An essential mark of a thoroughly critical mind is that such attachment has been definitely established.

Tendency to justify our beliefs instead of correcting them

In the second place, all our instinctive reactions of self-defense tend to be mobilized in support of our beliefs, just as they protect any other of our possessions. A challenge in the form of some contrary suggestion leads us to justify rather than to correct them. I may know perfectly well that my pet dog is nothing but a lazy, cowardly beast, without a virtue save his loyalty to me; but if anybody else should make such a suggestion, at once a hot defense complex is roused. I vigorously defend him with the highest praise, and nothing is farther from my intention than to engage in an objective inquiry to determine how far the insinuating remarks are true. The same principle, of course, operates in the case of our ideas about the universe and everything important in it—communism, free enterprise, birth control, the Freudian theory, the size of our navy, methods of taxation, the closed shop. Few of these matters have we really studied in any systematic fashion so as to reach intelligently justifiable views about them. Our beliefs have been imbibed, how or why we hardly know, but mainly in the quite irrational way described above—through docility to the insinuations of the newspapers, the wiles of a party orator, the prejudices of our parents and teachers, the vague sense of advantage to our group interests. But let a question be raised as to the soundness of our notions on these things, and at once we find ourselves filled with an illicit passion for them; we defend them just as we would defend a punched shoulder. The problem, how reasonable they really are, does not trouble us. We refuse to learn truth from a foe. And indeed the hotter and more violent defense is provoked in favor of ideas with the least intelligent justification—those acquired merely by subconscious yielding to the intellectual currents that play upon us. The reason for this supreme pitch of irrationality lies in the fact that beliefs reached through some thinking, however meager, we know can be doubted, while

those absorbed as uncritically as the milk from mother's breast are so firmly rooted in our entire make-up that to question them seems at first sight like scepticism carried to an insane degree. When they are assailed we instinctively rush to protect them, whereas we are more ready to reopen inquiry into those originally gained by some effort at impartial thought. This tendency to intellectual self-preservation, to defend our ideas rather than to find out the truth about the matters concerned, is technically termed *rationalizing*. It takes various forms in detail, all of which are destructive to honest thinking. The most popular of these is the search for an ostensibly good reason—one socially acceptable—for going on believing and doing what we already find ourselves believing and doing, how or why we hardly know. Our party should be continued in power because (such are the reasons we pick up and affirm) the country has made progress during its regime; the closed shop should not be conceded to labor, for an employer has a right to decide whom he shall hire. A clergyman of international reputation preached some years ago a sermon on the text: "Lay not up for yourselves treasures on earth, where moth and rust do corrupt," in which he pointed out that the modern enterprising business man does not really violate this prohibition; he places his funds in bank vaults specially constructed to avoid these undesirable consequences, or he invests them in stocks and bonds, which a simple box will amply guard!

Most of our rationalizing, it should be noted, is quite unconscious. We attempt to deceive other people about the motives that control our conduct because we have already deceived ourselves. Unable to face honestly the real feelings affecting us, because they are often so childishly irrational, we substitute for them motives that we can approve and know that others will approve. Such self-deception is therefore a form of pretense that can only be uncovered by the most strenuous and persistent self-examination. It is relatively easy to discover when exhibited in others, as the "debunking" recently become a popular practice shows, but it hides its role in our own thinking with marvelous dexterity.

Akin to rationalizing is the habit of covering up the inadequacies of one's thinking by a swirl of pretty-sounding words when discussing some important topic. Stuart Chase uses himself as a horrible example of this vice:

> The system called capitalism, for all its sprinkling with holy water in the nineteenth century, is at heart irreligious, without internal unity or public spirit, often a mere congeries of possessors and pursuers. When it adopted as its basic principles the competition of tooth and claw and the supreme duty of selfishness, all the holy water in the cosmos could not disinfect it. Great religious movements have usually been grounded in collectivism, in the brotherhood of man, leaving laissez faire, in the last analysis, a cold and ferocious anti-Christ.[1]

It is perfectly proper, of course, to use language as a means for expressing and evoking emotion as well as a means for accurately reporting facts. We need poets as well as careful and critical exponents of literal truth. But the two functions should obviously not be confused; nothing is really gained in any serious discussion by substituting high-sounding words for a real answer to a real problem. Since, however, the clever manipulation of words is a delightful exercise, seductive to hearer as well as to author, the temptation to engage in such substitution is often too hard to resist.

Other factors supporting adherence to tradition

Certain other characteristics of human nature, which support adherence to customary beliefs in general and the tendency to rationalize such adherence in particular, may be more briefly listed. One of these is intellectual laziness, our natural avoidance of concentrated effort. It is always easy to continue acceptance of a long familiar idea, whereas to replace it by a new one always means effort, and sometimes very hard work. Especially does a revision of beliefs meet strenuous resistance when a flock of other cherished notions is so bound up with them that serious doubt is likely to precipitate a collapse of the whole structure. The beliefs that the earth is the center of the universe and that all species of animals were separately created are historic examples of such basic ideas. It seems, so to speak, to conservative minds, like breaking a

[1] *The Tyranny of Words*, p. 363.

tacit treaty of peace, and wantonly plunging the world into intellectual chaos, to question doctrines on which the whole fabric of accepted science and philosophy rests.

Another factor is a semijustifiable preference for playing safe. Nothing is more apt to annoy eager radicals than the fact that many people still clutter their conduct with irrational practices even after they are quite ready to admit that there is no real evidence for the beliefs on which these practices are based. "There might be something in it, and it will do no harm to keep it up," is the rationalization offered if adherence to any such practice is criticized. This feeling perpetuates through long centuries such performances as the ritual ceremonies connected with planting and harvesting, in communities many of whose members realize perfectly well that there is no support for the supposition that these ceremonies really contribute anything to the production of crops.

Difficulty of securing a motive for persistent reasoning

It is, of course, obvious that the same factors which beget incorrect opinions and render their criticism so difficult, will operate to abbreviate, confuse, and prematurely end a train of critical thought once it has managed to get started, unless some vital interest is felt to be so dependent on it that an effective motive can continue to function. Otherwise the thinking, driven by insufficient energy, will soon fade and evaporate, or wander in irrelevant directions. Primitive people are notorious for their inability to concentrate on a problem for any length of time except, perhaps, when engaged in war or the hunt. It is only by a long and gradual process that persistent motivation can become attached to thinking not directly focused upon some practical need, and that an intellectual curiosity eager to find methods helpful in various sorts of perplexity can develop.

Thus far our consideration has been occupied with forces making for conformity with already accepted beliefs, which operate within each individual as such. But the one first mentioned—submissiveness to indoctrination—reminds us that the individual does not think in a vacuum. He is surrounded by social pressures of various kinds, and these are all too fre-

quently exerted with vigor in such a way as to insure that anybody who shows a tendency to think along novel and hence disturbing lines will be brought to terms. Here is another factor supporting adherence to established conventions.

Several points must be held in mind in the interest of understanding clearly why such repression is resorted to. For one thing, an entirely sound instinctive feeling underlies this practice. The community must be coherent and well disciplined, to insure success in securing the means of livelihood and in withstanding external attack by united effort. Whatever promotes lack of confidence, hesitancy, or social discord is rightly felt to be prejudicial to success. Now criticism of existing ideas, to say nothing of fostering a sceptical attitude generally, unquestionably does introduce these things; an obvious effect is the weakening of social unity and the encouragement of a rebellious individualism.

Social repression of novel ideas

The very foundations of community welfare are thus felt to be imperilled by any attack on traditional beliefs. Accordingly, in all ages except the rare ones in which intellectual initiative has itself, at certain points, become socially prized (of which more anon), the reformer in matters of belief is regarded as a menace to society, to be treated accordingly.

For another thing, love of power on the part of those in positions of political or economic leadership naturally finds expression, among other ways, in the suppression of novel ideas except in the cases where these promise advantage to their personal or group interests. Such men are eager to establish effective control over the behavior of their fellow citizens and they find it easy to rationalize this love of domination by assuming that such control is necessary to save the country from the ruin sure to ensue if the policies of their opponents were to be followed. Now control over people's action is temporary and precarious unless it also extends over the innermost convictions from which actions spring; moreover, nothing so tickles the vanity of a power-greedy mind than to find himself able to manipulate the basic habits of thinking and feeling among those who constitute the mass

Due to love of power

of his community. Leaders naturally exert this control by methods that promise to serve their ambitious purposes and to perpetuate their dominant position. If, as is often the case, they are sufficiently public-minded so that the welfare of the country is not by such means quite sacrificed to their own advantage, they are apt to be intellectually fossilized in many areas of their thinking, assuming that the good old ways are intrinsically superior to any alternative, and that a new idea is to be distrusted simply because it is new. Their minds are in these areas static and provincial, unwilling to recognize the necessity of adjustment to changed conditions, and the desirability of profiting by the experience of other communities.

The nature and force of propaganda

The general procedure by which such intellectual control is exerted on a large scale is now called *propaganda*. Thus conceived, propaganda includes any exercise of influence over the media of public information and discussion, such as newspapers, printed books and pamphlets, journals, the radio, signboards, and moving pictures, for the purpose of persuading people to adopt or preserve ideas which the propagandist wants them to believe. The forms with which America has been most familiar in the past are business advertising and political propaganda in the press, together with, in recent years, the motion picture. The invention and widespread use of the radio, however, have disclosed new possibilities of influence over public opinion. An audience of millions can be secured for propagandists who possess sufficient wealth or prestige. And since many people have become accustomed to listen to the radio while they attend to other tasks or are in an attitude of relaxation, which renders them very suggestible to any ideas persuasively presented, assertions constantly dinned into their ears are bound to have a seductive and cumulative effect. Unless our critical capacities are alert and on their guard, the mere persistent repetition of an idea is quite sufficient to make it seem reasonable—a principle of which all who wield the forces of propaganda are well aware.[2]

[2] They thus put to conscious use the factor of frequency in the association of ideas, above discussed.

Under dictatorial governments especially, and also democratic governments with wartime powers, it is almost impossible for anyone to avoid some molding of his judgment by the persons in control of the state; he is not allowed access to prohibited arguments or to all objective sources of information, and although, aware of this, he tries to maintain an appropriate scepticism, it is impossible for him to be on guard in all areas of his thinking at once.

Some of the specific techniques of the propagandist deserve mention, since they reveal so vividly the irrational forces in human nature to which appeal can be cleverly directed. The technique most constantly employed is *card stacking, i.e.*, selecting, in an argument, only those facts and principles which support the propagandist's conclusion, and trusting that his audience will not be alert and critical enough to think of considerations that point in a different direction. A quite frequent device is *name calling*—describing an opponent or his theory in terms that arouse hostility by their emotional associations, instead of in terms that are fair and unprejudiced. Efficient labor leaders have, for example, often been dubbed "anarchists" or "communists" by those whose interests are threatened by their activities. Another method is that of *dichotomization*,[3] which means dividing the scene contemplated by a discussion into the activities of heroes on the one side and villains on the other, the one group being assumed to be all good and the other to be all bad. During the war, for example, all who supported the cause of the United Nations were elevated to a pedestal of high virtue while all who failed to do so were treated as collaborators with the powers of iniquity. Still another is the technique of the *scapegoat*, illustrated by Hitler's persecution of the Jews before the war and the less ruthless adoption of the same procedure during the war by other countries. The principle behind this technique is: Cover up your own delinquencies and vices by blaming the evils that harass you on some group too helpless to fight back. By taking part in the maltreatment of such a group an intolerable sense of failure in the propagandist himself,

[3] Sometimes called "tabloid thinking"

and in his following, is replaced by delusions of righteousness, grandeur, and power. One more device may be mentioned, namely, that of *semantic confusion*, or distortion of language. A good instance of this is the tendency, on the part of individuals who exhibit many of the significant characteristics of fascism, loudly to describe themselves as devoted champions of democracy. They count on others being gullible enough to judge them by their words rather than by critical consideration of their acts. All these devices may, of course, be employed unconsciously, for the purpose of rationalizing one's own impulsive desires or blunders, as well as in conscious attempts to persuade others.[4]

The role of institutional religion

A rather distinctive form of social pressure on the individual is that exhibited by institutionalized religion. Some sects, at times, have given considerable scope to the kind of leadership typified in the prophet, who refuses to be bound by the past and is eager to win a novel moral and religious insight. Over the years, such sects have gained in influence, and they have encouraged a measure of intelligent freedom in their members. But, by and large, it is the priestly tradition that has reigned in institutional religion, the tradition insisting on the necessity of preserving unchanged the hallowed beliefs and practices of the fathers. Through the persuasive discourses of the priests, fear of penalties imposed by the gods is added to fear of living political and economic potentates as a force supporting intellectual conformity. Nothing, in their view, will be surer to draw devastating consequences than toleration of any individual who takes lightly the sacred traditions. Unquestioning belief in the creed of the ages and readiness to act implicitly upon it thus become the primary ecclesiastical virtues. The gods are held to be the invisible rulers of the community, dispensing prosperity and woe with mighty hand; it is uncritically but naturally supposed that every established custom must be precious to them; hence any re-

[4] These irrationalities, and others described in the present chapter, will be noticed again in the form which they assume when judged in the light of the criteria of correct reasoning. See below, pp. 277 ff., 503 ff., 733 ff.

vision of tradition is an insult to the deities, the certain prelude to dire punishment, delayed if not executed at once, in another world if not in this. Furthermore, the role of priest means a position of importance as a social authority. Fanaticism combined with love of power leads to frequent abuse of this position. The will of the gods can only be ascertained in special ways, of which the priests are custodians; true doctrines are only revealed through special agents, which are none but they or their initiates. They gain thus an intellectual authority very difficult to withstand, since it is their privileged position to wield invisible powers affecting the weal or woe of their fellows. If one ventures to question the truth of their pronouncements, to social ostracism in the present they may add the threat of endless damnation in the future.

This emphasis on the power of conventional ideas to maintain themselves, and on the social pressures which are resorted to for preserving them, must not, of course, be misunderstood. The point is not that such beliefs are always erroneous, and that notions revealing this or that individual's eagerness to rebel against established custom would be more likely to be right. Such a misunderstanding would be very serious. Mere impulsive rejection of standards built up by past experience, and readiness to adopt an idea because it is different and thereby frees one from unwelcome restraints, is just as irrational as uncritical acquiescence in conventional notions. Indeed, on the whole it is more irrational, because no widespread custom would ever have become such were there not much evidence for its value, whereas there may not be any substantial evidence at all for a theory hit upon merely because by its aid somebody can rationalize his eagerness to break away from restraints that persistent social forces have imposed. Mere antisocial rebellion is disintegrative to oneself as well as to one's community. The point is that conventional beliefs—to say nothing of propaganda on behalf of interested groups—always harbor some error as well as much truth, and that the forces in question tend to perpetuate the error along

with the truth, obstructing any effort critically to evaluate or rationally to revise the ideas in which it is embedded.

Clues to many important truths are obscure

Besides all these vigorous factors working toward the maintenance of error, there is a consideration not yet mentioned, which adds enormously to the difficulty of escape from its grip. The clues to the discovery of controllable causes of events are often very obscure, requiring both a high degree of intellectual power and a long course of baffled experience before they could be suggested to anybody's mind. The causal connection between a severe fall and the resulting pain offers little tax on one's memory once it has been observed in experience—but how should anybody seriously think of the bite of a mosquito as the cause of malaria? The relation between the two events is surely far from obvious. Only a very fertile imagination, fortified by long experience of the advisability of digging beneath surface sequences to hidden clues, would be apt even to consider the possibility of a connection. To ferret out important causal laws is thus an exceedingly difficult affair because the kind of fact which needs to catch attention may be very unobtrusive. It seems like a silly waste of time even to entertain some of the suggestions that later become thoroughly verified. Some, indeed, can hardly be considered without an equipment which man with his unaided senses does not possess. Lacking a microscope and the other appliances of modern medical science, how should anybody guess that a minute organism in the blood stream is the cause of typhoid fever? Yet it is through the discovery of such organisms that medicine has undergone one of the most beneficial revolutions of its entire history.

Forces that support right thinking

How, then, is it that we are not completely lost in error, that some dependable beliefs have easily been hit upon, and others, after a long struggle, have replaced earlier superstitions? Why are we ever able to think aright?

So far as concerns the fundamentals of human nature on which habits of correct thinking must in the end depend, the answer is twofold. On the one hand there are the factors in the connection of ideas which usually engender relevant sug-

gestions with a good chance of being right—the factors of frequency and means-end contiguity. When the course of our reasoning is dominated by other factors, these wait in reserve; if we get into trouble as, barring lucky accidents, is likely to be the case, we may fall back on them and be guided to a more successful outcome. In matters of direct bearing on life or welfare, as has been noted, our habits of thinking already betray the cumulative effect of this process. And the lesson is always capable of being applied over a wider range of problems; when this is done, distrust of the other forces leading to belief, and increased confidence in these, become themselves habits by which our subsequent reasonings are more and more influenced. On the other hand, an element in human nature cooperates in this process which thus far has been mentioned only in passing. This is *curiosity*—an interest in noting objects and relations quite apart from any bearing they might have on our immediate practical concerns. In solving any problem we always notice more than is obviously relevant to that problem, because of the presence of curiosity in our make-up. These apparently superfluous observations are stored in memory and may be useful as a source of suggestions in subsequent theoretical or practical problems. And curiosity is the main motive which we must count upon to carry us through when we seek to explain some puzzling event. But it is obvious from long experience that in the case of the bulk of mankind, including even leaders in political, military, and industrial affairs, the exercise of curiosity is too desultory and unsystematic to accomplish any valuable or permanent reform in their habits of thinking, not to say any extension of right methods of reasoning to matters which are not clearly of vital concern. How then has it come about that a persistent intellectual curiosity has arisen out of this sporadic inquisitiveness? And how has it united its force with the power to discriminate the more dependable factors affecting belief from the less dependable ones so as to generate a definite, consciously understood procedure for the progressive establishment and continuous correction of verified knowl-

edge? For a procedure of this sort—which we now call "scientific method"—has in course of time actually developed, and is now playing a significant role in human history. The answer is briefly as follows.

Descriptive sketch of their historical operation

When sufficiently trustworthy means of building shelter, of hunting and fishing, of carrying on war and agriculture, were invented, it became possible for more stable civilizations to arise than had appeared before, accumulating capital beyond the amount needed for immediate consumption. In such communities not all the energies of all persons were required to meet insistent practical needs; a leisure class appeared, supported in comfort by the rest and largely free to apply its interests in whatever directions happened to appeal. Now some of the members of that leisure class were persons with an especially live curiosity as well as unusual powers of concentration and constructive imagination. Instead of seeking political, industrial, or ecclesiastical power, or in addition to doing so, they became the first intellectual leaders of mankind. Theoretical problems intrigued them, and were attacked with patience and systematic inquisitiveness. They were attracted, for example, by the orderly motions of the celestial bodies and began to record observations of their behavior, gradually formulating general laws of the regularities thus disclosed; they studied the ancient annals and traditions of their peoples, noting the historical changes which had taken place; they traveled among other countries, observing the startling variations between the beliefs accepted among different peoples and thus becoming conscious that what anybody believes must, to a large extent, depend on the accident of his birth and education. With a few of these sages, such exercises in curiosity led to doubt as to the possibility of attaining demonstratively true beliefs. With others, however, a more hopeful consequence appeared. They discovered that more accurate knowledge than had previously been available could in many instances be successfully established. Especially did this prove to be the case in the field of mathematics, where definite results, so they found, might be demonstrated

to the satisfaction of anyone capable of understanding the problems involved. So in time they were led to compare this privileged discipline with other branches of study in which such success seemed conspicuous by its absence, and to note by contrast the factors which appeared responsible for its power. By such critical comparisons they gradually disentangled the more efficient from the less efficient methods of reasoning that people employ, and slowly found out some of the major requirements that any piece of thinking must meet if it is to have a good chance of attaining its goal. They thus gained a position which enabled them to undertake constructive criticism of the traditional notions prevailing around them, guided by a consciously formulated conception of what true knowledge is and how it can be hopefully and systematically pursued. A small but growing group of intelligent truth seekers came in this way to be at work in the world, characterized by alert attention to the most approved methods of reaching trustworthy results, and by scrupulous devotion to the tests of truth which persistent criticism had been able thus far to establish. To these tests theologies, histories, moral and political theories, technologies, and cosmologies were more and more required to conform through the influence of these men, and through the success which under their leadership was attained. In the activities of such thinkers we find a feeble but definite anticipation of the vast human enterprise now called "science."

The emergence of science

This word, which will be employed frequently in the following chapters, has both a broader and a narrower meaning, and these two meanings may well be distinguished at once. In its broadest sense, "science" includes any responsible search for the soundest conclusions that are obtainable at any given time, and for appropriate methods by which those conclusions may be progressively corrected. In this sense the men just described may be called scientists, and regarded as pioneers in the great task which present-day scientists are carrying on. But the same word also has a narrower sense, especially when employed in the plural. By the "sciences," people often mean

those particular branches of this enterprise which have clearly demonstrated their ability to reach results that command the agreement of all competent investigators in the field. Mathematics, physics, botany, geology, etc., are such sciences. In this narrower sense theology, history, the various branches of philosophy, and perhaps also social theory, are excluded from the scope of "science," because of their failure as yet to reach results of this sort, although the work of many thinkers in these fields would be fully scientific in the broader sense. Now the sciences whose success is undeniable have not only preserved their methods and handed on their results from generation to generation; by the triumphant practical application of their discoveries they have won, especially during the last two centuries, a growing measure of popular influence. Once this stage in their development has been reached, overhauling of traditional ideas and the progressive reconstruction of beliefs by approved techniques becomes itself, in some communities, an increasingly accepted practice, backed by a certain measure of social approval. This situation obtains in the more cultured portions of most urban communities of the Western world today, so far as concerns beliefs about nature, and even, to some extent, beliefs about man and society.

When science is viewed thus in historical perspective, it is seen to possess a unique value for every thinker eager to improve his habits of reasoning and to use his intellectual powers to the best advantage. In the broad meaning of the word, he will wish to be as fully scientific himself as he can learn how to be; he will contemplate the progress of rational inquiry with sympathy and understanding; he will be anxious to contribute to its advance in whatever ways he may; he will strive to gain familiarity with the achievements, in as many areas as possible, of those who most clearly follow the rules of responsible scientific procedure. In its narrower meaning, he will give such special attention to the methods of the distinctively successful branches of science as their success appears to justify; and when faced with problems which neither time nor technical competence enables him to attack

on his own resources, it is the accredited results of these branches to which he will turn to supply whatever needed information and guidance is available.

How, then, may we intelligently guard against the forces that support erroneous beliefs? Fundamentally, in two ways. First, by gaining a vivid awareness of their presence and nature, and by persistently forming the custom of looking for their possible influence on ideas which are suggested to our minds or which we have already adopted. Second, by understanding the factors which support right thinking, and the principles by which it must be guided, forming the habit of making the fullest and wisest use of them in all our intellectual operations. The latter of these two ways we shall now consider more fully. In doing so, we shall be offering a preliminary reply to the last question raised at the end of the opening chapter, namely, how reasoning must proceed if it is to reach as correct results as possible. The rest of the book constitutes an answer in greater detail.

EXERCISES

1. Explain the difference between reasoning and rationalizing.
2. Is propaganda always and necessarily bad from the standpoint of an interest in critical thinking? Justify your answer as fully as you can.
3. Just what is added when one qualifies "curiosity" by the adjective "intellectual"?
4. Making use of material from all the chapters to date, write an essay on "The Rational Habit of Mind."
5. Collect, for a week, instances of propaganda from newspapers and from radio broadcasts to which you listen. Which of the propagandistic devices mentioned in the chapter are illustrated?

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CHAPTER 6

GENERAL CONDITIONS OF GOOD THINKING

What are the conditions of correct thinking as revealed in scientific reasoning?

The major theme to which the preceding analyses have been largely introductory now confronts us. What are the essential conditions of correct reasoning? What marks distinguish the sort of reasoning that proceeds in the manner most appropriate to the attainment of true judgments from reasoning that does not? And how can we form the habit of consistently respecting these conditions in practice?

These questions might be attacked in several ways. But the outcome of the inquiry in the preceding chapter suggests that a careful study of the principles and methods exhibited in the sciences would best help us here. The pure sciences, so called, display the most successful procedures that have been discovered to date for the solution of theoretical perplexities, challenging our curiosity, while the applied sciences, e.g., engineering, show how the truths thus established may be most hopefully used in meeting practical difficulties. But in view of the distinction drawn in the preceding chapter between the broader and the narrower meanings of "science" it is necessary to indicate more exactly what is intended by this proposal. It implies, of course, that we should turn for help, in the first instance, to science in the narrower sense—science as composed of those branches of study whose claim to successful achievement stands quite undisputed. There is no intimation, however, in this that we should blind ourselves to possible limitations of these branches, or assume in advance of critical consideration that reasoning in all subjects—such as history, social ethics, or religion—must follow in detail a pat-

term derived from them. How far this is the case is a question to be faced in due time.

What is intended is rather this. The preceding chapters have brought to light certain qualities which all serious thinking must exemplify as far as it can—qualities necessary if it is to have the best chance of achieving its goal. Such qualities are scrupulous devotion to truth, patience and honesty, freedom from prejudice and partisanship, readiness for self-criticism, boldness of imagination, intolerance of confusion and inconsistency, and awareness that the tools and methods of inquiry stand in constant need of improvement.[1] Now the obviously successful sciences display these qualities at a high level of maturity; they succumb less than other branches of thought to the temptations always lying in wait to lead the mind astray. With what better teacher can we commence, then, in our need to learn what virtues human reasoning can possess at its best? Moreover, unless we have first gained familiarity, not only with what such qualities actually mean in systematic practice, but also with the distinctive procedures that seem responsible for the special achievement of these sciences, how can we compare them intelligently with those of the less successful disciplines? Until one has mastered the secrets of their success, he is in no position to tell how far and in what ways their guiding principles and canons can be properly applied in those areas where at present the precision, definiteness, and dependability that characterize the best scientific knowledge are conspicuous by their absence.

A brief classification of the sciences

In pursuing this study some classification of the various branches of inquiry will need to be adopted—a classification both fair to the obvious characteristics of science as a going concern, and likewise so devised as to throw as much light as possible on the present problem. Such a classification will then supplement, as a guide throughout our subsequent discussion, the general distinction between science in the broader and science in the narrower sense which has already been

[1] Later we shall analyze these qualities in a more systematic way. See chap. 14, pp. 279 ff., and chap. 24, pp. 564 ff.

introduced. First, that distinction itself will be recognized; we shall separate the branches of systematic thought which are generally regarded today as sciences from those which are not. Then within the former group a distinction will be drawn between the formal sciences, such as mathematics, and the factual sciences. This distinction rests, as we shall see, on the difference between syntactic and semantic meanings, noted in Chapter 2. Finally, among the last mentioned, it will be necessary to distinguish between the natural and the social sciences. This classification will be used in the following chapters to provide a principle of order in treating their various themes.

THE RATIONAL DISCIPLINES

- THOSE USUALLY RECOGNIZED AS SCIENCES
  - FORMAL SCIENCES
  - FACTUAL SCIENCES
    - NATURAL SCIENCES
    - SOCIAL SCIENCES
- OTHER TYPES OF INTELLECTUAL ENTERPRISE

Moving from left to right in this table, we shall begin with an examination of the aspect of reasoning which is revealed most clearly in formal science, proceed to the methods of the factual sciences, natural and social, and end with a consideration of the disciplines whose results seem less capable, at least at present, of conforming to scientific standards than those attained in the others. When the last of these fields is reached it will be necessary to emphasize the difficult problems of method which confront us there, whereas, in dealing with the earlier branches, already established procedures more readily approve themselves as essentially sound.

Certain conclusions reached in our analysis of a typical case of reasoning likewise suggest the same order of topics. Inquiries in formal science are less complex than those in factual science because they are complete at the end of the fourth step. The factual sciences all require processes of verification

at the fifth step, but those processes can be more confidently pursued in the case of the natural than in the case of the social sciences, and the results reached are more obviously sound. It surely seems wise to proceed from simpler to more complex themes, and from methods capable of more objective application to ones where we must be content with less assured results.

Preliminary statement of certain general conditions of effective reasoning—1. Health

The fact, however, that there are qualities exemplified by all responsible thinking, whatever its subject matter, implies that certain general conditions of good reasoning can be noted which apply to problem-solving of every sort. They can be stated in the light of the considerations brought out in the preceding chapters, without any detailed examination of scientific methods. The balance of the present chapter, which concludes Part I, will be occupied with these.

The first may seem to be a simple physiological matter, and to those who tend to regard mind and body as quite separate entities, rather irrelevant. This is the condition of *health.* Good health is just as favorable to good thinking as it is to the efficient performance of any normal organic process, such as the elimination of toxic materials or restful sleep, and the same relation obtains between bad health and bad thinking. We know full well that arduous intellectual toil is out of the question when we are suffering from a toothache simply because the local pain drags our attention from the problem in hand, and one of the signs of a resilient condition is an unwonted rapidity in the flow of ideas and a firmer grip on the conceptions we are struggling to master. But there is a more scientific basis for the assertion. The health of the entire body is uniquely dependent on the efficient functioning of the nervous system, in that every organ is regulated by reflex arcs linked through the spinal cord or the brain. Now all the evidence available points to the conclusion that the higher centers in the brain are exercised vigorously when active reasoning is going on. But metabolism there cannot take place so freely or be continued so long, if an unhealthy condition elsewhere requires more than its share of nerve energy. It is pre-

cisely the same principle that is noticeable whenever we try to think hard after a hearty meal, and find that either the stomach quite refuses to surrender the extra energies needed to handle its unusual burden, or else if we do concentrate on the intellectual task our dinner remains largely undigested till we give the mistreated organ a chance. Other things being equal, he thinks best whose entire organic life is in tune with itself and its surroundings to the highest degree.

Apparent exceptions to this rule occasionally confront us. Some of the greatest scientists, philosophers, and men of practical achievement have been the prey of chronic ill health. But their testimony is fairly unanimous that this was a handicap to large intellectual accomplishment and not an aid.

Special importance of normal sense organs

There are, of course, certain special phases of a healthy condition which are of peculiar importance to good reasoning. In regard to physiological matters, for example, it is especially important that the sense organs be in normal condition, for through them we gather the observations that clarify our problems and verify our hypotheses. Color blindness is as truly a handicap in thinking as it is in seeing. As regards distinctively mental phases, it is the alertness and zest for intellectual effort, naturally attending good health, that furnish essential conditions for effective problem-solving. One whose attention moves slowly or whose mind easily tires can hardly excel in intellectual competition. And the self-control involved in pushing a piece of hard thinking through to the end demands not only power to neglect distractions, but also self-possession in the face of temptation to excitement or worry. It is very important that one be free from all distortions such as require the art of psychoanalysts for their proper treatment, and likewise from minor dissociations displayed in the form of melancholy, fanaticism, and pathetic sighing for remote or impossible goods.[2]

2. Fluency and power of concentration

Consideration of these mental phases of good health leads readily to an analysis of certain intellectual characteristics im-

[2] The brevity of this sentence will not, I hope, obscure its importance. These mental maladies are very widespread.

portant for successful reasoning, that most people would not regard as included under the concept of health. Such are *fluency* and *power of concentration*, which appear primarily in a thinker's capacity steadily to control the entire course of a piece of reasoning by the nature of the problem he is endeavoring to solve. Our stock of these qualities is, perhaps to a considerable extent, determined at birth, yet there is evidence to show that improvement in their exercise is always possible. The significance of superiority in these respects is most obvious at the third step of a course of reasoning. It seems tolerably clear that people, roughly equal with respect to their knowledge in a given field, vary greatly in their ability to produce the appropriate suggestion at the right time. This variation is itself a complex matter in which at least two important and independent factors enter. On the one hand, the mere quantity of suggestions occurring within a given time may vary from almost zero to a very large number; the flow of ideas stimulated by a perplexity may trickle slowly, or it may swell into a rapid flood. In general, of course, the richer the flow of suggestions, the more likely is the individual to reach a solution of his problem, although there may be times when one is so overwhelmed by the wealth of his ideas that he cannot satisfactorily develop and test any of them. Again, people vary greatly in the degree to which the immediate problem controls the rise of suggestions and insures their relevance. With some, irrespective of their condition of health, attention wavers or quickly flags even in serious situations, so that ideas of no value for the perplexity faced insist on appearing. With others, concentration is much more complete and persistent, so that only when the problem has continued to baffle for a long time does its mastery over the course of ideas relax. In which of these directions the condition of fruitful reasoning lies is clear enough without specific statement.

What is meant by the rather vague term "genius" is probably more intimately connected with superiority in these qualities than with anything else. A scholar of the pedantic

type is a very different person; he commands as much, perhaps more, information, but he can only reproduce it freely in situations similar to those in which it was gained; and even then it lacks the spark of imagination needed to set the mass aflame and bring its elements into constantly new and suggestive combinations. To be in such a perpetual ferment of ideas is precisely to be a genius, and when this faculty is combined with wide knowledge and the patience for creative toil as of the artist, or for careful verification as of the scientist, the man who possesses it leaves his mark on the world's history. The same general contrast on a smaller scale is frequently revealed in the drawing room. The ready conversationalist, good at repartee, whose wit is always entertaining, is simply the fellow who in the common themes of polite discourse is always able to produce the right suggestion at the right time. Far at the other end of the scale is the rather dull chap whose imagination so far lags behind that of the crowd that everybody else anticipates his ideas before they occur to him.

3. Control of each stage of reasoning so that it facilitates the later stages

Concentration means, then, in part, control of every step of an act of inquiry subsequent to the second, so that it will be relevant to the nature of the problem as there defined. Another condition of fruitful reasoning at once appears when we ask what principles for the guidance of concentration can be specifically derived from the analysis of an instance of reasoning into its temporal stages. The primary principle is clearly this: *Each step must be performed in such a way as to aid the successful performance of the later steps.* Let us recall again the distinctive function of the second, third, and fourth steps so as to see what this principle means in practice.

The second step consists in the precise formulation of what the problem is, and indicates what kind of solution will accordingly be acceptable. It thus provides the framework into which the activity of the subsequent steps must be fitted; by establishing the proper universe of discourse it shows what suggested solutions would be relevant, which of their various implications promise definite aid, and what sort of verifying

observation or experiment would yield a conclusive result. Evidently, unless this formulation is appropriate, clear, and adequate, the entire process and product of reasoning will be likely to lack these characteristics. It is essential, therefore, in any given kind of perplexity, for a good thinker to know how to realize these virtues in formulating his problem, so that pertinent and promising suggestions may be readily elicited and the subsequent stages of thought be guided as smoothly as possible. In the case of the second step little can be said as to what this involves beyond listing the general conditions of effective reasoning with which we are now occupied. In the case of the third step, however, it becomes quite clear that the distinctive conditions of its wise performance are determined precisely by the fact that it is followed by the two steps which succeed it.

Two essential marks of a good suggestion

This obviously means that there are two essential marks of a good suggestion. First, it should be such as to permit deduction of definite implications; and second, the implications should be such as to furnish clear and unambiguous guidance for verifying observation or action. A suggestion without these characteristics is at most a barren intuition; it cannot possibly function as suggestion in any complete piece of problem-solving. If, however, reasoning is superior to intuition, it is highly important that suggestions be of such a kind that the reasoning process can continue into its later stages with them. Yet many people seriously entertain suggested solutions of problems which violate one or both of these two requirements. The first is violated by appealing to the creative purpose of God to explain the differences between the various species of organic creatures.[3] Since the divine purposes are recognized as inscrutable to human minds, nothing can be deduced from such an idea; we cannot say that if God has created something, then such and such specific facts will be found in it. It is primarily for this reason that modern science, while not denying the validity of a

[3] This was the generally accepted explanation of these differences prior to the appearance of the Darwinian theory of evolution.

religious attitude toward the world, has found it necessary to abandon appeals to God to explain this or that particular occurrence. The second is violated by such suggestions as the one historically made in answer to Galileo's discovery that the surface of the moon is irregular, namely, that these irregularities are filled with a transparent glassy material.[4] Since this material by hypothesis is invisible, its presence cannot be verified by observation. How, then, tell whether the suggestion should be adopted or rejected?

Moreover, in general, a suggestion encouraging numerous deductions, some of which point to novel and unique verifying observations, is on these accounts more fruitful for intellectual progress than one revealing these characteristics in lesser degree. According as the deductions are more numerous and the experiments novel, the more likely is the suggestion to lead to an unexpected outcome in the form of possible answers to other problems; while if an implied circumstance is unique, not following apparently from any other suggestion, our verifying observation can establish a given explanation against all competitors. This is important because there are often groups of facts explicable by any one of several solutions, as long as no one of them can be seen to point to some unique verifiable consequence. For example, the fact that under ordinary conditions heavy bodies fall faster than light ones of the same size seemed explicable by several different theories until it was discovered that in a vacuum the acceleration is the same for all bodies. Then one solution was established to the exclusion of all others thus far conceived.

The same principle, of course, applies to the fourth step. As will be seen in the next chapter, an indefinite number of implications can be drawn from any suggestion. But only those that indicate some specific observations or acts whose performance will confirm or disprove the suggestion are of any use.[5] Thus, to return to the problem first employed as

[4] See below, pp. 401–403.

[5] Consideration is restricted here, of course, to problems in which a fifth step is required. In formal science any relevant implication may be of value.

illustration in Chapter 2, it would be possible to deduce from the suggestion, "This path leads to a spring," the implication, "This path does not fail to lead to a spring." Such an inference, however, would help not a whit in indicating what to do to test the suggestion. Few people fall into serious violations of the principle in this fashion, however.

4. Command of the best available tools

A fourth condition of successful reasoning is *command of the best tools* that human experience has forged for our aid. He who is not able to use to the full the most potent instruments thus far developed in the course of humanity's intellectual pilgrimage is under a terrific handicap in struggling through a challenging puzzle. Others better equipped will outstrip him, and he will find in time that much energy that a more varied and thorough knowledge would have effectively applied has been wasted.

These are in part intellectual

These tools are of two sorts, intellectual and mechanical. By the former are meant the ideas, concepts, words, in which we perceive, recognize, define, and order the constituents of our experience. All our problems have to be formulated in the meanings fixed by these tools; we cannot proceed in thought at all without using such tools as we have, and in the best way we know how. But it makes a vast difference whether the best way we know how is also the best way available or not.

Here is the fundamental reason for education on its informative side. In the matter of technical education, for proficiency in the skilled trades and professions particularly, the value of accurate knowledge is now popularly recognized. "Get the man who knows," is a slogan generally respected in questions of practical concern. This is not universally the case, of course. In the most important profession of medicine, quackery still abounds, and hosts of people turn to magic rather than to science. Yet there is some excuse here in the fact that scientific medicine is still in its youth; there are many matters on which no verified laws for remedial guidance can be formulated. Where practical pressure is strong, and dependable knowledge available, no argument is now needed

to enforce the principle. But it is quite universal. Whatever problem we face, and whatever kind of solution we want, if we have at our command the best available knowledge of the field in question, we shall be more able, other things being equal, to locate the relevant facts and pursue the inquiry in the most promising terms.

This point may be rendered more concrete by considering our needs at the second and third steps of a piece of reasoning. When, in the example recently employed, I perceived a certain interesting part of the landscape as a path, and asked the definite question, "Where does it lead?" I was stating a present perplexity in words whose meaning was fixed by similar experiences in the past. Had I not known what kind of thing a path is, I could certainly not recognize this present fact as a path, or ask any specific questions about it. We must be able to identify the objects on which our problems converge, and guide our questions about them by some knowledge of what we may expect them to be like; otherwise we shall be completely helpless in their presence. And it makes all the difference in the world how accurate is the knowledge on which we thus depend.

As far as the third step is concerned, suggested solutions of a problem are, and can be, nothing but notions which have been acquired in the past, and are now resurrected under guidance of the laws of connection among ideas. Clearly, it is essential to good reasoning at this step that one have at his command a large storehouse of ideas interconnected in many and flexible ways. Mastery, in fact, of the best knowledge already available on any type of problem is the main condition of the occurrence of promising suggestions.

In part mechanical

In regard to the mechanical tools, it is of the greatest importance to recognize that in the history of thought they have been equally necessary. Without them such observations as we engage in must be done with our unaided senses, and these, at best, are quite unable to reveal many facts crucial for adequate diagnosis of our difficulties or verification of suggestions. Primitive man develops his senses to exceeding acuteness com-

pared with civilized man; by keen sight, hearing, and smell, he detects signs of an enemy or intimations of prey quite imperceptible to most of us, and follows swiftly and noiselessly a trail which to the dulled senses of the pampered modern would be nothing but unbeaten forest. But the limits to fruitfulness of observation in this form are quickly reached, for the facts which often need to come to light for the proper statement and solution of our puzzles are quite beyond the reach of anything but an exceedingly delicate instrument. Increasing recognition of this fact has led civilized man to invent finer and finer tools to supplement the testimony of the senses, and to abandon (except in special situations) the attempt to rival the savage in the keenness of inborn tools. The history of science could be written in terms of the progressive invention of more powerful instruments for exact observation. The story of modern astronomy is largely the story of the telescope, beginning with the simple tube and lens with which Galileo startled his generation by announcing the mountains on the moon, the phases of Venus, the spots on the sun, and the satellites of Jupiter. The story of modern biology is largely the story of the microscope, by which for the first time the minute structure of living matter betrayed its secrets, and by whose development the most momentous discoveries have been made in the age-long struggle with disease. And the point could be equally well exemplified from every science. A large number of these instruments are, of course, instruments of measurement. This is shown by the suffix "meter," with which so many names of our technical instruments end; exact observation means observation sharpened not only to locate the hidden facts that would otherwise not appear, but also to reveal their quantitative relations.

The importance of both illustrated

A single illustration of this theme will suffice. Suppose a doctor essaying to diagnose a serious case of illness with none of the tools invented by science to aid him in the task. The man has fever, perhaps, but no thermometer gives its measure; the doctor must needs guess the best he can by the mere feel of the man's forehead to the palm of his hand. And his

judgment will inevitably vary enormously because the temperature of his hand changes with many factors. The lungs are perhaps in the last stages of decay, but he has no X-ray to apply, no stethoscope even to aid him in sounding their condition. He can roughly count the pulse and inspect a specimen of the blood, but he will not be able to see what vitally needs to be seen. If typhoid bacilli happen to be in the blood stream, he has no means of discovering their presence. The blood pressure may be dangerously high, but he has no way to test it; vision may be distorted, but he cannot tell how much; serious chemical changes in the products of the liver, kidneys, and other glands may have taken place, but he can only crudely conjecture them, if able to entertain such a suggestion at all. Should an operation be needed, his instruments are those of a primitive carpenter, and he has no way of rendering his surgery painless or of guarding against infection. The catalogue could be continued almost endlessly. Only the most prominent general possibilities and the most popularly familiar medical instruments have been mentioned.

But this would be merely a part of his plight. Parallel to the lack of mechanical instruments, there would be the same lack of the necessary intellectual instruments in terms of which to clarify his task, conceive appropriate suggestions, and trace their implications. He would have no conception of infection, because the causes of infection are beyond his ken; he would have no promising idea of many critical diseases, such as typhoid fever, because the underlying key to such a complex of symptoms, the presence of bacteria in the blood stream, is an occurrence of which he has never dreamed. Nay, even the notion of disease itself—that is, of a group of symptoms bound together by some basic condition and curable by a single line of organized attack—would be absent. The history of medicine reveals that the earliest civilized doctors, among the ancient Egyptians, treated what we should call the symptoms of a disease rather than the disease itself. There was one remedy for headache, another for chills, a third for fever, a fourth for quickened respiration, etc., all of which were applied without

any reference to one another or to any central condition as their cause. The reason is simply that nobody had yet imagined that there might be any such central causal condition. Even the practice of recording the history of a case, so as to suggest laws of the temporal development of typical diseases, did not appear till the time of the Greeks. It is not too much to say that a modern doctor would feel totally helpless could he picture himself reduced to this level in the history of his science. He would see little value in having such doctors at all.

Some guiding corollaries

Few doctors today would dare to practice without a fairly adequate knowledge of at least the main principles of their science and the possession of the most constantly needed instruments; and they would know to whom to send a patient for the more specialized diagnosis in which they are not expert. But in the fields where exact knowledge is still less established than it is in medicine, or where human suffering is not so directly associated with its lack, a tendency still prevails to be satisfied with second-best knowledge, or even with nothing but a quite haphazard guess. In politics or morals, for example, people often resent the mere intimation that there might be knowledge on such matters beyond their present ken and which might make a vast difference in their method of dealing with them. Everyone who wishes to be a good thinker must take the responsibility of equipping himself with a general knowledge of varied fields and a detailed knowledge of at least one considerable branch of some field; he should know where to turn for accurate information on any type of problem which he lacks; and he must form the habit of raising the question before dealing with any important task, whether there may not be knowledge or physical tools available which might make its pursuit far more promising than it would be without them. Unless the need of immediate action is very great or the matter of small importance, it is best as a general rule to spend some time in the search for such knowledge.

5. Readiness to correct the past

But as a fifth condition, *realization that the best knowledge of the past may still prove inadequate* is vital. We always clarify our problems and pursue possible solutions, at first, in

terms of such knowledge, but continued bafflement may force us to entertain suggestions reaching far beyond what the best information hitherto available would justify. If this possibility is held in mind at the start, the character of the whole thought sequence will be affected accordingly, and reasoning at each of its steps will become more tentative, flexible, and alert. Even observation of facts will be profoundly affected. It will be suffused with an attitude otherwise absent, the attitude of readiness to see what even the clearest sighted have overlooked as well as what others have bidden us expect, and perhaps to make an original discovery such as would enable our intellectual heirs to attack their problems more hopefully than we have been able to attack ours. It is naturally very easy to overlook important facts, for one tends to see in any situation only what past experience and knowledge lead him to anticipate. When a teacher shows a new botanical specimen to his pupils he knows full well that unless he tells them what to look for they will not be apt to notice the most interesting and significant facts about it. And a clever magician can easily compel us to overlook the motions that would give away his astounding performances even when we are doing our utmost to observe them. Readiness to make discriminations that one has never made before is a fundamental requirement if he is to clarify a novel problem successfully and elicit adequate suggestions for its solution.

Emphatic need of such readiness at step three

It is at step three of a piece of reasoning that this readiness to correct the past is most important. In examining the way of intuition it was noted that there is a natural human tendency, at least in matters of no immediate practical concern, to accept uncritically the first plausible suggestion that comes to mind. The most essential habit to be formed, if one is to protect himself against this tendency, is the habit of tentativeness—that is, of welcoming whatever suspense is necessary if the implications of a suggestion are to be carefully drawn and the suggestion put to a responsible proof. The great human weakness, the primary source of error in thinking, is oversuggestiveness —readiness to accept solutions without proper testing, because

they are emotionally congenial, because they are dinned into our ears by a skillful propagandist, because they are supported by tradition and authority, or for some other inadequate reason. It is of the highest importance to recognize that a suggestion has no logical claim upon assent till it has received adequate confirmation.

The meaning of "hypothesis" in science

When a scientist dealing with factual matters conceives a possible solution of a problem that challenges him, he views it not as an intuition, but as an *hypothesis*. This registers the fact that it is tentative—a candidate for acceptance—not a conclusion already definitely adopted. If it is a more complex idea which can only be adequately tested by many years of work on the part of many people, and if some preliminary evidence is secured in its favor, it may be called a *theory*. Should the hypothesis or theory be verified, it takes its place in the system of science as a *law*—that is, a regular relationship in a certain sphere of nature which we are justified in believing as true and adopting as the basis of further inferences. If the law thus established is so basic in a science or group of sciences that much or all of the other knowledge contained in them tends to organize about it and take a form deducible from it, it may function as an *axiom* [6]—that is, as a principle so well grounded that only persistent facts quite irreconcilable with it could compel its abandonment or radical modification. Thus the doctrine of the moon's gravitation, when it first occurred to Newton's mind, was a hypothesis; the idea of universal gravitation, when confirmed by evidence drawn from the moon's orbit, a theory; when the latter became verified by a study of the motions of many kinds of bodies, it stood as a law; and when seen to be so central in astronomy and mechanics that other knowledge in them could be fruitfully organized about it, an axiom. But no law, not even the most firmly held and seemingly certain axiom, may ever be properly regarded as absolutely final and unchangeable. Further facts may appear

[6] Since the traditional conception of axiom has now largely disappeared, I believe that it might be fruitful to reintroduce the word in the sense here given.

which, when fairly faced, demand its revision. The present radical changes which Newtonian physics has been compelled to undergo furnish an arresting illustration of this circumstance. If even the most assured results established by the most careful and painstaking methods of science are provisional rather than absolute, opinions which rest upon no such solid basis are much more likely to prove inadequate if not downright false. Accordingly, the priceless habit of tentativeness in opinion, and readiness to face new truth, for which the notion of hypothesis in science so definitely stands, is in the case of such ideas much more vitally needed.

The example just drawn from the history of medicine enforces the importance of this condition of good reasoning as much as it does the need of using at any time the best resources at hand. It is clear that if some of those whose lives were devoted to medical theory and practice had not dared to think in this spirit, the best doctors today would be no more scientific in their methods or effective in their results than the Egyptian doctors, or rather, even the rudiments of scientific procedure which the latter reveal as compared with the primitive medicine man would not have been gained. To meet every situation both in the light of the most competent knowledge and pertinent tools yet devised, and also in the light of possibilities not yet made actual by anyone—this is a hard combination to realize, but it is essential if thinking is to attain its highest level of effectiveness. There is usually more to be learned from others, and always more to be found than they have found, about any object of human interest.

6. Accommodating inquiry to the available time

A sixth mark of good thinking is *the habit of accommodating the time spent in reasoning to the importance of the problem*. This is necessary if thinking is to be wisely adjusted to the whole of life. The preceding considerations must not be supposed to imply that whenever one faces any problem he is responsible to achieve the best result that it is humanly possible to achieve. This would be quite fantastic. The appropriate guiding principle is that to some perplexities it is silly or worse to devote much effort, while others deserve the very

hardest application that can be given to them. How to tell, then, which perplexities should be placed in either class? Well, no general rule can be laid down; it depends on the individual's situation, interest, and the responsibility under which his particular abilities place him. Thus for one to spend time trying to find out how to build a small flower garden would be rather foolish, if almost any arrangement of flowers gives him pleasure. But the same would not be true of one who made of his garden a veritable hobby, still less of the professional horticulturist whose accepted social task is to discover and teach the best answers to such problems.

Merely temporal considerations often play a decisive part in an act of reasoning. If my problem is to decide how to spend an unexpected afternoon of leisure, it would obviously not be the part of wisdom to consume most of the afternoon pondering the question. Far better abandon the attempt to deal with the matter reflectively, and simply toss a coin, than let time decide by negating all the possibilities otherwise open. Time will decide every question for us, if we do not decide it in some more intelligent way. And of course many difficulties have to be solved rapidly and are so important that we must needs mobilize in whatever time avails the best thought at our command, even though its accomplishment may in retrospect seem very meager. Accident, or threat of accident, important decisions that must be made by a certain time but which affect our entire future career, are cases of this sort. When they come, our only resource is to bring to bear at once whatever knowledge and ideas we can muster, and think out a more fully defensible way of dealing with the issue in our hours of leisure—provided we survive. With problems of curiosity we are often able to take our time, but practical problems usually come with a definite temporal pressure. To give the proper amount of time to the proper things is a difficult art, but so fundamental an art that every thinker must do his best to acquire it.

While it holds true, however, that it is just as silly to spend too much time on an unimportant problem as it is to spend too

little on a really significant one, yet our inheritance from an impulsive race of animals causes us to fall into one of these errors far more frequently than into the other. We often decide matters hastily when we are not pressed for time and the issue deserves more lengthy consideration. As the preceding discussion has indicated, whenever it is clear that quick judgment is not necessary, and the matter is of moment, circumspection and delay are desirable.

7. Acceptance of responsibility to protect the social conditions necessary for effective reasoning

There is one more major condition of a general nature. It is clear from the portrayal in the preceding chapter of the social conditions which foster and protect erroneous notions, and from the practices of totalitarian governments in contemporary times, that a reasoner may be threatened at any time with the establishment of political or other institutions which will seriously obstruct his ability to use his reasoning powers to the best advantage. Those who direct these institutions may deny him access to the relevant facts; they may pour into his ears a flood of misleading propaganda unbalanced by contrary considerations; they may violate on principle his right to speak the truth as he sees it. Obviously, in view of this danger an intelligent thinker will fully accept the responsibility to play his part in *preserving the social conditions which are necessary to genuine freedom of thought and inquiry,* and if they should unhappily be lost he will do whatever he can to aid in their reestablishment. Thinking is not good thinking unless it expresses, among other things, an alertness to secure and protect the social matrix requisite if further effective thinking on the part of all in the community is to be freely engaged in.

This means that a good thinker must be alive to the social changes going on around him and to the policies proposed by his fellow citizens, being prepared to point out in what respects they threaten to weaken freedom of thought and expression; that he must be on constant watch against those who love and abuse power over others; that he must carefully discriminate between the pretense and the reality of devotion to liberty; that he must do his part to keep freedom from de-

generating into antisocial license; and that he must be ready, when such dangers become critical, to participate actively in movements aiming to bring them under control.

In general, it means that he must be an intelligent champion and protector of democracy in all human relationships, for we mean by democracy precisely that pattern of social life which maintains full freedom of inquiry as a cornerstone in the structure. In a democracy, growing toward more complete realization of its guiding ideal, reasoning finds the best environment for its effective functioning; in a nondemocratic social order it is inevitably under a serious handicap and may be unable to accomplish more than a small fraction of what it might otherwise achieve. A wise thinker cannot be indifferent to the virility or weakness of democracy as practiced in his community or in the world at large.[7]

Summary of Part I

In concluding Part I it will be well to resurvey the ground with which these six chapters have been occupied. After identifying thinking in its broadest sense, the first chapter discriminated the kinds of thinking which may be right or wrong, and exhibited the distinctive importance for a study of right thinking of the particular kind called reasoning—the systematic endeavor to solve a problem. In the second chapter this type of thinking was analyzed into its successive stages, and certain concepts fundamental to our enterprise were explained. The third chapter took advantage of this analysis to show the superiority of reasoning over other methods developed by Nature for meeting novel situations, and its virtue in comparison with the method of intuition, frequently followed by men and women in preference to it. Why reasoning is not infallible, and what the main factors in human nature are that account for the rise of inadequate beliefs, were the questions next attacked. The forces that perpetuate them, together with those that support progressive improvement in man's intellectual quest, were briefly described in Chapter 5. And, finally,

[7] In Part IV, pp. 620–629, 633 f., we shall engage in a fuller consideration of this theme.

the present chapter, after outlining the course of the investigations to follow, has described certain general conditions of good reasoning which deserve emphasis as conducive toward the solution of all kinds of problems.

EXERCISES

1. Surveying your experience for the past week, note on what occasions the mere factor of time settled your problems. In how many of these was nothing of importance lost by such a solution?
2. Describe and explain three important cautions which you would have in mind if it were your duty to cross-examine an eyewitness to an important event.
3. Write an essay on "Intellectual Habits: Their Use and Abuse."
4. Criticize the following hypotheses:
    a. The flowers on this grave were disarranged by some ghost.
    b. A stone falls when released because it wants to reach its proper place.
    c. John committed this crime because the devil tempted him.
    d. The grass grows upward because of a vital urge toward the sunlight.
    e. The stars in their courses make music for the angels, inaudible to men.

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PART II

THE FORMAL STRUCTURE OF REASONING

CHAPTER 7

THE NATURE OF IMPLICATION

Introductory considerations

From these conditions of good reasoning which can be stated without detailed analysis of the structure or methods of the sciences, we pass to conditions which cannot be formulated without such an analysis. In treating them, the order suggested by the classification in the preceding chapter of the various fields of investigation will be followed. It will be recalled that in this classification the formal sciences came first, then the factual sciences; and of the latter the so-called natural sciences preceded the social. Finally appeared the branches of thought not generally recognized today as sciences. The main advantage to be derived from following this order is that each of the later themes depends, in one important way or another, on the earlier ones; hence a statement of its appropriate methods presupposes an understanding of the methods examined earlier. No such relation of dependence obtains, however, in the reverse order.[1]

In the case of the relation between formal and factual science, analysis of the distinctive features of the former, and of the lessons which can thus be taught for the guidance of reasoning, naturally precedes a similar study of the latter. It does so because the conditions of formal correctness apply to all sorts of reasoning without exception, whereas the methods distinctively appropriate to factual science are not universally applicable; they do not apply to mathematical thinking, being pertinent only to the kind of reasoning in which our endeavor is to explain some puzzling event or to use the explanation for

[1] Except in one important respect, which cannot be adequately stated till we come to Part IV. See below, pp. 625 ff.

practical ends. The nature of a formal relation appears most distinctly when one analyzes the fourth step of any piece of reasoning. After it has been clearly apprehended one can see (as we shall in the next chapter) that all the steps are bound together by certain formal relations which must be respected if reasoning is to have any chance of solving its problems successfully. The structure of any course of reasoning, taken as a whole, thus reveals a formal aspect while providing the key to its understanding most openly at the fourth step. Moreover, the solution of problems in the factual sciences always involves—indeed, heavily emphasizes—the fifth step, that of the systematic verification of hypotheses. A statement of its distinctive methods, therefore, assumes as a matter of course that one who uses them knows how to respect the formal requirements of correct reasoning at the earlier steps. This more detailed part of our study, then, begins with an examination of the formal aspect of thinking and a formulation of the conditions of correctness which apply to all reasoning because of its presence. Our aim is to master the main lessons which may be learned from formal science for the sake of bringing our reasoning on any subject under more effective control. And, as would be expected from what has just been said, the key to this enterprise consists in an analysis of the implicative relation which is the distinctive feature of reasoning at the fourth step.

The essential meaning of the implicative relation

Consider two instances of the drawing of implications from a suggestion, using the linguistic form which is most simply and naturally resorted to for this purpose.

1. If the brace supporting this window box gives way, then the box will fall.
2. If Oklahoma City is due west of Memphis, then Memphis is due east of Oklahoma City.

Two important questions need to be asked about these assertions. First, just what is meant by the "if . . . then" which connects the antecedent and the consequent of each of them? And the answer is: It is meant that the antecedent and the

consequent are so related that the consequent will be true in case the antecedent is true. There is no assertion that either antecedent or consequent is as a matter of fact true; to take the first of these instances, there is no assertion that the brace has given way, or will give way, or that the box is destined to fall. The meaning is simply that *in the event of* its becoming true that the brace gives way, then it will *also* become true that the box will fall. In other words, the essential nature of the implicative relation is this: It is that relation in virtue of which the truth of the consequent is warranted by the truth of the antecedent, so that we may confidently infer the consequent from the antecedent.

Some needed definitions—*true, false, proposition*

Now this answer contains no words that have not appeared in the preceding chapters, but it is requisite to clarify some of them, since they are now to be used with a more precise meaning than has been needed hitherto. The word "true" has been employed; we must pause over it long enough to emphasize two considerations.

In the first place, just what does it mean? In Chapter 1 we defined it as meaning, essentially, agreement with the realities to which a judgment claiming truth refers. This is the meaning that has in general been traditionally assumed, and thus far it has led us into no difficulty. But we shall now give the word a narrower meaning. As will gradually appear in our subsequent analyses, there are at least three different kinds of realities with which a judgment asserted in the course of any given piece of reasoning may claim to agree: There are the relevant formal patterns whose study we have now embarked upon; there are the appropriate facts disclosable by some process of verification; there are also pertinent values. Now modern thought has found it necessary to distinguish these three kinds of realities rather carefully; the procedures by which one tells whether a judgment agrees or disagrees with the reality concerned are different according to which kind of reality is involved. As a result of realizing this need, it has seemed desirable to restrict the words "true" and "false" to cover agreement and disagreement in dealing with the second of these three

kinds only. Hereafter, then, we shall mean by a "true" statement its agreement with the verifiable facts to which it refers, and by a "false" one its disagreement with those facts. When we are thinking of agreement or disagreement with formal relations, or with values, other terms will be used, which will be introduced as we need them.[2]

In the second place, just how are "true" and "false" related to each other? It might seem superfluous to raise this question, but it is important to be very clear about it. The answer is that in their proper context these adjectives are intended to be exhaustive and mutually exclusive—that is, a statement must either agree with the facts referred to or disagree with them; there is no third alternative. We may not know which is the case, but one or the other is the case. The principle that an assertion must be either true or false is the basic Law of Excluded Middle, as applied to assertions. Thus, an assertion that is not true is false, and one that is not false is true.

But just what is it that must be either true or false? Thus far, assuming the broader use of these adjectives which is now to be abandoned, it has been taken for granted that any assertion, or judgment, or belief is properly characterized by one or the other of them. But if their meaning is to be limited in the manner just specified, the answer to this question must be expressed in a different, more technical term, which may be introduced in the following manner. Let us think, for the moment, of facts whose factual character has been established in the most dependable way, namely, by reasoning. In their case, what is found to be true or false is always a hypothesis proposed in the third step of the process of reasoning as its solution. Let us derive a noun from the participle "proposed" and call a *proposition* anything capable of being found to agree or disagree with fact in this way. Any proposition, then, will be either true or false. And, considered in this context, any proposition will also be capable of serving as ante-

[2] The term to be used in the case of formal relations will be introduced later in the present chapter; the one to be used in the case of values in chap. 27. See below, pp. 657 ff., 667.

cedent in the fourth step of a piece of reasoning, since it is the role of that step to connect the occurrence of the hypothesis with its verification in the fifth step by determining what consequents would be true if the antecedent is true, *i.e.*, what implications can be drawn from the antecedent. Hence this term has proved natural and helpful in a study of the implicative relation, too. If "proposition" is thus understood, any proposition can in the nature of the case fill the two roles which, as was noted in Chapter 6, any good hypothesis must be able to fill—namely, (1) serve as premise from which implications can be drawn, and (2) guide some verifying action or observation aiming to find out whether or not it squares with the facts to which it refers. It is the first of these two roles that we are now to study in some detail.

It is worth noting at once that there are compound propositions as well as simple ones—that is, propositions containing other propositions as their constituents. Just as "The brace supporting this window box has given way" and "The window box is falling" are both propositions, since one can easily think of problems in which they would appear as verifiable hypotheses, so the compound assertion: "If the brace gives way the window box will fall," is also a proposition, since it too might function as a proposed solution of a factual problem; someone might raise the question, what else would happen in case the brace should give way, and this proposition would offer an answer.

From these considerations it is evident that the same group of words may stand for either a sentence, an assertion expressing a judgment or belief, or a proposition. When it is taken merely from the grammatical point of view, it is a declarative sentence. When it is taken in its relation to somebody who affirms it, it is an assertion expressing a judgment. When it is taken as a possible solution of a problem dealt with by reasoning, and hence as ready to play the roles which any such entity must play, it is a proposition.

Inference and implication

The word "infer" has also been used; its distinction from and connection with "implication" must be carefully noted.

Inference is a mental process; it is the passage in reasoning from an antecedent to its consequent. *Implication* is a logical relation between propositions; it is that relation in virtue of which the truth of one proposition is warranted by the truth of another.[3] The two are, of course, closely connected. Implication is the relation between two propositions which justifies an inference from the first to the second. Inference is correctly performed only when a genuine implication obtains between antecedent and consequent. But it is essential to avoid confusing them. For instance, one would never infer from "Oklahoma City is west of Memphis," to "Oklahoma City is west of Memphis"; since antecedent and consequent are identical, it would be quite superfluous to pass in thought from one to the other. But it is entirely correct to say that "Oklahoma City is west of Memphis" implies "Oklahoma City is west of Memphis," for it is evident that the truth of the antecedent does warrant the truth of the consequent. There may thus be implicative relations between propositions which will never be used as guides to inference,[4] just as there are many cases of inference where there is no implication—namely, all the inferences that are erroneous.

This difference between inference and implication leads to a further important distinction, in virtue of which it will no longer be necessary to depend on the loose terms "antecedent" and "consequent." When, in any case of implication, one is considering the logical relation as such, the antecedent is the *implicans* and the consequent its *implicate*. When, on the other hand, one considers their actual or possible employment in a piece of inference, the antecedent is a *premise* while the consequent is its *conclusion*. Any inference may be mistaken, the conclusion not really following from its premise; but if the truth of an implicate does not really follow from the truth of its implicans it is not an implicate at all.

[3] Heretofore, following common usage, this word has been more loosely employed. Often it has meant, for example, "a consequent inferred from an antecedent." Hereafter it will be restricted to the technical meaning above defined.

[4] Except, of course, when the nature of formal relationships itself constitutes the problem studied.

Now a second question must be asked about our two illustrative instances. How do we know that the truth of the conclusions is warranted by the truth of the premises? Suppose that someone, hearing or reading these inferences, should challenge us to justify them—should intimate that we might be reasoning erroneously in drawing them. How would we meet this challenge?

How does the truth of the premise warrant that of the conclusion?

In the first case our justification would be: "Why, any unsupported object heavier than air near the surface of the earth falls; therefore if the support be removed from this window box it will fall." In the second case we would say: "Whenever one place is west of another the other must obviously be east of the one. It couldn't be otherwise." Let us examine these two ways of justifying an inference, noting their differences and also their similarities.

The first justification involves an appeal to certain facts which, it is assumed, somebody has observed and recorded; they are summarized and generalized in the scientific law regarding the gravitative attraction of the earth. Whoever draws the inference has also identified some particular fact as belonging to the class of facts to which this general law applies—that is, he regards the window box as an object of the sort that should be affected by the earth's gravitation. Finally, there is the supposition that what holds generally about a matter of this kind will hold of a new instance which is of the same kind; what is true universally about unsupported objects must be true about this particular object if it becomes unsupported.

The second mode of justification is quite different, in that no appeal to observable facts seems to be necessary here. The very meaning of the phrase "west of" tells us, contemplating it, that if one place is west of another the other must be east of the one. "West of" and "east of" are intrinsically so related that a truth expressed in terms of one of these phrases may, with obvious legitimacy, be inferred from a truth expressed in terms of the other. Another technical word must be introduced here: "Oklahoma City is west of Memphis" not only implies "Memphis is east of Oklahoma City"; there is an implicative relation of a special kind between these two proposi-

tions. It will be called *strict* implication. Given the truth of the implicans, the implicate not only *is* true; it *must* be true, let facts—geographical or any other—be what they may. The connection between them, in other words, is logically necessary. That this is no overstatement is evident if we suppose "Oklahoma City is west of Memphis" to be false instead of true—not to agree with the geographical fact which it purports to describe. Then, clearly, "Memphis is east of Oklahoma City" must be false also. These two propositions are both true or false together; no fact can be conceived in virtue of which one of them might be true and the other false. This circumstance shows definitely that their connection is determined by their own meaning, and is entirely independent of any facts which we might describe by affirming one or the other of them.

Strict versus material implication

Now let us return to case 1 in the light of this "must be" which seems to characterize the implicative relation exemplified in case 2. "If the brace supporting this window box gives way, then the box will fall." Is there any such necessity present in the relation between premise and conclusion here? Well, the conclusion does not follow merely from the meaning of the words used in the premise. As has just been seen, the relation between them can be justified only by appeal to the facts described by these clauses. And the preceding chapter has shown that, in the past, exceptions to laws about the behavior of facts that had previously appeared universally true have occasionally been met. In view of these experiences are we not forced to admit that it is always possible for future facts to exhibit different ways of behaving from those which our past familiarity with similar facts would lead us to expect? There *may* appear exceptions even to such a well-attested generalization as the law of gravitation—which means that it cannot be pronounced strictly impossible that sometime a window box might lose its support, and yet not fall. If this is so, a profound difference obtains between inference 1 and inference 2, consisting in the presence of an intrinsic necessity connecting premise and conclusion in 2 and its absence in 1. The implicative connection illustrated in case 1 is factual

merely; what is meant is that if the premise is true the conclusion, merely as a matter of fact, is true. No claim is made that it must be, that it cannot but be, true. Following current usage, this relation will be called *material* implication, and its difference from strict implication needs always to be kept in mind. It may be noted that whenever two propositions are related by strict implication they are also related by material implication, whereas the reverse principle does not hold. Whatever must be true is true, but not all propositions that are true are so because of any logical compulsion. So when, given the truth of a premise, a certain conclusion is necessarily true, it is also true as a matter of fact; but if the truth of the conclusion is implied by the truth of the premise merely as a fact, one may not assume that it is also implied as necessarily true.

But suppose that case 1 is enlarged by adding to its premise the assertion which was offered when the question, how the inference is to be justified, was faced: "Any unsupported object heavier than air near the surface of the earth falls." Suppose we assume that this added assertion is true, and combine it with the original premise into a single compound proposition which now functions as premise of the inferred conclusion. In that case our inference becomes the following: "If any unsupported object heavier than air near the surface of the earth falls, and if this window box becomes such an object; then it must fall." Now a transformation has taken place. With this change, case 1 has become like 2, as the introduction of the word "must" in the conclusion indicates. A strict implication now connects premise and conclusion, and the latter follows from the very meaning of the former just as "Memphis is east of Oklahoma City" followed from the meaning of "Oklahoma City is west of Memphis." The truth of the premise still depends on external facts in both cases, but the relation between premise and conclusion no longer depends on such facts. If the former is true the latter must be true.

The central questions of Part II

It appears, then, that anyone's reasoning at the fourth step of a problem-solving process employs as its guide either the

relation of material implication or that of strict implication. And it is further evident that whenever the assertion of a material implication is to be justified, or tested for its correctness, it must be transformed so as to become a case of strict implication—a case claiming that premise and conclusion are bound together by logical necessity. What sort of affair, then, is this relation of strict implication? What is the secret of the necessity which it exhibits? How does one tell, when engaging in any piece of inference, whether he is conforming to its demands or not? In the case of different kinds of inference, are there corresponding forms of strict implication to which a thinker can appeal in order to check the correctness of the inference? Are these forms, should there prove to be such, systematically interconnected, and if so in what manner?

These are the questions to which answers will be sought in Part II. And it is already clear that whatever answers are reached will play two roles in our study as a whole. On the one hand they will constitute a solution of the problem: What distinctive conditions must be respected in virtue of the formal structure of an act of reasoning? On the other hand, they will provide an introductory understanding of the nature of formal science and the contribution which it can offer for the guidance of human thought. For the essential difference between formal and factual science lies precisely in the circumstance that while, in the latter, implicative inference is only a part of the process which remains quite incomplete in the absence of appeal to factual material, the former consists merely in tracing a chain of necessary connections of concepts without any dependence on questions of fact. Inferences in factual science must meet a twofold standard—(a) Are the conclusions strict implicates of the stated or assumed premises, or are we deceived in believing them to be so? (b) Do they agree with the observable facts which they purport to describe? Formal inferences need to satisfy the first of these tests alone. If they satisfy it they hold good irrespective of what facts may or may not occur.

The rest of the present chapter will lay the foundations for

an answer to these questions, and the ensuing chapters will build the superstructure in such detail as seems necessary.

Again there is need of a few more technical terms before we advance further. A true proposition has been defined as one which agrees with the relevant facts. In view of the distinctive nature of implication, and especially of the formal necessity which makes strict implication radically different from facts, it is desirable to have a special term by which to refer to the agreement of an inference with the implication whose nature it must respect if it is to be correct. The term which has gained currency for this purpose is "validity." An inference is *valid* if such agreement obtains—that is, if its premise does imply the conclusion. It is *invalid* otherwise.

The conception of *validity*

It is essential to keep in mind, however, that truth and validity are related in a definite way. An implication obtains between two propositions, and therefore the inference of one from the other is valid, when and only when the truth of the second is warranted by the truth of the first; implication is significant for thinking because of this circumstance. When one engages in a piece of serious reasoning he is always seeking truth. But the truth of a proposition may be discovered either directly or indirectly. Sometimes its presence or absence may be told by directly observing the facts which it describes. Sometimes this can be done by seeing that it is implied by another proposition (simple or compound) which is known to be true. Ordinarily the latter method is more economical than the other, hence preferable whenever it can be used. (Just consider how timesaving it is to be able to affirm at once that Memphis is east of Oklahoma City when it has been established that Oklahoma City is west of Memphis, without needing in both cases to go through the process of factual verification.)

Its relation to truth

With the difference and interconnection of truth and validity both before us, it will occasion no confusion to note that a *conclusion* is both true or false and valid or invalid. When it is pronounced true or false, what is meant is that it is correct or incorrect as a description of the facts which it purports to

describe. When it is pronounced valid or invalid the reference is to its being or not being implied by the premises from which it is drawn. And it is evident from the above discussion of the relation of implication to truth that there is only a single necessary connection between its status in one of these respects and its status in the other. Let us see how this is so, remembering that a validly drawn conclusion may be either true or false; likewise one invalidly drawn.

1. Valid inference and true conclusion:
   Oklahoma City is west of Memphis, hence Memphis is east of Oklahoma City.
2. Valid inference and false conclusion:
   President Truman is younger than Mr. Stettinius, hence Mr. Stettinius is older than President Truman.
3. Invalid inference and true conclusion:
   Some sufferers from disease are liquor drinkers, and some jail inmates are liquor drinkers; hence some jail inmates are sufferers from disease.
4. Invalid inference and false conclusion:
   The Capitol building is larger than the White House, hence the White House is equal in size to the Capitol building.

In the case of the invalid inferences in this list, the premises happen to be true. But those premises might have been false; one can draw inferences from a false proposition as easily as from a true one if he understands what it means. The most instructive case in the list, however, is the second. Here the conclusion is validly drawn but false; notice that the premise from which it is drawn is false too. Try to conceive a case where the conclusion is validly inferred and false but whose premise is true! This attempt will always fail because validity has been defined as requiring that the conclusion be implied by the premise, and this means, in turn, that there is such a relation between them that if the premise is true the conclusion is true. Hence the impossibility of validly deducing a false conclusion from a true premise.

The quest for validating forms

With these technical terms available, we may come to closer grips with the central questions of Part II which were formulated on page 112. We wish to find the secret of the logical

necessity essential to the relation of strict implication. And in view of its intimate connection with the difference between correct and incorrect inference, we wish to exhibit that implicative necessity in such a guise as will provide systematic tests to be applied to our inferences. It is best to commence with the latter of these two pursuits, since its outcome will supply materials needed for an answer to the former. Already it is evident that inferences are not all of the same kind. The illustrations discussed earlier in the chapter are of two kinds, each revealing important differences from the other, and it will soon appear that there are distinguishable species within each of these kinds. If each of these is capable of being valid or invalid, there must be a corresponding pattern of strict implication which provides a *validating form* for that type of inference—that is, a form such that any inference of the sort in question which exemplifies it is valid, while any which fails to exemplify it is invalid. The phrase "logical form" is also used as a synonym for "validating form," and the systematic study of such forms is often called "formal logic."[5]

Such a form illustrated

What would such a validating form be like? Well, consider again the first of the two illustrations of inference given on page 104, as expanded so that a strict implication obtains between premise and conclusion. "If any unsupported object heavier than air near the surface of the earth falls, and this window box becomes such an object; then it must fall." Suppose that, without regard to whether the propositions affirmed are true or false, we introduce certain variations into the premise of this deduction, making also what changes in the conclusion are necessary in order to keep the inference valid.

1. If any unsupported object heavier than air near the surface of the earth falls,
   And this window box becomes such an object;
   Then it must fall.

[5] I wish I might avoid any controversial definition of the word "logic." As the reader who pursues this subject further will discover, some identify logic with what is here called formal logic; others would add to its field a critical consideration of methods appropriate to the explanation of facts; while still others would allow it to include the entire realm dealt with in the present book. I shall use it in the broadest sense.

2. If any unsupported object near the surface of the earth rises,
   And this window box becomes such an object;
   Then it must rise.
3. If any meteor falls,
   And the object at which I am looking is a meteor;
   Then it must fall.
4. If any ghost makes frightening gestures,
   And the object at which I am looking is a ghost;
   Then it must make frightening gestures.
5. If any truth wins out in the end,
   And this doctrine is true;
   Then it must win out in the end.

Comparing any two of the first three cases, we find obviously that there is much in common between them; they have the same logical structure. On careful examination it is clear that cases 4 and 5 have the same structure too, although this is less obvious because there is not so much in common between them and the first three. Let us try to detach the valid structure which these five cases share from their individual differences and express it clearly.

> If any (entity of a certain kind) (acts in a certain way),
> And (this particular thing) is (an entity of that kind);
> Then (this particular thing) must (act in that certain way).

Here is an instance of what is meant by a validating form for a certain type of inference. It is just the implicative structure exemplified in common by all valid inferences of that type. And their validity evidently depends upon the fact that they share it in common, for one can see that any inference exemplifying that structure must be valid. Given the truth of any such premise, the truth of the corresponding conclusion necessarily follows.

And concisely symbolized

In the interest of brevity, and in order to reveal still more clearly and precisely the nature of this validating form, let us represent each essential part of it by an appropriate symbol. It will be requisite for the reader to master these symbols so that he can use them freely as devices by which to state any lessons that may be reached in the detailed analysis of strict

implication on which we have embarked. We are familiar in algebra with the use of symbols to represent unknown quantities; the symbols now to be introduced play a similar role here.

Leaving aside for the time being the meaning of the words "if" and "any," we may begin with the phrases first appearing in these five inferences. What is shared in common by: "unsupported object near the surface of the earth," "meteor," "ghost," "truth," in virtue of which they may be referred to as "entities of a certain kind"? In technical terms, each of these is a *class* consisting of an unspecified number of individual *members*. That is, "meteor" refers to the class constituted by all particular meteors that have existed or may exist; and each of those meteors is a member of that class. Similarly, "ghost" indicates the class constituted by all particular ghosts, and likewise with the other class-names "unsupported object" and "truth." Each of these refers to all entities of a certain kind, and this is precisely what is meant by saying that they indicate all members belonging to a given class. We shall use the early letters of the alphabet—*a*, *b*, *c*, etc.—to symbolize any classes about which we may be reasoning, and the end-letters —*x*, *y*, *z*—to symbolize any particular members of such a class that may be referred to.

What, next, is shared in common by the ideas: "falls," "rises," "makes frightening gestures," and "wins out in the end," in virtue of which one can refer to all of them as "acting in a certain way"? Well, the phrase is probably self-explanatory, but in technical parlance each of these modes of action is a *property* [6] which may be exhibited by the members of a class. Thus "falling" is thought of as a property exhibited by such classes as unsupported objects near the surface of the earth and meteors; "making frightening gestures" as a property characteristic of ghosts; and "winning out in the end" as a property possessed by truths. Let us symbolize any such mode of action, or property, which the members of a class

[6] This word is here used in its modern sense rather than strictly in that of Aristotelian logic.

may display, by letters in the middle of the alphabet—*f*, *g*, *h*, etc.

We may now express in these symbols the validating form which the above five inferences exemplify.

> If any *a* has *f*
> And *x* is an *a*,
> Then *x* has *f*.

Later, this process of symbolic representation will be continued until no English words are left in such formulations, but for the purpose of introducing the notion of validating form these transformations are sufficient. This expression constitutes a symbolic equivalent of the abstract description given on page 116. It must be understood, of course, that while in this formulation *a* may refer to any class whatever, it must refer to the same class in the two parts of the inference in which it occurs, just as in algebra any particular symbol must refer to the same quantity throughout any given problem; similarly, *x* in the premise and the conclusion must refer to the same class-member, and *f* to the same property. Otherwise it is evident at once that the form has no validity. If in the same formula a second class needs mention, it will be symbolized by *b*, and a third by *c;* similarly, a second class member will be indicated by *y*, and a second property by *g*, etc.

Another validating form symbolized

For purposes of further clarification let us examine in the same way the second of our original pair of illustrative instances.

> If Oklahoma City is west of Memphis,
> Then Memphis is east of Oklahoma City.

Compare this with two other cases belonging to the same kind of inference:

> If Chicago is north of Kankakee,
> Then Kankakee is south of Chicago.

> If Miami is southeast of Tampa,
> Then Tampa is northwest of Miami.

In these inferences one is reasoning about geographical locations or areas, and the directions in which they may stand to each other. The general structure which they all exhibit may be expressed in ordinary English as follows:

> If (one place) lies in a certain direction from (another place),
> Then (the other place) lies in the opposite direction from (the one).

How is this validating form to be symbolized? Let any two places be represented by the symbols x and y, and any geographical direction by Rd (meaning "the relation of direction"). The opposite direction will then be symbolized by $\mathrm{Rd_o}$. The validating form may then be expressed as follows:

> If x Rd y, then y $\mathrm{Rd_o}$ x.

A more general form which includes it

Indeed, this validating form readily suggests one which is more general still. The relations with which we have just been concerned are those of geographical direction merely. But the principle thus symbolized applies to many other relations as well. If one thing is greater than another, then the other must be smaller than the one; if it is older, then the other must be younger; if it is more beautiful, then the other must be less beautiful, etc. Any relation which has an opposite—a *converse* relation, in technical parlance—exhibits the same structure. Symbolizing now by R any relation which has a converse, its converse by $\mathrm{R_c}$, and any two entities capable of standing in such a relation by x and y, we have the following expression for a more general validating form:

> If x R y, then y $\mathrm{R_c}$ x.

What is the secret of implicative necessity?

With these validating forms in mind, each concisely symbolized, let us attack the postponed question: What is the secret of the necessity which binds implicate to implicans in each of these cases? Just why must an inference that exemplifies any of these forms be valid?

Earlier in the chapter it was noted that truth and falsity are

mutually exclusive and exhaustive properties of any proposition; if a proposition is true it is not false, and if it is false it is not true. Now we must express this principle precisely and symbolize it. Assert any proposition at random; then it follows that the denial of that proposition must be denied. This is the same as saying that the truth of any proposition strictly implies the falsity of its denial, which would be expressed in a contradictory proposition. Illustrations:

1. If John's blue suit is equal in value to his overcoat,
   Then John's blue suit is not unequal in value to his overcoat.
2. If most novels are interesting,
   Then most novels are not uninteresting.

In these inferences the premise is flatly contradicted by use of the prefix "un" in "unequal" and "uninteresting,"[7] and then this contradiction is denied as false, the denial being symbolized by the word "not." Clearly any such denial is necessarily implied by the original proposition. We shall symbolize any proposition under consideration by one of the letters p, q, or r, and the contradictory of any such proposition by p', q', or r'. Then the principle just analyzed will be expressed as follows:

If p, then not p'.

Answer: the principle of noncontradiction

Suppose that the validating forms presented earlier are examined in the light of this principle. Its essence is simply that if anything is asserted it may not at the same time and in the same realm of discourse be denied—or, which is equivalent, its contradictory may not be asserted. This is the basic Law of Contradiction, as applied to propositions. It is illegitimate, in brief, to contradict oneself, and the assertion, hence, of any proposition as true implies the rejection of its contradictory as false. In some situations, of course, self-contradiction is permissible; we should obviously not wish to insist that in all circumstances the poet, the dramatist, or the teller of fairy tales must avoid inconsistencies. But when the serious pursuit of

[7] Linguistic usage sometimes requires other prefixes, such as "im-," "in-," "non-," "a-."

truth is in question no such freedom can be allowed, and for the simple reason just pointed out. The acceptance of a proposition as true is incompatible with its simultaneous rejection as false, and therefore with the acceptance of its contradictory as true. One could make no headway toward reaching an answer to any problem, at any stage of an act of reasoning, if he failed to respect this principle. Suppose that I identify a puzzling object as a path and also as not a path; how could my inquiry proceed further? If I am drawing the implications of a suggestion, how would I know what to do for its verification were I to deduce from it both that the path should have a certain direction and also that it should have a different direction? What would I have accomplished at the end of a course of reasoning, if my conclusion is that a certain solution is to be adopted and that the same solution is to be rejected? The principle that in seeking truth one must avoid self-contradiction in his assertions would seem to be sound intrinsically and very fundamental.

Consider now the earlier validating forms, such as: If x Rd y, then y Rd_o x. Let us try to retain the premise and deny the conclusion: If Oklahoma City is west of Memphis, then Memphis is not east of Oklahoma City. Is it not clear that this would be a self-contradiction just as much as though the conclusion were: Oklahoma City is not west of Memphis? For "west of" and "east of" are intrinsically related in virtue of their meanings, just as "west of" and "not west of," or "true" and "false." To say, then, that x Rd y, and to deny that y Rd_o x, is essentially the same as to say that a proposition is true and in the same breath that it is false. Similarly with the validating form: If any a has f, and if x is an a; then x has f. Suppose that this conclusion is replaced by its denial: Then x does not have f. Is it not clear that this now contradicts the premise? For, if this conclusion is to be maintained, and inconsistency avoided, one of the two propositions combined in the premise must now be denied. If a window box does not fall, and it is still the case that any unsupported object falls, then the window box cannot be unsupported. Or, if the window box does

not fall, and it is still the case that the window box is unsupported, then it cannot be true that all unsupported objects fall.

The principle that two contradictory propositions cannot both be true is thus disclosed as the basic principle underlying any validating form, and providing the key to an understanding of strict implication. When one validly affirms the truth of a conclusion as strictly implied by the truth of a premise, the relation between the two is always such that refusal to affirm that conclusion would contradict the previous affirmation of the truth of the premise.

A validating form is thus a partial or complete tautology

To express the same principle in positive terms, a validating form is always a case of partial or complete *tautology*. Two propositions are tautologous when they are formally equivalent. Obviously, when this is the case with a premise and its conclusion, refusal to accept the conclusion is the same as refusal to accept the premise, and would contradict acceptance of the premise. That this is the situation with the principle of noncontradiction as formulated above is readily seen when it is noted that two negatives are equivalent to an affirmative—denying the falsity of a proposition is equivalent to affirming the truth of that proposition. Hence

If p, then not p'; is equivalent to:
If p, then p; which is clearly a tautology.

A validating form, on the other hand, such as:

If any a has f, and x is an a;
Then x has f—

is a partial tautology. Since x, y, z, etc., are the members which, taken together, constitute class a, a complete tautology here would be:

If any a has f, then x, y, z, etc., all have f.

With the conclusion: x has f, the tautology is partial only, since this merely affirms about one member of a what the premise affirms about all members of a.

Now in popular parlance "tautology" often means "needless repetition." Does it mean that here? No; the significance of implication in reasoning would be seriously misunderstood were we to suppose so. How it can be significant and at the same time tautologous will be brought out best if we recur at this point to the difference between inference and implication, and introduce one more technical term—"entailment."

Inference and implication again

Inference always occurs in someone's attempt to solve a problem, and is guided by the requirements of that problem, whereas implication, being a formal relation between propositions as such, is not a psychological affair at all. Hence it will not surprise us to note that any pair of premises strictly imply many possible conclusions besides the conclusion ordinarily drawn from them. Let us take the premises already employed on more than one occasion—If any *a* has *f*, and *x* is an *a*—and list a few of the conclusions that might be validly drawn should anyone be disposed to infer them. The conclusion earlier drawn was

1. Then *x* has *f*. Other propositions also strictly implied are
2. Then *a* has *f*.
3. Then *x* is not a non-*a*.
4. Then either *x* has *f* or it has *g*. (If a member of a class has a certain property, then it obviously must belong to the larger class which has either that or some other property.)
5. Then *x* has *f*, and either this tree is an oak or it is not an oak.
6. Then *x* is an *a*, and *q* implies not *q′*.

It is doubtless unnecessary to expand this list further. Here is clear indication that one may not interpret the necessity with which an implicate follows from an implicans as meaning that, given an implicans, he is bound to any particular implicate. If a certain pair of propositions are true, an indefinite number of other propositions can be seen by deduction to be necessarily true on their warrant.

Why, then, in any valid inference, does one select the particular implicate that he does? Because it is the one in which at the time he is interested; it is the implicate most obviously relevant to the problem with which he is concerned. Hence

he infers it and neglects the others. And why, in our validating forms, did we choose just one implicate instead of listing at least several of the implicates that follow from the implicans? Because, in all but special cases, when an inference assuming that kind of implicans is embarked upon, the problem is such that this implicate is the one that promises to help solve it. Now a validating form is not an attempt to formulate the complete network of implication in which any given implicans is imbedded. Its role is to be applied as a test of validity to a certain kind of inference. So wherever inferences are normally interested in a particular sort of conclusion rather than in others that are formally permissible, their validating form justifiably selects that specific implicate and omits the rest. From among the complete or partial tautologies available, this is the one chosen.

Implication and entailment

Still, if they are tautologies, why does reasoning need any of them? It wants to make progress, not just to repeat in a conclusion what is already contained in the premises. The answer is that although a valid conclusion is formally contained in its premises we do not see whether it is or not until we have put the premises together and considered what can be drawn from them that is relevant to our problem. They do not combine of themselves. Thus it is by using tautologies that reasoning makes progress at the fourth step toward its goal.

The circumstance that valid inference employs tautologies and at the same time engages in no needless repetition is registered in the term "entailment." [8] Consider the last two of the implicates just listed, strictly implied by the implicans "If any *a* has *f*, and *x* is an *a*." The first part of each of these propositions is obviously implied; on what ground can we add to it such a proposition as, "This tree is either an oak or not an oak," and hold that the two parts together are strictly implied by the implicans in question? Because "This tree is either an oak or not an oak" necessarily holds anyway; no

[8] I borrow this word from Mr. G. E. Moore, although in his usage it is not disentangled from what is here called "strict implication."

fact can be conceived, in the presence of which it is capable of being false. Any tree must either be an oak or not an oak. Hence such a proposition is strictly implied by any other proposition under the sun; given any proposition whatever, this proposition is necessarily true. The same is the case with "q implies not q'." But nobody would ever be interested in such implicates as these, except a formal logician whose concern is the systematic study of implication. No thinker, confronted by any other kind of problem, would make progress by using them; since they must be true, given any premise that might ever come to anybody's mind, they offer no special help in reaching a judgment about the particular premise with which he is occupied. They are equally pertinent (and therefore equally useless) in all problems save those of formal science as such. Now a conclusion implied by a premise is also *entailed* by that premise, when it is specifically relevant to the problem of which that premise (or part of it) had presented itself as a possible solution. In practice this means two things.[9] On the negative side, an entailed conclusion will omit all implicates of its premise that are equally implied by any other premise (and are therefore equally valuable or useless in all problems), as well as implicates that are merely identical with the whole or a part of the premise (and therefore do not enable reasoning to progress beyond the point it has already reached). On the positive side, it must contain some common elements with its premise, since otherwise continuity between the original problem and the conclusion would be lost. Thus the validating forms which have been examined in the present chapter are validating forms of entailment as well as of implication, and in the following chapters the same restriction will control the analysis.

It will be well to relate this distinction to others with which it is closely connected, by a summary statement. Inference is the mental process of drawing a conclusion from a premise. If the inference is valid, that conclusion is a logical implicate

[9] Barring, of course, problems concerned with the nature of formal relations, in which any implicates may be relevant.

of the premise as implicans. In case the implicate is true merely as a matter of fact, given the truth of the implicans, the relation is one of material implication; in case it is necessarily true, the relation is one of strict implication. And an implication is also an entailment when the meaning of the implicate is specifically relevant to the premise, and carries the inquiry forward on the problem which is under attack.

The following chapters examine in order the more important kinds of inference in which reasoners engage, setting up in each case appropriate validating forms, and noting the main errors against which it is especially helpful to be on guard. They will begin with types of inference in which implicates are derived from propositions taken as unanalyzed wholes, passing later to types in which the validating form cannot be stated without some analysis of the propositions involved into their elements.

EXERCISES

1. Give as clear an explanation as you can of the syntactic meaning of each of the following words which have been used in the chapter: formal, factual, implication, inference, antecedent, consequent, premise, conclusion, implicans, implicate, proposition, true, false, validity, validating form, symbol, class, member, property, converse relation, contradiction, entailment.
2. State in the case of each of the following inferences whether you think it is valid or not:
   a. Comets must be material, for they obey the laws of gravitation and all substances which obey this law are material.
   b. Tom is a better scholar than James, for Tom is better than Hugh, and James is at least no better than Hugh.
   c. Nothing is better than wisdom; a crust is better than nothing; hence a crust is better than wisdom.
   d. You voted for Governor R., and Governor R. appointed Judge T.; hence you must be in favor of Judge T.
   e. Some statesmen are also authors, for Gladstone, Disraeli, Roosevelt, and Wilson were both statesmen and authors.
   f. In moral matters one may not stand still; hence if one does not advance he is sure to fall behind.
   g. You are not what I am; I am a man; therefore you are not a man.
   h. If God created the world and God only creates what is good, then the world must be good.

i. If truth is stranger than fiction, then fiction is not as strange as truth.
j. All boys are mischievous, so all mischievous individuals are boys.
k. If grass is green, then it is either blue or not blue.
l. Gold is valuable; hence it is not invaluable.

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CHAPTER 8

SOME MOLECULAR VALIDATING FORMS

The molecular analysis of propositions

Of the validating forms used for purposes of illustration in the preceding chapter only one—that expressing the law of contradiction—can be stated merely by considering implicans and implicate as unanalyzed propositions. In the others both implicans and implicate were analyzed into their logical elements, each of which was specifically noted. This difference is shown in the symbolic expression of these forms. The principle involved when a proposition is contradicted was stated in terms of p and p'—p symbolizing the implicans, and p' the implicate, taken as unanalyzed wholes. In the other cases each proposition was broken up into its parts which were separately symbolized. Thus, in the form earliest discussed, a class, one of its properties, and one of its members, were all separately discriminated and given a symbolic designation; the form could not have been stated without doing this. Similarly, in another illustration the geographical places involved, together with the relations of direction between them, had to be specifically taken into account and symbolized.

Two general kinds of validating form have thus already been implicitly distinguished. The first, illustrated by the form employed when the idea of contradiction was examined, consists of *molecular* validating forms, and the study which issues in the establishment of such forms is called the molecular analysis of propositions. The second, illustrated by the other cases, consists of *atomic* validating forms, and the sort of inquiry by which they are reached and justified is called the atomic analysis of propositions. The origin of these adjectives

is easy to explain. The physicist, for many of the purposes of his science, finds it unnecessary to take into account the properties of matter which require for their exhibition analysis into constituents smaller than the molecule, while the chemist typically must take into account the internal structure of the molecule and the nature of the atoms of which it is composed. The analogy with the formal analysis of propositions which now occupies us is obvious.

We begin with the molecular examination of propositions. From what has just been said it is evident that this will consist in a systematic consideration of those relations which may obtain between unanalyzed propositions in virtue of which the assumed truth or falsity of one has a bearing on the truth or falsity of the others. Two propositions may, of course, be logically *independent*. They are so when the truth or falsity of neither is limited by the assumed truth or falsity of the other. "Syracuse is east of Buffalo" and "This is a rainy autumn" are obviously independent propositions. But with many pairs of propositions independence does not obtain. How may p and q be so connected that the truth-status of one is affected by the truth-status of the other? What validating forms involving these connections can be set up, and how shall they be symbolized? What main fallacies, *i.e.*, logical errors, is one likely to fall into when engaging in inferences tested by these forms? Such are the questions to which answers will now be sought.

The logical constants

In practice, what this inquiry consists in is an exploration of the meaning of the so-called "logical constants," and of the way in which they are interrelated. These logical constants are the connectives by whose presence propositions affect each other's truth-status. Four of these connectives play a quite fundamental role in this regard. In order to understand why they do, and why the list includes just these four, we must return to the analysis of a complete piece of reasoning in Chapter 2. The connectives in question are links by which the successive propositions which constitute a chain of reasoning are so bound together that they all become relevant to

the problem confronted by the reasoner and guide it toward a solution. Let us see how this is so.

Consider the second step of a course of reasoning, as illustrated by the problem there selected. Endeavoring to clarify the object that perplexed me I said: "This is a path, *and* it is a narrow one, *and* it leads to some place which people wish to reach." As suggested solutions came to my mind at the third step, I expressed their connection with each other by saying: "The path leads to the lake, *or* it leads to a spring, *or* it is a short cut to another part of the road." Developing the implications of these suggestions at the fourth step, I said: "*If* it is a short cut to another part of the road, *then* it will have the appropriate direction." And in the course of carrying out the final step I found myself forced to conclude: "It is *not* the case that it is a short cut, *and* it does *not* lead to a spring." Only when the chain of propositions had reached this point, by being bound together in this way, was it possible to establish a definitive solution, in the presence of positive evidence supporting the remaining hypothesis—that the path led to the lake.

It is evident from such a typical case that the connectives, "and," "or," "if . . . then," and "not," play an indispensable role in the sequence of propositions by which a course of reasoning moves toward a solution. And it is because each plays a constant role in all problem-solving enterprises, no matter what variety these may show in detail, that they are called logical constants. They express stable ways in which one proposition affects the ability of another to provide an acceptable answer to the problem faced or to contribute toward its discovery. Clearly, then, no one can reason correctly unless he knows how to use these constants aright—knows just how each affects the truth-status, under stipulated conditions, of the propositions it connects. So, we are now going to ask: What does each constant strictly imply about the propositions which it connects, when it is employed in the context of any course of reasoning?

We shall want to carry forward this analysis by the aid of

appropriate symbols. Hence we shall need from the very beginning a symbol for the relation of strict implication itself, in virtue of which, given the truth of a proposition p, we know that another proposition q is necessarily true. It will be examined later along with material implication and has, indeed, already been explained. But the accepted symbol for it is $\prec$. To indicate briefly, then, that p strictly implies q we shall write

$$p \prec q$$

The relation of conjunction

Let us begin with the idea expressed by the word "and." When two propositions are thus connected they stand in the relation of *conjunction*, and the conjunction is symbolized by a dot ($\cdot$). The compound proposition $p \cdot q$ is called the "conjunct" of p and q, and the meaning is that both p and q are to be taken as true.[1] For example, when this connective is used at the second step of a piece of reasoning, what is intended is that each of the items that thus clarify the problem is accepted as sufficiently trustworthy for that purpose. In another problem they might become doubtful, but here they are taken for granted as true. Now this connective does not directly affect the truth-status of the propositions involved—two independent propositions, thus connected, remain independent—but it often does so indirectly. No one would conjoin two propositions unless, even though independent, they were relevant to the same problem. Hence the relation of conjunction is intimately bound up, in meaning, with the other connectives which reasoning about a problem must employ, and the truth-status of conjoined propositions is affected by that circumstance. For example, if the conjunct $p \cdot q$ is false, then it must be that either p or q is not true. If it is not the case that "Mary cried and stamped her feet," then it must be true that "Either Mary did not cry or did not stamp her feet." One of the important logical uses of this

[1] There are other connectives which are equivalent logically but express varying psychological attitudes toward the relation, such as "but," "although," "nonetheless."

relation, soon to be frequently illustrated, has been brought out in the preceding chapter. Often two propositions, when conjoined, strictly imply a conclusion which neither by itself could imply. "Any *a* has *f*" and "*x* is an *a*," taken in conjunction, strictly imply the conclusion "*x* has *f*." But this conclusion does not follow from either of the premises alone.

Consideration of this relation makes it evident that in such a validating form as was presented on page 118, containing the two premises just mentioned, it is entirely a matter of convenience whether one regards the form as having a single premise, or two. Two simple propositions are necessary strictly to imply the conclusion, and if we regard them in their twoness it is natural to say that the reasoning has two premises. But since through the connective "and" they become a single compound proposition, it is also legitimate to regard them as constituting, together, a single premise.

Logical operations

At this juncture brief attention must be given to the notion of a logical "operation." Any act of combining one factor of logical significance with another, removing one from another, or substituting one for another, is an *operation*. Thus, corresponding to the relation indicated by any logical constant there is the operation of placing the factors involved in that relation.[2] When we connected p and q, for example, so that they illustrated the relation of conjunction, we were engaging in the operation of *conjoining* them. Another operation with which the reader has already become familiar is that of *negating*, expressed in ordinary language by the word "not." To negate a proposition p is the same as to deny it; one modifies it by the factor denoted by "not" and the result, symbolized, is p'.

The relation of disjunction and the operation of disjoining

A second relation which affects the truth-status of propositions connected by it is the one ordinarily expressed in English by the words "either . . . or." The technical name for this relation is *disjunction*, and the operation of placing propositions in that relation is called "disjoining." Take the com-

[2] Hence constants and operations in formal logic are intimately bound together, whereas constants and operations in algebra are not in the same way.

pound proposition: "Either Tom screamed or one of the other boys did so." If p represents the proposition "Tom screamed," and q the proposition "One of the other boys screamed," this compound proposition—technically termed a "disjunct"—will be "p or q." We have noted how this is the natural connective to employ in stating the possible solutions which come to one's mind at the third step of reasoning about a problem. The meaning is that one of the two (or more) propositions which are disjoined in the compound proposition is assumed to be the true solution, but we do not know as yet which of them it is. "Either p is the case or q is the case," is what the disjunct affirms. And the proper way to deny or contradict a disjunct is evident on a moment's consideration; the denial of "Either Tom screamed or one of the other boys did so" is "Neither did Tom scream nor did any of the other boys do so." The contradiction of "either . . . or" is thus expressed by the use of conjunction and negation; in symbols, $p' \cdot q'$.

Two possible interpretations of it

Now the bearing of disjunction on the truth-status of propositions thus connected depends vitally on which of two possible interpretations is given to it. And since each interpretation, on occasion, is appropriate, we are really occupied with two distinct relations here. So if one's reasoning is to be guarded from error, the distinction between them must be carefully made. When "either p or q" is asserted (whatever propositions p and q may stand for), does the assertion exclude the possibility that both p and q may be true, or not? Consider our chosen example in the light of this question: "Either Tom screamed or one of the other boys screamed." Does this mean to rule out the possibility that two boys screamed—Tom and one of the other boys also?

The answer is that in this and many other cases it is impossible to tell without further indication from the context of the assertion. In some cases, of course, the disjoined propositions are such that it would obviously be impossible for both to be true together, as for example: "Either Mr. L. is in New York or he is in Boston." But this is not the situation in many cases, and in some of these, at least, it is clear that the alterna-

tives are not intended to exclude each other, as in the proposition: "Either Sinsabaugh committed this crime or he derived advantage from it." The assertion of this disjunct would hardly mean to rule out the possibility that perhaps Sinsabaugh did both; indeed, the fact that a man derived benefit from a crime would in many situations be sufficient to create or deepen suspicion that he committed it.

Appropriate terms are needed by which to take explicit account of this difference. When disjunction is interpreted in the former of these two ways, the possibility being excluded that p and q may both be true, we have a case of *strong* disjunction. When it is interpreted in the latter of these ways, not excluding that possibility, it is an instance of *weak* disjunction.[3] Strong disjunction, then, will be the second relation in our list, and weak disjunction the third.

The strict implications of strong disjunction

Now suppose that a disjunct is conjoined with another proposition affirming or denying the truth of one of the constituents disjoined in it, and the conjunctive proposition thus resulting is considered as an implicans. What implicate, that in such an inference one would usually be interested in drawing, strictly follows from this pair of premises? "Either Tom or one of the other boys screamed, and Tom screamed." Well, this depends on which of the two interpretations above distinguished is assumed. Let us first assume that we are dealing with a case of strong disjunction, and illustrate the implications involved by an instance in which that assumption is plausible. In such a case four strict implicates of the kind sought are discoverable. The disjunctive proposition will be called the *major* premise, and the proposition conjoined with it the *minor* premise.

[3] Some writers deal with this difference by restricting the term "disjunction" so that it means what is here called strong disjunction only. Weak disjunction they refer to as mere "alternation." Some define disjunction as meaning simply the denial of the conjunctive proposition—not both p and q. But both weak and strong disjunction as above defined are frequently used in inference, and their implications need to be mastered. Denial of the conjunctive, when nothing more is intended, is relatively rare except in formal systems.

Sarah is either driving toward town or toward the beach (the two being supposed to lie in different directions),

1. And she is driving toward town;
   Then she is not driving toward the beach.
2. And she is not driving toward town;
   Then she is driving toward the beach.
3. And she is driving toward the beach;
   Then she is not driving toward town.
4. And she is not driving toward the beach;
   Then she is driving toward town.

Letting p represent "Sarah is driving toward town," q the disjoined proposition, "Sarah is driving toward the beach," and Ⓥ the relation of strong disjunction, we shall symbolize each of these four implications as follows:

1. $p \circledvee q \cdot p \; : \!\prec\; q'$
2. $p \circledvee q \cdot p' : \!\prec\; q$
3. $p \circledvee q \cdot q \; : \!\prec\; p'$
4. $p \circledvee q \cdot q' : \!\prec\; p$

Notice that in these expressions the symbol for strict implication is set off from the symbols representing the premises by two dots. This is a convenient way of indicating that both propositions at its left, together, stand in the relation of implication to the conclusion. Otherwise it might seem that only the second premise is asserted strictly to imply the conclusion drawn. The double dot thus performs the same function that might be performed by the use of parentheses:

$$(p \circledvee q \cdot p) \prec q'$$

If the situation should arise in which one has a more complicated formula still, a double dot appearing within the expression which, as a whole, implies a conclusion, three dots should be used to indicate that the whole expression stands in the implicative relation to what follows.

The implications of weak disjunction

Suppose it is now assumed that the relation is one of weak disjunction, namely, that "p or q" means, when fully and precisely expressed: "Either p is true, or q is true, or both

are true." Employing the illustration mentioned on the preceding page in whose case this interpretation is plausible, we shall conjoin four propositions in the same manner, and see what happens.

Either Sinsabaugh committed the crime or he derived advantage from it,

1. And he committed it;
   Then he did not derive advantage from it.
2. And he did not commit it;
   Then he derived advantage from it.
3. And he derived advantage from it;
   Then he did not commit it.
4. And he did not derive advantage from it;
   Then he committed it.

On careful examination it appears that now inferences 1 and 3 are invalid; they are inconsistent with the present interpretation of "either . . . or," namely, that both of the disjoined alternatives may be true. But inferences 2 and 4 are still valid. For when the minor premise negates one of the propositions disjoined in the major premise it also necessarily negates the possibility that both propositions may be true. Hence one may validly infer in this case that the remaining alternative must be true. Supposing it to be the case that either Sinsabaugh committed the crime or derived advantage from it; then if his having committed it is denied, the conclusion may be drawn that he derived advantage from it; and if his having derived advantage from it is denied, one may validly infer that he must have committed it. The relation of weak disjunction being symbolized by $\vee$, the only correct validating forms are:

2. $p \vee q \cdot p' : \prec q$
4. $p \vee q \cdot q' : \prec p$

Its concrete significance

The concrete significance of the validating forms just analyzed is brought out when we note that they are exemplified in the total process of any piece of reasoning at steps three,

four, and five. Hypotheses p, q, and r, let us say, occur as possible solutions of the problem faced—that is, until other hypotheses are introduced one assumes that the correct solution is either p or q or r. Now, as a result of the deductions in step four and whatever further verification is engaged in, p and r are eliminated, *i.e.*, shown to be false and hence denied. Then, tentatively at least, q must be taken as true. Of course, it might turn out later, in the light of further evidence about q, that the major premise has not exhausted all the relevant possibilities. Maybe q is not the correct solution either; perhaps it will finally turn out to be s or t. But the process of considering and eliminating hypotheses, so far as any group of them is concerned at any given time, must, if it is done properly, conform to the pattern above described.

And consideration of disjunction in this situation throws light on the difference between weak and strong disjunction, and shows why both relations have to be recognized. We should like, when formulating hypotheses, to be able to state them in such a way that they are mutually exclusive, for in that case more implications can be derived from them than are permissible otherwise. We know then that if p is true q must be false, and if q is true p must be false, which we cannot know if they are not mutually exclusive. But sometimes two hypotheses that occur to us as alternatives are both found to be true. For example, in the illustration employed in Chapter 2, it might have turned out that the path led to a spring and was also a short cut to another part of the road. Neither of these alternatives precludes the other. In view of this possibility, weak disjunction, with its more limited implications, must be recognized, and assumed to provide the meaning for "either . . . or" when we do not know that the propositions involved exclude each other.

Implication, strict and material

We must now examine more fully a relation between propositions, obviously affecting their truth-status, which has perforce been employed all along. This relation is that of *implication*, and we have noted how it fills its natural role at the fourth step of a piece of reasoning. It has already been defined,

and the important difference between *strict* and *material* implication has been considered. Where implication is strict, the implicate must necessarily be true if the implicans is true. No such "must" is present, however, when the implication is merely material; the intent in that event is simply to affirm that as a matter of actual fact the implicate is true provided that the implicans is true.[4] The fourth relation in our list will then be strict implication, and material implication will be the fifth. The two kinds of implication share, of course, a certain common character, otherwise the same word would hardly be appropriate for both. This is that the conclusion is affirmed to be true if the premise is true. We have already been using the symbol for strict implication; the currently accepted symbol for material implication is $\supset$.

Conditions of validity in the case of material implication

Let us examine more fully than the preceding chapter could do a rather frequent role that material implication plays in our inferences, and see under what conditions such inferences are correctly drawn. It was observed there that when a second proposition of a certain kind is conjoined with a proposition affirming a material implication, the two together constitute a premise (or, when it is convenient so to regard them, a pair of premises) from which a conclusion follows by strict implication. "If any unsupported object falls, and this window box becomes an unsupported object, then it must fall." This inference will now be expressed symbolically. "A thing is an unsupported object" will be represented by p, and "a thing falls" by q. The entire inference then will be symbolized as follows:

$$p \supset q \cdot p : \prec q$$

It will be noticed that p in the minor premise here is not exactly identical with p in the major premise; in the one case it refers to a window box being an unsupported object, while

[4] The conception of "material implication," with its symbol, was systematically presented by Whitehead and Russell in their famous *Principia Mathematica*, published in 1910–13. The phrase "strict implication," and the symbol by which it is represented, were introduced by C. I. Lewis in his *Survey of Symbolic Logic*, 1918.

in the other case it refers to anything being an unsupported object. Were we considering propositions atomically this difference would have to be taken into account. But from the molecular point of view it may be neglected.[5] The minor premise states that the implicans of the major premise is exemplified, either in some actually occurring or in some contemplated instance.[6] The gist of the inference is simply this: If p is true then q is true, and p is true in such and such a particular instance; therefore q must be true in that same instance. Let us investigate systematically the conditions of validity in this kind of inference, considering cases where the minor premise affirms p to be false as well as those in which it is said to be true, and cases in which it makes affirmations about q instead of about p. The sort of inference with which one is thus dealing has traditionally been referred to as "hypothetical" inference; it is very frequently used in both popular and scientific reasoning. Now there are obviously four ways of combining any assertion of material implication with another premise in this fashion; which of them are valid, which not, and why?

If a man is blind, then he needs a guide,

1. And Mr. H. is blind;
   Therefore he needs a guide.
2. And Mr. H. is not blind;
   Therefore he does not need a guide.
3. And Mr. H. needs a guide;
   Therefore he is blind.
4. And Mr. H. does not need a guide;
   Therefore he is not blind.

Symbolically expressed, with p standing for "a man is blind," and q for "he needs a guide":

1. $p \supset q \cdot p \;:\; -\!\!\!3\; q$
2. $p \supset q \cdot p' \;:\; -\!\!\!3\; q'$
3. $p \supset q \cdot q \;:\; -\!\!\!3\; p$
4. $p \supset q \cdot q' \;:\; -\!\!\!3\; p'$

[5] In logical strictness, the principle of specification is here assumed.

[6] Here, the major premise is defined as the one asserting the material implication.

The resulting validating forms

On examination it is evident that only the first and fourth cases are valid. Whenever p is true, q is true; this is what the major premise affirms. Hence, given an instance of p being true, we must also have an instance of q being true; this is case 1. Hence, also, given an instance in which q is not true, in that instance p cannot be true. For if p were true, q must have been true instead of false. This is case 4. But cases 2 and 3 are invalid. The reason for their invalidity is that the major premise does not affirm that p is the only condition of q being true. It says that q is the case whenever p is, but maybe it is the case under other circumstances too. Thus, in the present instance, the truth that any blind man needs a guide does not preclude the possibility that there are other people who need a guide too—a visitor in a strange city, for example. Hence one may not legitimately infer that if a man is not blind he doesn't need a guide (2), or that if he needs a guide he must be blind (3). Only 1 and 4 are valid, therefore, and only the corresponding abstract formulations are genuine validating forms.

The fact that only two of these inferences are valid is registered in contemporary terminology by describing p as in these cases the *superimplicate* of q, and q as the *subimplicate* of p. And from the above analysis it is clear that when one proposition is superimplicate of another its truth warrants the truth of the other, while its falsity leaves the other indeterminate. Contrariwise, when one proposition is subimplicate of another its falsity warrants the falsity of the other, while its truth leaves the other indeterminate.

But certain traditional phrases have acquired the sanction of long custom for describing these four cases of hypothetical inference. Taking the valid forms first, 1 is a case of *affirming the antecedent.* For the minor premise affirms that the antecedent of the major premise is true, as exemplified in some particular instance. Case 4 is that of *denying the consequent,* since the minor here denies that the consequent of the major is true, at least in the instance contemplated. The two valid forms for this kind of inference are, then:

1. Affirming the antecedent: $p \supset q \cdot p : -\!\!3\ q$
4. Denying the consequent: $p \supset q \cdot q' : -\!\!3\ p'$

Since the other forms are invalid, any inference that illustrates either of them commits a fallacy. The two fallacies that are distinctively possible in hypothetical inference are:

2. Denying the antecedent (of the major premise by the minor), and
3. Affirming the consequent.

Since whatever is true of material implication is also true of strict implication, the rules just formulated apply also to inferences which are similar but in which the major premise is a strict implication.

They also apply when the major premise is a strict implication

If a triangle is equilateral then it is isosceles,
And this triangle is equilateral.
Hence it must be isosceles.

In symbols, $p -\!\!3\ q \cdot p : -\!\!3\ q$.

The reader will check on the other three ways of completing such an inference.

It is essential to keep in mind that since either antecedent or consequent of any major premise in such inferences may contain the word "not," the mere fact that the minor premise is negative in form does not indicate whether it affirms or denies the corresponding proposition in the major. Obviously, if the antecedent is a negative proposition, its affirmation will be negative in form, too. The reader may supply his own illustrations of this point.

The importance of the valid forms of hypothetical inference may be indicated by pointing out that affirming the antecedent is the logical procedure by which one tests the correctness of any attempt to anticipate an event not yet observed, whether for ordinary practical aims or for the scientific purpose of predicting the outcome of some experiment. Something now happens, either in the ordinary course of nature or through

In what problems do we employ inferences tested by these forms?

our purposeful interference. How do we intelligently anticipate its consequences? Only if it can be identified as an instance of the antecedent of a material implication, which affirms that whenever that sort of event occurs another sort of event (denoted by the consequent) can be depended on to occur. Given a case of p, in this way we justly predict a case of q. Denying the consequent is the form which is implicitly used whenever a hypothesis is rejected as inconsistent with evidence now observed. If such and such a hypothesis is true, a reasoner has said, then such and such would be the case; but I find now that it is not the case, hence I conclude that the hypothesis is false.

The hypothetical inferences above illustrated were called traditionally *mixed* hypotheticals, the word "mixed" indicating that the minor premise and conclusion are simple, not hypothetical propositions. Occasionally one makes use of a *pure* hypothetical inference, in which case all three propositions involved are hypotheticals, the consequent of the first becoming the antecedent of the second, and dropping out in the conclusion. An instance of a pure hypothetical inference would be:

> If the kettle is boiling the water is ready,
> And if the water is ready we can pour in the tea;
> Hence, if the kettle is boiling we can pour in the tea.

$$p \supset q \cdot q \supset r : \mathrel{-\!\!3} : p \supset r$$

The dilemma

A kind of inference which has long been of interest to rhetoricians as well as to students of logical form, and which has attracted a considerable amount of popular attention, employs both material and strict implication, along with disjunction. It is technically called the *dilemma*. In a broad sense a dilemma means any situation which is unpalatable to those caught in it because they find themselves compelled to choose between alternative courses of action, all of which lead to unpleasant consequences. In the narrower sense which is alone relevant to a study of right and wrong thinking, a dilemma is

a piece of inference in which hypothetical and disjunctive propositions are so conjoined that they claim to compel logically a choice among two or more alternative assertions and to lead thus to a conclusion. It is almost always a controversial device, attempting to coerce an opponent into the acceptance of a result inconsistent with his previous contentions or otherwise repugnant to him. For this reason dilemmas are more often defective than sound. The premises may be unjustified by the facts which they purport to describe, or the conclusion may not strictly follow from the premises. The last named error is not as common as the other, however; most dilemmas are formally valid.

Let us now describe a dilemma more precisely from the standpoint of logical form. Its major premise consists in a conjunct of two hypothetical propositions, and the minor premise is a disjunctive proposition affirming the antecedents or denying the consequents of the major. Then the conclusion affirms the consequents or denies the antecedents respectively. Such a dilemma must be either destructive or constructive, and either simple or complex; each of these possibilities will be illustrated and defined.

Complex dilemmas, constructive and destructive

Consider the following dilemma:

> If a woman is good-looking, higher education is superfluous; if she is not, it is inadequate.
> But either a woman is good-looking or she is not.
> Therefore either higher education for women is superfluous or it is inadequate.

This is a complex constructive dilemma. It is *constructive* because the disjunctive premise affirms the antecedents of the hypotheticals while the conclusion affirms the consequents. It is *complex* because the major premise consists of four distinct propositions, and the conclusion is a disjunctive proposition. Let us assign a symbol to each of these propositions:

p = A woman is good-looking.
q = Higher education for women is superfluous.
r = A woman is not good-looking.
s = Higher education for women is inadequate.

The validating form for a complex constructive dilemma may then be symbolized as follows:

$$p \supset q \cdot r \supset s \cdot p \vee r : ⥽ \cdot q \vee s$$

Contrast this dilemma with the one below:

> If this speaker is a politician he is attempting to win votes, while if he is a minister he is aiming at the moral improvement of his hearers.
> But neither is he attempting to win votes, nor is he seeking the moral improvement of his hearers.
> Therefore, neither is he a politician nor is he a minister.

Here we have a dilemma which is like the former in being complex, but it is destructive instead of constructive; it is *destructive* because the minor premise denies the consequents of the major premise while the conclusion denies the antecedents. Its validating form may be thus symbolized:

$$p \supset q \cdot r \supset s \cdot (q \vee s)' : ⥽ \ (p \vee r)'$$

Simple dilemmas

A dilemma is *simple* instead of complex when the major premise consists of only three distinct propositions, and the conclusion, for this reason, is a simple proposition. Here is an instance of the simple constructive dilemma:

> If you say what is just you will be hated; and if you say what is unjust you will be hated.
> But either you say what is just or you say what is unjust.
> Therefore you will be hated.

In this case the two hypotheticals in the major premise have the same consequent, and hence only three propositions are involved. When, accordingly, the minor premise affirms both antecedents, the conclusion is a simple proposition affirming their common consequent. The validating form is thus symbolized:

$$p \supset q \cdot r \supset q \cdot p \vee r : ⥽ \ q$$

In the presence of these illustrations and definitions, specific comment on the simple destructive dilemma is hardly necessary. It will, however, be illustrated and its validating form symbolized.

> If God created the world it would be good, or there would be a clear explanation of its badness.
> But neither is it good, nor is there any clear explanation of its badness.
> Therefore God did not create it.

Here the two hypotheticals of the major premise share a common antecedent; fully expressed, the first premise is: If God created the world it would be good, or if he created it there would be a clear explanation of its badness. The validating form:

$$p \supset (q \vee r) \cdot (q \vee r)' : \dashv p'$$

Ways of rebutting a dilemma

In what ways may a dilemma be rebutted by an opponent who is not willing to accept the conclusion into which the wielder of the dilemma hopes to coerce him? One way is to *escape between its horns*. This means to point out that while the minor premise asserts that two possibilities exhaust the available alternatives there are, in fact, more than two. For instance, in the third of the above illustrations an opponent might point out that one may say many things about which no question of justice or injustice would arise.

Another way to rebut a dilemma is to *take it by the horns*—that is, to maintain that one or both hypothetical premises is illegitimate. The first of the above dilemmas, for example, could easily be taken by the horns; an opponent would point out that a normal woman has other purposes in life besides catching a husband. In fact, when a dilemma can be taken by the horns, a third and very effective way to reveal its unconvincing character is to concoct an opposing dilemma, whose major premise is at least equally plausible, but whose conclusion points in a quite different direction than the con-

clusion of the original dilemma. Thus one might counter this inference condemning higher education for women by the following:

> If a woman is good-looking, higher education will increase her charm; if she is not, it will give her an attraction of its own.
> But she must be either good-looking or not.
> Therefore higher education will either increase her charm or give her an attraction of its own.

When a pair of opposing dilemmas confronts us in this fashion, discussion of the subject matter involved has of course reached a stalemate. What is needed is further evidence or careful analysis, enabling us to decide rationally between the contrasting major premises or to replace both by a more justifiable description of the facts.

The relations of equivalence between propositions

Another pair of relations between propositions, affecting their truth-status, may be best examined at this juncture. Both are forms of *equivalence*. Now two propositions are equivalent when each implies the other. This means, of course, that given p one may validly infer q, and given q one may validly infer p. But we meet different properties according to whether the mutual implication involved is material or strict. In the former case the equivalence is material merely; in the latter it is strict equivalence. Following general usage, material equivalence will be symbolized by $\equiv$, and strict equivalence by $=$, since the properties of the latter are the same as those of mathematical equality. Strict equivalence will be our sixth relation; material equivalence the seventh.

$$p \equiv q, \text{ then, means} : p \supset q \cdot q \supset p$$
$$p = q \text{ means} : p \prec q \cdot q \prec p$$

From these definitions the essential difference between the two kinds of equivalence will be apparent. When strict equivalence obtains between two propositions, either can be derived merely by formal examination of the other, guided by the law of contradiction. A very simple instance of strict equivalence is the case where q is the denial of the contradictory of p.

Men are mortal ⥽
Men are not immortal ⥽
Men are mortal.

Another illustration may be taken from geometry.

A plane figure is triangular ⥽
It is three-sided ⥽
It is triangular.

Of this pair of propositions each may be deduced from the other by necessary implications of the geometrical definitions involved, without appeal to any factual observation. In fact, any mathematical equation is an instance of strict equivalence.

Material equivalence is exemplified in the following propositions:

This man is Mr. Robert Wadleigh $\supset$
He is the tallest man in the United States $\supset$
He is Mr. Robert Wadleigh.

Here each of the propositions implies the other merely as a matter of fact; no one could derive either from the other by formal considerations alone. It just happens to be the case that a given individual has this name and is also the tallest man in the United States.

Why is it ever important to discover *p*'s and *q*'s such that not only does *p* imply *q* but also *q* implies *p*? Well, consider what we are trying to accomplish at steps four and five of a piece of reasoning. A suggested solution to our problem has come to mind but we cannot test it directly. We see, however, that if it is true something else must be true which we can test directly. *p* cannot be tested; but if *p* then *q*, and *q* can be tested. Now observe the situation that we are in if *p* is merely the superimplicate of *q*, the two propositions not being equivalent. Suppose we find that *q* is true; can we conclude that *p* must be true? No, for there may be some other antecedent of which *q* is a consequent; to draw such a conclusion would be to commit the fallacy of affirming the consequent. Suppose we

Its importance

find that q is false. Then, indeed, we can conclude that p is false, but this establishes no hypothesis as true; it only means that we must look for the truth among other possible hypotheses besides p. But if p's and q's can be discovered that are equivalent, q implying p as well as p q, then our situation in these two steps is very different. Then we know that q can be true *if and only if* p is true; hence if we find q to be true by some appropriate test, we can confidently conclude that p is true. An essential part of the methodology of factual science is concerned with the procedures by which such material equivalences can be discovered; they will be studied with some care in Part III.[7]

But as helpful present illustrations of both material and strict equivalence between propositions, we may examine the problems of accurate definition and division.

Definitions as illustrating equivalence

Considered in its formal or syntactic requirements merely, a correct definition must exhibit the relation of equivalence as just described, or, more strictly, it must appear as a formula from which two equivalent propositions may at any time be derived. Examine the following definition as given in Webster's *Dictionary:* "Nausea is any sickness of the stomach . . . with a desire to vomit." Here the word defined (in technical parlance, the *definiendum*) is "nausea"; the phrase employed in defining it (technically, the *definiens*) is "any sickness of the stomach with a desire to vomit." Now if this definition is correct, whatever is true about nausea will also be true about sicknesses of the stomach with a desire to vomit; that is, for any proposition that is asserted about nausea there will always be an equivalent proposition that could be asserted about sicknesses of the stomach with a desire to vomit, and vice versa. A simple illustration of this derivation of a pair of such propositions from a definition is: "This is a case of nausea" becomes equivalent to "This is a sickness of the stomach with a desire to vomit." Granted the correctness of the definition, either of these propositions implies the other.

And this illustration likewise indicates the test by which

[7] See below, chap. 16, pp. 324–332.

one tells whether definiendum and definiens are equivalent or not; that test is substitution. In any proposition in which either definiendum or definiens occurs, substitute the other; if the resulting proposition is always true when the original proposition is true, and false when the original is false, then the two are shown to be equivalent. If not, then they are not equivalent. The one essential formal requirement of a good definition is, hence, substitutability of definiens for definiendum, displayed in the capacity of the definition to yield pairs of equivalent propositions in this way.

When is the equivalence strict? When material?

But is the equivalence thus derived from a correct definition strict or material? Well, this depends on the intent of the one who adopts it. When the intent is to describe accurately in the definiens a certain class of facts which the definer has in mind, the equivalence is material. This is presumably the situation with the illustration just employed. It is true as a matter of fact, but not by any formal necessity, that the malady conventionally referred to as nausea is truly describable as a sickness of the stomach with a desire to vomit. Such definitions are called *real* definitions.

How can there be strict equivalence between definiendum and definiens, *i.e.*, a situation in which one can be necessarily derived from the other? The answer is: When the purpose of the definer is not to describe anything but merely to indicate how he is going to use the word or phrase defined. He determines its meaning in terms of other words by his own arbitrary prescription, so far as his employment of it is concerned. Such determination by prescription occurs frequently in scientific writing. Suppose that a scientist wishes to avoid frequent repetition of a lengthy phrase describing a group of complex operations. The way to do this is arbitrarily to define some word as equivalent to this longer phrase. He is not claiming to give a correct description of anything; his intent is merely to establish a convenient substitution. And since nothing more than his decision is required to establish it, so far as his writings are concerned, the equivalence exhibited by such a definition is strict, not material. Until he withdraws the

definition, replacing his prescribed definiens by some other, the equivalence is formally secured.

Definitions of this kind are called *verbal* or *prescriptive* definitions—that is, they merely express the purpose of prescribing to what phrase some word or phrase is to be regarded as equivalent.

Division also demands equivalence

Division also, if correct, exhibits the relation of equivalence. The central purpose behind any division or analogous operation is to indicate with logical precision the subclasses of which a more inclusive class is composed.[8] This is done by employing the principle of strong and exhaustive disjunction; if anything is a member of the inclusive class it is either a member of this, or that, or of the other subclass, and no two of these subclasses are supposed to overlap. Conversely, if it is a member of any of the subclasses it is a member of the inclusive class.

In the formal sciences operations which apply this principle are quite familiar. One may, for example, divide the class of triangles into the three subclasses: equilateral, isosceles but not equilateral, and scalene. Here the equivalence is obviously strict, since it can be determined from the very definition of "triangle" that any triangle must belong to one of these three subclasses. Symbolizing "this is a triangle" by p, "this is an equilateral triangle" by q, "this is isosceles but not equilateral" by r, and "this is scalene" by s, the following equivalence obtains:

$$p = : q \ovee r \ovee s$$

That is, $p \strictif : q \ovee r \ovee s$, and $q \ovee r \ovee s : \strictif p$. Similarly, the class of propositions may be divided into true or false propositions by analysis of the definition of "proposition," and in many formal systems each of these subclasses is further divided in the same way; true propositions are either factually or necessarily true, and false propositions either factually or necessarily false.

[8] The less precise word "classification" is sometimes used as a synonym for "division"; more often it means the converse process of organizing subclasses under more general ones. It would be preferable to use it only in the latter sense.

In all cases where the terms of the division are not necessarily determined by the nature of the class divided, the equivalence which results is material only. For example, the Weather Bureau in its forecasts and reports often divides days into three subclasses—clear, partly cloudy, and cloudy days. Obviously, this division does not follow merely from a formal analysis of the meaning of "day." It simply is a fact that from the point of view of weather prediction any day can be classified under one of these three heads. A correct division of this sort is symbolically expressed as follows:

$$p \equiv : q \, ⓥ \, r \, ⓥ \, s, \text{ etc.}$$

Here p means being a member of the inclusive class—in this case, day. Then q, r, s, etc., mean respectively being a member of one or another of the various subclasses—in this case, clear, partly cloudy, and cloudy days.[9]

Equivalence between the logical constants

But the present chapter has itself revealed some strict equivalences not yet systematically noted. Are the relations between propositions, which it has examined, themselves formally interrelated, so that the truth or falsity of a compound proposition employing one of them determines the truth-status of a compound proposition with the same constituents employing the others? Yes, they are, and this circumstance has been anticipated in passing, as when it was noted that the contradictory of a disjunctive proposition is most simply expressed as a conjunctive proposition, and vice versa. The more important of these interrelations may be best brought out by asking what other propositions are formal equivalents of any compound proposition employing one of these relations.

Examine again: "If a man is blind, he needs a guide." As before, the antecedent will be symbolized by p and the consequent by q. We are saying, then: $p \supset q$. And it has been noted already that this proposition is equivalent to one about p' and q'; if it is true it follows that if a man doesn't need a

[9] The theme of definition and division will be dealt with more fully below, in chap. 24.

guide he is not blind, *i.e.*, that $q' \supset p'$. From this, by the same process, we can return to the original proposition. If $q' \supset p'$, then $(p')' \supset (q')'$, which (since to deny the contradictory of a proposition is to assert that proposition) is the same logically as $p \supset q$. Hence $p \supset q$ and $q' \supset p'$ are strictly equivalent—$p \supset q : = : q' \supset p'$.

But the original proposition is also equivalent to one stated in terms of "either . . . or." Assuming it to be the case that if a man is blind he needs a guide, it must also be the case that either he is not blind or else he needs a guide—that is, $p \supset q : = : p' \vee q$. Similarly, our earlier consideration of any disjunctive proposition has shown that if it is true, a proposition of material implication follows. If it is the case that "either John is sick or else he is resting," it must be the case that "if he is not sick he is resting." Hence again it is clear that $p \supset q$ and $p' \vee q$ are equivalent.

To what conjunctive proposition is the proposition, "If a man is blind he needs a guide" equivalent? Well, let us consider how this proposition might be most simply contradicted. Clearly, we would contradict it by saying "A man can be blind and not need a guide—*i.e.*, $p \cdot q'$. Hence the denial of this contradictory, which would be $(p \cdot q')'$—"It is not the case that a man can be blind and not need a guide"—must be equivalent to the original proposition. In other words, $p \supset q : = (p \cdot q')'$.

Summarizing these equivalences, we have:

$$p \supset q : = : q' \supset p' : = : p' \vee q : = (p \cdot q')'$$

We shall now examine three relations between propositions which bring out more directly than has thus far been done the significance of the idea expressed by the word "not"—the last of the logical constants listed at the beginning of the chapter. When, testing a proposed solution of a problem, one affirms that a given proposition is not true, or not false, what is he implying with respect to other propositions with which it may be connected?

The relation of contradiction

Let us begin this part of the analysis with the relation of *contradiction*, briefly discussed in the preceding chapter. It will then be the eighth in the list of relations now being considered. Suppose that two propositions p and q are given, and that q happens to be identical with the contradictory of p, that is, with p'. What bearing does this relation have on the truth-status of the propositions so related? Well, by analysis it appears that in that event the assumed truth of p strictly implies the falsity of q and the falsity of p strictly implies the truth of q. Let p, for example, be the proposition, "Johnny fell down." Then q will be, of course, "Johnny did not fall down." Now, if it is true that "Johnny fell down," then it must be false that "Johnny did not fall down"; and if it is false that "Johnny fell down," then it must be true that "Johnny did not fall down." The same strict implications clearly obtain if we assume the truth or falsity of q, and inquire about the corresponding status of p. If "Johnny did not fall down" is true, then "Johnny fell down" must be false—that is, the truth of q implies the falsity of p. Similarly, if "Johnny did not fall down" is false, then "Johnny fell down" is true; q as false implies that p is true.

The relation of contrariety

What fallacies is one likely to fall into through failing to understand the relation of contradiction correctly? A full answer to this question the entire present Part is providing, but one such fallacy in particular must be noted here. It consists in mistaking a ninth relation—that of mere *contrariety*—for one of contradiction; two propositions are treated in inference as though they contradicted each other when they are really only contraries. And what does it mean for two propositions to stand in this latter relation? It means that the truth of one strictly implies the falsity of the other, but that the falsity of one does not strictly imply the truth of the other.

$$p \prec q', \text{ but } p' \text{ does not } \prec q$$
$$q \prec p', \text{ but } q' \text{ does not } \prec p$$

That is, when this relation obtains, p and q do not exhaust the alternatives as they do when the relation is that of contradic-

tion; there is a third possibility, namely, that both p and q are false. Contrast, for example, these two groups of inferences:

a. If anything is not mortal it is immortal.
   If anything is not consistent it is inconsistent.
   If anything is not living it is nonliving.
   If anything is not material it is immaterial.
   If anything is not continuous it is discontinuous.
   If anything is not necessary it is unnecessary.

b. If anything is not good it is bad.
   If anything is not beautiful it is ugly.
   If anything is not fortunate it is unfortunate.
   If anything is not constructive it is destructive.
   If anything is not lovable it is hateful.

Notice that in all these cases, if the premise were affirmative and the conclusion negative, the inferences in both (a) and (b) would be valid; "anything is good" strictly implies "it is not bad" just as clearly as "anything is mortal" strictly implies "it is not immortal." At least, this is so if we add the phrase, "in the same respect and at the same time," which is meant to be understood. But in group (a) the negative premise strictly implies the conclusion drawn; if it is false that anything is mortal then it necessarily follows that it is immortal. In these cases q is identical with p'; the relation is one of contradiction. In group (b) however, this is not the case. If it is false that a given thing is good it does not necessarily follow that the thing is bad, even in the same respect and at the same time; if it is false that it is constructive there is no implication that it is destructive. For here the contrasting propositions are not contradictory, but only contrary to each other. There is a third alternative. It is possible for an entity to be neither good nor bad, but neutral in value; likewise to be neither constructive nor destructive but merely preservative of what has already been established.

Hence it is essential to discriminate meticulously between the relations of contradiction and contrariety if a thinker is to avoid error in his inferences. And the principle which should guide the discrimination is evident; contrariety is the weaker

of the two relations. In both cases one is warranted in inferring from the truth of one of the pair of opposing propositions the falsity of the other; but only in case the relation is one of contradiction may he properly infer from the falsity of one the truth of the other. Thus, whenever two propositions are inconsistent with each other but it is not clear that they are genuinely contradictory—that is, that q is identical with p'—they must be treated as merely contrary. In this way any fallacy which might result from confusing the two relations will be avoided.

The relation of subcontrariety

A tenth relation, that of *subcontrariety*, should be added to the list of molecular relations affecting the truth-status of propositions. It obtains between any pair of propositions whenever, given the falsity of either, the other must be true, but when both may be true. The following pair of propositions exemplifies this relation:

> John is absent from school.
> John is away from home.

The relations of contradiction, contrariety, and subcontrariety assume a particularly interesting form when they appear in the atomic analysis of certain kinds of propositions.[10]

EXERCISES

1. Identify the relation obtaining between each pair in the following list:
   a. This figure is a square.
      It is a rectangle.
   b. The boys are not at the swimming hole.
      They are not at the candy store.
   c. Henry is taller than James.
      Richard is James' brother.
   d. All of these girls are pretty.
      One of them is not pretty.
   e. Lines A and B are parallel.
      Lines A and B are such that they will never meet, however far extended.
   f. Wishes are horses.
      Beggars will ride.

[10] See below, pp. 172 ff.

g. The sixth president of the United States was John Quincy Adams.
   The sixth president of the United States was a son of the second.
h. Brewster lives east of Chicago.
   Brewster lives west of Chicago.

2. Characterize each of the following as hypothetical inferences, disjunctive inferences, or dilemmas. Put them in exact form for testing their validity—that is, restate any phrase or clause that may need restatement in order to reveal the essential structure of the inference, and supply any missing premise. Are they valid or invalid? If the former, describe the structure of the argument in detail—as a case of denying the consequent, as a case of a simple constructive dilemma, as a case of weak disjunction, etc. If the latter, describe technically the reason for their invalidity.
   a. If the sun shines it will get warm. The sun has just begun to shine, therefore it will get warm.
   b. If he had performed the errand I should not have received this letter. But I have received the letter, therefore he did not perform it.
   c. Either I forgot having paid this check, or else it was forged. But it is evidently not forged, hence I must have forgotten paying it.
   d. If a man is a Protestant, he believes in the right of private judgment in religion. This man believes in the right of private judgment in religion, hence he must be a Protestant.
   e. I recall that Mr. W. is either a Senator or a lawyer. It appears that he is a lawyer. Therefore he is not a Senator.
   f. If a body moves, it must either move in the place where it is, or in the place where it is not. It cannot move where it is (for then it would no longer be there), nor can it move where it is not (for it is not there to do it). Therefore a body cannot move. (Argument of the Greek philosopher Zeno.)
   g. General C. is either in California or Florida. He is in California. Therefore he is not in Florida.
   h. That creature is either an owl or a bat. The shape of his wing shows that he is not a bat. Therefore he is an owl.
   i. If the clouds do not disappear there will be a beautiful sunset. But they are disappearing. Therefore there will not be a beautiful sunset.
   j. Had you taken the right medicine your fever would be gone. It is not gone, so you could not have taken the right medicine.
   k. A friend is a dubious blessing. For if he is wealthy he will induce you to make questionable investments, while if he is poor he will borrow money from you.

l. The Greek Sophist Protagoras is reported to have made an agreement with a pupil Euathlus, to teach him the art of pleading at law, on condition that one-half the fee was to be paid when the instruction was completed, the other half when Euathlus had won his first case in court. Euathlus paid the first half, but put off beginning practice. Protagoras finally brought suit for the remainder of the fee, offering this dilemma to justify his position:

If Euathlus loses this case he must pay me, because that will be the judgment of the court; if he wins he must pay me as the contract provides.

But he must either lose or win.

Therefore in any case he must pay the fee.

Euathlus countered with the following dilemma:

If I win the case in court, I will not have to pay, for such will be the judge's decision; if I lose it, according to the contract I will not yet have to pay.

But I must win or lose.

Therefore in any case I will not have to pay.

The student should study this pair of dilemmas in detail, stating clearly just where the trouble lies.

m. If the minor premise of a dilemma affirms the antecedents of the major, it is a case of *modus ponens;* if it denies the consequents, it is a case of *modus tollens.*

But the minor premise of a valid dilemma does one or the other of these.

Therefore a valid dilemma is either a case of *modus ponens* or of *modus tollens.*

3. Give three equivalents for each of the following propositions:
   a. If this correction is made, your paragraph will be intelligible.
   b. Either these boys are hungry or else that meat tastes good.
   c. That a girl can be beautiful and not attract men is false.
   d. If you don't hurry you will miss your train.
   e. Henry neither entered the house nor did he go to the garden.
   f. Either Henry didn't take the car or else he failed to park it at the usual place. (In the last two cases the student will find it helpful to ask what propositions are contradicted by these propositions.)

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CHAPTER 9

THE ATOMIC ANALYSIS OF PROPOSITIONS

The atomic analysis of propositions

The preceding chapter has been occupied with a study of relations that may obtain between propositions taken as unanalyzed wholes, and with the discovery of certain validating forms which are useful in checking the correctness of inferences employing these relations. In terms of the phraseology there introduced, our concern thus far has been with a molecular analysis of propositions and with lessons significant for the guidance of reasoning which such an analysis readily enforces.

But by no means are all validating forms disclosed by such a procedure. Some of the most important forms can be exhibited only by an atomic analysis of propositions—that is, by noting the constituents of which a proposition is composed, and the kind of internal structure which it exemplifies. For instance, consider the following inference:

> Northampton is east of Albany, and
> Boston is east of Northampton;
> Hence Boston is east of Albany.

This is a valid piece of reasoning, but its validity cannot be shown merely by applying the principles discussed in the preceding chapter. Of course, since it is valid, a relation of strict implication obtains between the two premises, taken in conjunction, and the conclusion. But the reason why that relation obtains cannot be revealed merely by a molecular examination of the three propositions involved. However, atomic analysis of the premises of this inference quickly reveals an internal

logical pattern by reason of which the conclusion drawn is necessarily implied.

Some illustrations of propositional structure

The first need met in embarking on such an analysis is that of finding out how many different internal patterns a simple proposition may exhibit, and what the distinctive properties of each are. The following list illustrates all the main kinds of propositional structure of which an elementary exposition needs to take account.

1. The piano is in the parlor.
2. Weeds grow.
3. John is running.
4. This flower is fragrant.
5. The man on your left is a full-blooded Indian.
6. A few bombs struck the stone buildings.
7. All objects in this shed are garden tools.
8. Truth is stranger than fiction.
9. x is between y and z.
10. Some automobiles are poor hill climbers.

It will be well to commence an examination of this list by making use of a grammatical distinction. Notice that even the briefest of these statements consists of two parts whose function is quite different. This is the second: "Weeds grow." The first word here denotes that about which something is being asserted. In grammar, this is called the subject of the sentence. The second word expresses what is asserted about this subject; it is called the predicate. If one runs through the rest of the list under the guidance of this distinction he finds the same two parts discoverable in all, and the line between them falls directly to the left of the verb in each case. Now a step has been taken which is foundational for a logical as well as a grammatical analysis. The subject part of the sentence always contains a *substantive*—that is, a word or phrase referring to some entity or entities capable of being considered without assuming the existence of other things. The entity may be an individual person or thing, as in cases 1, 3, 4, 5, 8, and 9; it may be a group of individuals belonging to the same class, as in 2, 6, 7, and 10. It may be an abstract entity, as in 8 and 9;

while it may be perceivable through one or more of our senses, as in all the other cases. But the predicate part of these statements falls under no single type. Neglecting for the time being the significance of the little words "is" and "are," which will need consideration later, let us initially classify the important portion of the predicates under the following heads: (a) a prepositional phrase, as in 1, 8, and 9; (b) an intransitive verb, as in 2 and 3; (c) a predicate adjective, as in 4; (d) a predicate substantive, as in 5, 7, and 10; and (e) a transitive verb with its object, as in 6.

Logical classification of the predicates

For the purposes of formal analysis, however, this grammatical division of the predicates may be somewhat simplified. There appears, for instance, no need to insist on any essential difference between (a) and (e). The intent of each of these propositions is to affirm that the entity referred to in the subject stands (or has stood) in a certain *relation* to another substantive (or other substantives) indicated in the predicate—in 1, 8, and 9, the latter being the object (or objects) of the preposition; in 6, the object of the verb. This common structure may be somewhat awkwardly exhibited by transforming the transitive verb of 6 into a relational phrase: A few bombs were strikers of the stone buildings. That is, just as in 1 the piano is affirmed to stand to the parlor in the relation of being in it, and in 8 truth is affirmed to stand toward fiction in the relation of being stranger than it, so in 6 the bombs are affirmed to have stood toward the stone buildings in the relation of striking them.[1]

Likewise, there is no essential difference between (b) and (c). Their likeness is rendered more obvious verbally if in proposition (2) "Weeds are growing" be substituted for "Weeds grow." Then it becomes apparent that the function of the predicate in 2, 3, and 4 alike, is to attribute a certain nonrelational property to the subject of the proposition; it is

[1] The reader will observe that when the word "relation" is used in discussing formal science it is essential to keep in mind whether one is dealing with an atomic or a molecular analysis. In a molecular analysis the entities related are propositions; in an atomic analysis they are elements found within a proposition.

a matter of linguistic custom that in some cases this is naturally done by employing an intransitive verb, in others by a predicate adjective. And words denoting such a property are not substantives; when one says "growing" or "fragrant" he must assume some entity to which they may be attributed.

Thus instead of five different kinds of predicate there are left only three which appear to be different in ways which a formal analysis of propositions must take into account. At this point, however, a distinction emerges which is of little concern to the grammarian but very important in dealing with some problems of formal science. What is the meaning of "is" in proposition 5? Well, the subject of the proposition is an individual, while the predicate names a class to which it and various other individuals belong. The purport of the "is" would seem, therefore, to consist in pointing out that the individual referred to by the subject *is a member of* the class indicated in the predicate. How now, about propositions 7 and 10? Here the noun in the subject (with, in 7, its modifying phrase) itself refers to a class—"objects in this shed" in the one case, "automobiles" in the other. And the meaning of the word "are" in these cases is more precisely rendered if it is replaced by "are included in." Proposition 7 affirms that the entire class of objects in this shed *is included in* the class of garden tools—that is, that every member of the former class is also a member of the latter class—proposition 10 that an unspecified portion of the class of automobiles is included in the class of poor hill climbers.

Four kinds of proposition

Let us list the four kinds of proposition which, according to the above discussion, need to be discriminated for purposes of atomic analysis, and provide appropriate symbols for representing them.

1. *Relational Propositions,* exemplified by 1, 6, 8, and 9. Each of these affirms that the entity (or entities) referred to by the subject stands (has stood, or will stand) in a certain relation to another entity (or other entities) indicated in the predicate. Following the procedure already used in Chapter 7, any such relation will be symbolized by R, and its converse relation by

R_c. Where the entity which stands in the relation is an individual it will be symbolized by x, y, or z; where it is all or part of a class, by a, b, or c. Thus 1 and 8 may be symbolized by x R y, 6 by a R b.

2. *Class Membership Propositions,* exemplified by 5. Such propositions affirm that the individual referred to by the subject is a member of the class indicated by the predicate. Current custom uses the symbol ϵ to signify "is a member of," and the method now familiar for distinguishing individuals from classes will be adhered to. Thus any proposition of this kind may be symbolized: $x \epsilon a$.

3. *Propositions of Class Inclusion,* exemplified by 7 and 10. These affirm that the whole or part of the class referred to by the subject is included in the class denoted by the predicate. Now class inclusion within a proposition has the same formal properties as the relation of implication between propositions, but the two need to be distinguished, since one involves molecular analysis merely; the other, atomic. Hence the symbols for implication, turned around from right to left, may be used to represent class inclusion. If the inclusion of the subject class in the predicate class is merely factual, as is the case in the above instances, the proposition may be symbolized thus: $a \subset b$; if it is formally necessary, a ⥼ b would be the appropriate symbol.[2] An illustration of the latter situation would be: All squares are quadrilaterals; here the inclusion affirmed is necessitated by the formal nature of the two terms.

4. *Attributive Propositions,* exemplified by 2, 3, and 4. Any such proposition attributes a certain nonrelational property, indicated by the intransitive verb or predicate adjective, to the entity or entities referred to by the subject. Now f, g, or h has been used as the symbol for any such property, hence an appropriate way of representing attributive propositions is obvious—proposition 2 will be symbolized by $a\ f$; propositions 3 and 4 by $x\ f$.[3]

The interconnection of class membership, class inclusion, and attributive propositions

But at this point, with these four types of proposition carefully discriminated, a bit of unexpected encouragement ap-

[2] When we come to detailed analysis of these propositions, however, a different symbolism will be used, based on the distinction between subject, predicate, and middle terms. See below, pp. 186 ff., 210 ff.

[3] The more usual method of symbolizing such a proposition is $f(x)$ or f_x. For reasons of symmetry in the present context the above method is employed instead.

pears. In a complete analysis of the formal structure of simple propositions, the distinctive differences of each of these types would have to be carefully respected throughout. But for the purpose of an elementary study of atomic validating forms, a certain simplification is allowable. That is, the differences between class membership, class inclusion, and attributive propositions may be neglected, if certain cautions regarding the first-named type are kept in mind. This possibility is due to the fact that propositions of these three types show partial equivalences which will now be explained. First, an attributed property may at any time be replaced by a class which, for most purposes, is logically identical with it. Thus "Flowers are fragrant" may be treated as though it were "Flowers are fragrant things"; an attributive proposition, without any change in meaning that affects our subsequent analysis, has become a proposition of class inclusion. Or, "This flower is fragrant" may become "This flower is a fragrant thing," and an attributive proposition has become one of class membership. Second, a class membership proposition may be considered as one affirming class inclusion, by the simple expedient of treating the individual referred to by the subject as though it were a class containing but a single member.[4] Since we shall not give detailed attention to the distinctive properties of class membership propositions, this expedient will be adopted. Thus, "Aristotle was a wise man" will be transformed into a proposition of class inclusion by regarding "Aristotle" as denoting a class of Aristotles which happens to have only one member. A proposition of any of these three kinds may, then, for the present treatment, be dealt with as a proposition of class inclusion.

But in certain respects which may not safely be neglected, class inclusion has different formal properties from those ex-

[4] This procedure leads to no error when there is only one class membership premise in a given inference or when, if there is more than one, they have a common subject term. Otherwise the procedure may lead to an invalid conclusion. Such cases will rarely be met, but their possibility should be kept in mind. Certain other problems arising from this interpretation are dealt with below. See pp. 167 f., 247.

hibited by a typical relational proposition. Hence it is necessary for us to distinguish systematically two sorts of atomic structure in a proposition: the structure required for the validation of inferences employing relational propositions, and the structure to which one must appeal for the validation of inferences concerning the inclusions and exclusions of classes with their attributes [5] and members. The study of the latter, exhibiting its principles in some orderly scheme of interrelationship, is called the *calculus of classes;* a similar study of the former may be appropriately described as a *calculus of relations.*[6]

It is evident now that the further pursuit of our task will require a systematic analysis of the atomic structure, on the one hand, of relational propositions, and on the other, of the three remaining kinds of proposition as they function in our inferences. Certain features of such an analysis, however, are common to both these types of atomic structure. The present chapter will conclude with an exposition of these quite general features that are discoverable in any kind of proposition, and with practice in discriminating their implications.

Subject and predicate terms

In the first place, any proposition has two or more *terms.* In the propositions: $a \mathrm{R} b$, $x \mathrm{R} y$, $x \epsilon b$, $a f$, and $a \subset b$, a and x symbolize *subject terms.* In these five cases b and y symbolize *predicate terms.* And f may at will be replaced by b, denoting the class determined by the circumstance that all its members possess the attribute f. Thus it will be evident that a term is any determinate entity or group of entities which may stand at either end of any of the sorts of connection assertible by a proposition. The subject term (or terms) denotes the entity or group of entities about which the affirmation is made. It may refer to an individual, certain specified individuals, or a class. And, when attributive propositions are reinterpreted as propositions about classes, the predicate term also refers either to an individual, certain specified individuals, or a class. It

[5] The noun, "attribute," will be used hereafter to mean "nonrelational property."

[6] This phrase is reserved by some for the more technical developments of such a study.

must be kept in mind, however, that the predicate term can properly denote an individual only in relational propositions; in other cases it must refer to a class.

Thus far, the analysis has primarily contemplated propositions with a single subject and a single predicate term. And propositions of class inclusion may always be construed in such a way that this is the case. Relational propositions may, however, have any number of terms, and this possibility has been indicated by parentheses in the preceding paragraph. Proposition (9) in the list given earlier in the chapter illustrates this situation—"x is between y and z." This proposition has three terms, and there might have been more. In such cases the usual method of symbolizing the proposition is to begin with the symbol for the relation and then follow it by the symbols for the terms in their proper order. Thus the accepted way of symbolizing a three-term relation is $R(x,y,z)$. Indeed, to preserve a parallel symbolic form, writers sometimes indicate two-term relations in the same manner, employing the symbol $R(x,y)$ instead of $x \mathrm{R} y$.

Quantifiers

In the second place, any proposition whose subject term denotes a class is *quantified*. By this is meant that what is asserted is asserted either about the whole membership of that class or about some portion of it. The former case is indicated by the presence of a *universal* quantifier preceding the subject term, such as "all," "every," "any," etc., and the proposition is said to be universal in quantity. The latter case is revealed by the presence of a *particular* quantifier such as "some," "many," "a few," etc., and the proposition is said to be particular in quantity. Consider the following propositions:

All the boys in this class are brilliant students.
Eighteen per cent of the voters in this state are illiterate.
Some New Yorkers are inhabitants of Long Island.
No diagonal of a square is commensurate with its side.
A few turkeys in this lot are not expensive.

The first and the fourth of these are clearly universal. The word "no" in the fourth is equivalent to "not any"; it is nega-

tively universal, while the first is affirmatively so. The others are particular in quantity. But notice that in their case the quantifiers vary in what they imply; certain among them are very *vague* in the quantity they indicate, while others are quite *definite* in this respect. "Some" is the vaguest; it means "at least one." "Eighteen per cent of" is quite definite; it indicates exactly how large a part of the subject class it is about which the assertion is made. "A few" falls between these two particular quantifiers; it is more definite than "some," but less so than a numerical fraction. "Many," "most," etc., are similar in this respect.[7] Thus, before submitting any proposition to a logical operation, it is necessary to note whether it is universal or particular, and if the latter, how vague or definite the indication of quantity is. For of course the implications of the propositions differ correspondingly.

Wherever a verbal indication of quantity is used, as in each of the above propositions, this is quite easy. But there are cases in which no such indication is present, and it is necessary to reflect a moment to make sure of the meaning with respect to quantity. Thus:

> Cats are fighting in our back yard

is evidently a particular proposition; it means some cats, not all cats. Whereas,

> Cats instinctively catch mice and birds

is universal. The relation is meant to be affirmed of all cats. If careful consideration of a proposition still leaves its quantity ambiguous, and study of the context gives no clear light, one may not safely use it as a premise until the ambiguity has been cleared up.

It has been noted that propositions about individual entities often enter into the same piece of inference with propositions about classes. Now the former are not quantified, but

[7] Some writers recognize these distinctions by calling propositions quantified by "some" *basic* particular propositions; those quantified by "many," "most," etc., *plurative* particular propositions; and those quantified by a definite number or per cent, *numerical* particular propositions.

traditional rules of validity covering such inferences assume that all premises can be interpreted in terms of quantity.[8] So, it is necessary to decide what interpretation is correct in this situation. The problem especially concerns class membership propositions when functioning in inference with class inclusion propositions. Consideration shows that the quantity of any such proposition must be regarded as universal. The individual entity is, in such situations, treated as a class of one member, and whatever is affirmed about that member is therefore affirmed about the whole class.

Los Angeles is a populous city.
Socrates was an Athenian citizen.

Particular use of "all" and "every"

At certain points linguistic usage is hostile to the requirements of clear thinking. Universality and particularity are matters of the meaning of a proposition, and are, therefore, not infallibly indicated by any set form of words. This is obvious especially in the case of "all" and "every." When these are employed in a negative proposition, one must be on his guard, as will be seen from the following:

All that glitters is not gold.
Every man that does such things is not crazy.

Clearly these are both particular, not universal, propositions. The first is equivalent to: "Not all that glitters is gold," *i.e.*, "Some things that glitter are not gold"; the second to: "Some men that do such things are not crazy." We may deplore such ambiguities of language, but when they have become rooted in common speech there is no way but to accept them and to guard carefully against deception by them. The way to do this is to form the habit of asking just what meaning the proposition is intended to convey.[9]

Logical quality

Thus it appears that propositions always have a logical

[8] See below, p. 247.
[9] It is well also to form the habit of expressing such a proposition in unambiguous form—using the quantifier, "No."

quantity, that this is either universal or particular, and that in the latter case the quantity is either vague or more definite. But a further distinction must be made besides that in terms of quantity. Certain of the above propositions *affirm* something of the subject in the predicate; others *negate* something instead of affirming it. The latter are the two following:

> A few turkeys in this lot are not expensive.
> No diagonal of a square is commensurate with its side.

Of these, one is universal, the other particular, but both are negative instead of affirmative. This negation is indicated by the word "not," or by "no," which is equivalent to "all . . . not." That is, a few turkeys in the lot are *excluded* from the class of expensive things instead of being included in it; the diagonal of a square is *denied* the relation "commensurate with" to its sides. Now this character of a proposition, by which it may be affirmative or negative, is called its *quality*, and besides affirmative and negative there is no third possibility with respect to quality.

It will be well to clear up, at this point, a possible difficulty that may occur to the reader. As the discussion of contradiction and contrariety in Chapter 8 has indicated, "not," from the molecular point of view, means the denial or contradiction of the proposition it qualifies as a whole. But from the viewpoint of atomic analysis, it means a denial that the predicate term is connected with the subject term in the way that would be asserted if it were absent. Hence, for atomic analysis, it becomes one of the internal elements of the proposition, just like the subject and predicate terms themselves.

The copula and its logical nature

What element, if any, in the structure of a proposition is represented by the words "is" or "are"? These are traditionally regarded as expressing the *copula* of the proposition. Well, in all cases one may no doubt say this—that these words fill the role of setting the subject and predicate *in meaningful connection* with each other, although not in all cases are they indispensable to this end. "Flowers fragrant," for example, would naturally be interpreted to mean the same as "Flowers are

fragrant."[10] But there is a second role that is ordinarily, though not universally, filled by these words. It is evident that a proposition does not always need to be *asserted.* It may be weighed, doubted, enjoyed, or just contemplated on occasion. However, when there is no intention of asserting it, the connection is usually expressed in some other way than by employing "is" or "are" in the simple fashion of the copula. Instead of saying "*x* is taller than *y*," one says "*x* may be taller than *y*," or wonders "whether *x* is taller than *y*," in such cases. Now this role of assertion is clearly not an element *within* the proposition, since it affects the status of the proposition as a whole. The other role, that of uniting subject and predicate in some significant and relevant way, is most conveniently regarded as such an element, since, in that case, atomic analysis is able to take account of every word or phrase that is likely to appear in the accurate verbal expression of a proposition.[11]

Summary list of logical elements of a proposition

Analysis of the internal structure of a proposition, so far as it can be pursued without reference to the distinctive character of relational propositions and propositions of class inclusion, is now complete. It is evident that every proposition has five logical elements, each of which must be carefully taken into account, from the atomic viewpoint, in determining its truth or falsity and its bearing on the truth or falsity of other propositions. These elements are: subject term, predicate term, quantifier, copula, and indication of quality. In the case of relational propositions there is a sixth element also, namely, the relation, which is absent in the others. A proposition may be such that each of these six elements is represented by some specific word or phrase, as in the following case:

Some of	the coins	are	not	in	the purse.
Quantity	Subject term	Copula	Quality	Relation	Predicate term

Several of the elements may not be so represented, as in the proposition:

Ripe mangoes are delicious.

[10] In many languages no word corresponding to "are" is employed in such cases.

[11] There are still other meanings of "is" and "are," as, for example, to indicate the relation of identity or that of equality.

Since this is not a relational proposition, it contains only five elements, but two, even of these, find no explicit verbal expression. The quantity (universal) is understood without the presence of any quantifying symbol, and since the proposition is affirmative no word is needed to indicate the quality. But the five elements are all present; they are determined by the logical nature of the proposition, not by a merely grammatical analysis.

The square of opposition

The bearing of the distinctions of quantity and quality on the validity of inferences is so important that before proceeding further it is well to engage in some practice designed to reveal their implications vividly. One of the best studies which afford such practice is a systematic examination of what has been traditionally called the *square of opposition.* The guiding problem, answers to which are visually illustrated in the square, is this: Given a certain proposition as true or false, what is implied by it about the truth or falsity of propositions otherwise identical, but differing in quantity, quality, or both? To prepare for such a study, symbols must be assigned to each of the kinds of proposition which the quantitative and qualitative analyses require one to recognize. And as a result of those analyses it is evident that any given proposition must fall under one of these four heads: universal affirmative, universal negative, particular affirmative, or particular negative. To avoid wearisome repetition of these phrases they are customarily symbolized by the four letters A, E, I, O (the first two vowels of the Latin words *affirmo* and *nego*), as follows:

> A universal affirmative proposition is an A proposition.
> A universal negative proposition is an E proposition.
> A particular affirmative proposition is an I proposition.
> A particular negative proposition is an O proposition.

The student of formal science must become so familiar with these symbols that a given proposition can be described at once, as to quantity and quality, in terms of them. Of course, one must not forget the difference between the implications of a vague particular proposition and those of a more definite

one, but this difference can be taken into account without complicating the scheme by further symbols. In the absence of specific indication, I and O propositions are to be taken as vague particulars, with no more definite quantifier than "some."

Restated in terms of these symbols, the problem now to be faced is: What is implied by the truth or falsity of an A proposition about the truth or falsity of the E, I, or O proposition containing the same subject term, predicate term, and relation (if the proposition be a relational one)? What is implied by the truth or falsity of E about A, I, and O; and similarly,

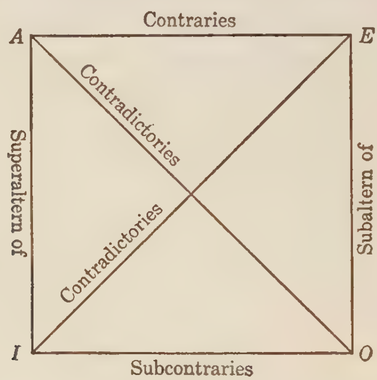

FIG. 2.

what by the truth or falsity of I or O? Clearly, the result of comparing any two such propositions in this way must exemplify one or another of three possibilities: If A is true, then E must be either true, false, or indeterminate—the last-named adjective meaning that one cannot tell from the truth of A whether E is true or false. And so with each of the other cases. The square of opposition charts the pattern of these various implications is shown in Fig. 2.

Contradictory propositions

The concepts "contradictory," "contrary," and "subcontrary," have already appeared in the molecular examination of propositions. Their meaning remains the same here. To say that A and O are contradictories is to say that if either is taken

as true, the other must be held false, and that if either is taken as false, the other must be held true. If, for example, it is true that "All the pictures are on the wall," then it must be false to say that some of the pictures are not on the wall. If it is false, on the other hand, that all the pictures are on the wall, then it must be true that some of them are not on the wall. Likewise, if it is true that some of the pictures are not on the wall, it must be false to say that all of them are there; if it is false that some of them are not on the wall, it accordingly follows that all of them are there—that is, that the corresponding A proposition is true. The student may analyze in similar fashion the contradiction between E and I.

Contraries and subcontraries

A and E are contraries, not contradictories. This means that if either is taken to be true the other must be held false, but that both may be false.[12] That is, it is impossible to argue, as in the case of contradictories, from the falsity of one to the truth of the other. Thus, if it is true that all the pictures are on the wall, it must be false to say that none of them are on the wall. Similarly, if the latter proposition is true, the A proposition cannot be true. But if it is known to be false that all the pictures are on the wall, one can only affirm that some of them are not there; he cannot assume that none of them are there. For it is obvious that some might be there, and some not. The same, of course, holds with reference to A if E is assumed to be false.

In the case of subcontraries, the opposite situation obtains. The falsity of one implies the truth of the other, and both may be true. Thus, if it is false that some of the pictures are on the wall, it must be true that some are not there—indeed, one knows more; that none of them are there. But if it is true that some of them are on the wall, the O proposition cannot be held false, for evidently some might be there, and some not.

Contrary and contradictory fuse in propositions about individuals

It is important to note that no distinction can be drawn between contrary, contradictory, and subcontrary in the case of propositions about individuals, for to speak of the subject of such a proposition at all is to speak of the whole of it. Here,

[12] Cf. above, pp. 153 f.

then, the only opposition is between affirmation and negation; consequently, each must be the full contradictory of the other. If Socrates is the wisest man in Athens, then it must be false to say that he is not the wisest man in Athens; also, if it is false to hold him the wisest man in Athens, the proposition that he is not the wisest man there must be true.

Super- and subalternation

The other relation, that between the universal and the particular of the same quality, remains to be examined. Notice that it is differently described according to whether one passes from the universal to the particular or from the particular to the universal. The particular is the subaltern of the universal; the universal is the superaltern of the particular. Without taking into account, for the purpose of the present practice, a distinction later to be emphasized,[13] it is clear that whatever is true of a universal is true also of the particular of the same quality, but that what holds true of the particular may or may not hold true of the universal. Hence the superaltern in the square of opposition is a special case of the superimplicate; the subaltern, of the subimplicate.[14] Thus, if it is true that all the pictures are on the wall, it must be true that some of them are on the wall; but if one knows that some of them are there, whether all of them are or not is indeterminate. In common speech "some" is often used to imply "some, but not all." This is not, however, its exact logical meaning; that is simply "at least one." "May be even all" is quite consistent with it. The same applies to certain of the more definite quantifiers. "Many," for example, should be taken to mean, strictly, "a large relative number, and possibly all"; "most," to signify "more than half, at least, and perhaps all." When the quantifier is a number or per cent, however, the implication may vary with the intent of the asserter as indicated in the context. Usually one is to assume, in such cases, that a careful investigation has been made, as a result of which it has been found that the predicate can be truthfully

[13] Cf. below, pp. 226 ff., 231 f.
[14] Cf. above, p. 140.

affirmed merely of the portion of the subject class that is specified. But such quantifiers may be used more loosely. If the I proposition is taken to symbolize an indefinite particular, on the other hand, the corresponding A must clearly be pronounced not false, but indeterminate. It is also clear that if the particular is false, the universal must be false too, whereas if the universal is false the particular is indeterminate. If "some of the pictures are on the wall" is false, then it is surely false to say that all of them are on the wall; but if the latter is false it does not follow that some of them may not be there. The student may work out the relations between E and O in similar fashion.

The implication of *most*, *a few*, *many*, etc.

An important point to be noted, not symbolized on the square, is that where the relation between a vaguer and a more definite particular quantity is under consideration, the same implication obtains between the definite and the vague as obtains between the universal and the corresponding particular. Thus, if it is true that "most Christians are not Orientals," it must also be true that "some Christians are not Orientals." If many of my class are sophomores, it may be deduced that some of them are sophomores. But if a vague particular is given as true, any more definite particular is indeterminate. If all that is known is that some of the class are sophomores, it does not follow that many, most, or even a few of them are sophomores. Perhaps only one in a very large class is a sophomore.

EXERCISES

1. Classify each of the following as relational, class membership, attributive, or class inclusion propositions. Express the attributive propositions as ones of class inclusion. Indicate in each case the subject term, the predicate term, the quantity, the quality, and the relation (if there is one).
    a. John is a Democrat.
    b. These boys are healthier than those.
    c. All men are mortal.
    d. Russia is a communist state.

e. Whales are mammals.
f. Most mountain lakes are cold.
g. Some blocks are cubes.
h. Many roots are not edible.
i. A few Indians massacred the inhabitants.
j. Love is stronger than hate.
k. All rivers flow downward.

2. Given as true the proposition: None of the food is spoiled; are the following true, false, or indeterminate?
   a. Some of the food is spoiled.
   b. Most of the food is not spoiled.
   c. All of the food is spoiled.
   d. Half the food is spoiled.
3. Given as false the proposition: Some service is not voluntary; what is implied about the following?
   a. No service is voluntary.
   b. Some service is voluntary.
   c. All service is voluntary.
   d. Most service is not voluntary.
4. Given as true the proposition: Some beggars are not fakes; what follows about:
   a. Some beggars are fakes.
   b. A few beggars are not fakes.
   c. Many beggars are fakes.
   d. All beggars are fakes.
   e. No beggar is a fake.
   f. Most beggars are not fakes.
5. What kind of opposition is illustrated as we pass from each of these propositions to the one which follows it?
   a. All turkeys are more expensive than ducks.
   b. Some turkeys are more expensive than ducks.
   c. No turkeys are more expensive than ducks.
   d. All turkeys are more expensive than ducks.
   e. Some turkeys are not more expensive than ducks.
   f. Some turkeys are more expensive than ducks.
   g. All turkeys are more expensive than ducks.
6. Given as true the proposition: Most of the days in the dry belt are clear; what follows as to:
   a. All days in the dry belt are clear.
   b. No day in the dry belt is clear.
   c. Most of the days in the dry belt are not clear.
   d. Some of the days in the dry belt are not clear.
   e. Some of the days in the dry belt are clear.

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CHAPTER 10

RELATIONS AND CLASSES

The distinctive structure of relational propositions

Now let us turn, first to relational propositions and then to propositions of class inclusion, and consider in some detail those features of internal structure in which they differ.

In the case of the former, the distinctive element is, of course, the relation. What kind of thing is a relation, so far as concerns its formal nature as disclosed in atomic analysis?

As a preliminary to answering this question, it will help us to consider a list of relational propositions exhibiting several different kinds of relation. In order that the characteristics of the relations may be brought out vividly, and distraction by other factors avoided, we shall choose propositions whose quantity and quality do not obtrude, and shall replace subject and predicate terms by appropriate symbols, while expressing the relations in ordinary English words.

1. x is north of y
2. x is in tune with y
3. x is between y and z
4. x is father of y
5. x is ashamed of y
6. x is equal in area to y
7. x sent y to z by w
8. x voted for y

Their classification according to degree

Perhaps the first thing that a reader will note in contemplating such a list is the number of terms involved in each case. In most of these propositions there are only two terms, but in one case (the third) there are three, and in another (the seventh) there are four. And it is clear that these differences are implied by the nature of the relations concerned.

The relation "between" is of such a character that no intelligible proposition is formed if we say merely "*x* is between *y*"; three terms are required to yield an intelligible meaning. The same is true of the relation "sent . . . to," and if a further preposition such as "by" is added, four terms are evidently needed. In the other cases, a meaningful proposition is present with two terms only. Now this property of a relation, in virtue of which a certain number of terms is required in order to make a proposition employing it significant, is its *degree*. A relation which demands only two terms is, in degree, a *dyadic* relation, the adjective here being derived from a Greek root meaning "couple." Thus "north of," "in tune with," "father of," etc., are dyadic relations. A relation which demands three terms, such as "between," is *triadic;* one requiring four terms is *tetradic;* five terms, *pentadic;* and so on. For convenience' sake it is usually sufficient to refer to any relation requiring more than three terms as *polyadic,* that is, as a "many-termed" relation.

The referents and relata of a relation

Consideration will make it evident, in the next place, that the nature of a relation imposes certain restrictions on the kind of terms which can significantly enter it as well as on their number. It determines an appropriate universe of discourse. Terms, for example, which enter the relation "north of" must be geographical points, areas, or entities occupying them; were they anything else the proposition would be meaningless. Similarly, the subject term entering the relation "father of" must be a male organism, and the predicate term must be an organism of the same species or a hybrid. Two words have become technical for denoting this circumstance, in the case of dyadic relations. The terms which are capable of significantly filling the role of subject term in a proposition using a given relation are called the *referents* of that relation, while those capable of filling the role of predicate term are called its *relata.* Thus, male organisms constitute the referents of the relation "father of"; entities located in, or occupying space on, the earth's surface constitute both referents and relata of the relation "north of."

Confining our attention now specifically to dyadic relations, which are by far the most common and important ones, we must note three principles in accordance with which any such relation may be classified. All are illustrated in the above list.

Reflexiveness of relations

One such principle is that of reflexiveness. Consider propositions 2 and 6. These relations are such that any term capable of significantly entering them must stand in the relation concerned to itself—x R x and y R y necessarily hold. Such relations are called *reflexive* relations. Certain other relations, with equal obviousness, are quite incapable of holding between any referent or relatum and itself. "North of" and "father of" are such relations; they are called *irreflexive* relations. And of course some relations are indeterminate with respect to reflexiveness—that is, they are such that a given referent or relatum may or may not stand in that relation to itself. These are called *nonreflexive* relations. This is the case with the relations in propositions 5 and 8. One capable of shame may be ashamed of himself, or he may not; and one capable of casting a vote may vote for himself or he may not.

Symmetry

Another principle for classifying any dyadic relation is that of symmetry. Notice that the two reflexive relations in the above list are also such that if any referent stands in that relation to any relatum, the relatum must stand in the same relation to the referent. In their case, $x \text{ R } y \mathrel{-\!\!3} y \text{ R } x$. Relations of this kind are called *symmetrical* relations. Most reflexive relations are also symmetrical. If a reflexive relation be such, however, as to imply a disjunction, it may not be symmetrical. Consider the relation, "as tall as." It is clearly reflexive, and if the adverb "just" were prefixed to it, it would be symmetrical too. As it stands, it implies "either of the same height as, or taller than," and this is not a symmetrical relation: x might be as tall as y, and yet y not be as tall as x. Relations such that, if any referent stands in that relation to any relatum, it is impossible for the relatum to stand in that relation to the referent, are called *asymmetrical* relations. In the above list, propositions 1 and 4 exemplify asymmetrical relations. Other instances are "above," "preceded," "more

beautiful than," "explorer of." Relations such that, if any referent stands in that relation to any relatum, the relatum may or may not stand in that relation to the referent, are called *nonsymmetrical.* The relations in propositions 5 and 8 are obviously *nonsymmetrical;* other instances are "hit," "greeted," "fond of."

Transitivity

A third principle, and the most important of all from the standpoint of the validity of relational inferences, is that of transitivity. A relation is *transitive* when, if a term stands in it to a second term, and the second stands in the same relation to a third term, the first must stand in that relation to the third. A symbolic definition is simpler; a relation R is transitive if $x \text{ R } y \cdot y \text{ R } z : \supset x \text{ R } z$. Of the relations in our list, it is evident that "north of," "in tune with," and "equal in area to" are transitive.

x is north of y.
y is north of z : $\supset$
x is north of z.

x is in tune with y.
y is in tune with z : $\supset$
x is in tune with z.

x is equal in area to y.
y is equal in area to z : $\supset$
x is equal in area to z.

Other common transitive relations are "less than," "more probable than," "descendant of," "contained in." A relation such that, if $x \text{ R } y \cdot y \text{ R } z$, x cannot stand in the relation in question to z, is called *intransitive;* proposition 4 in the list before us illustrates such a relation. And relations such that, if $x \text{ R } y \cdot y \text{ R } z$, x may or may not stand in the same relation to z, are called *nontransitive.* "Ashamed of" and "voted for" are clearly nontransitive as well as nonreflexive and nonsymmetrical.

It will promote familiarity with the formal character of each of these three principles of division to observe: that when

considering the reflexiveness of a relation we need have in mind only one term; that when we consider its symmetry it is helpful to think of two terms; and that the meaning of transitivity is clarified most simply by referring to three terms. The last named contemplates an inference in which at least two relational premises jointly are required to entail the conclusion drawn. In fact, the symbolic definition of "transitive," as given above, expresses succinctly the validating form that is applicable to the many common inferences consisting of three relational propositions and employing the same transitive relation in each of them.

In the present volume we shall not take the space to go into the further problem: What validating forms apply to inferences when one premise employs a transitive relation and another premise connects subject and predicate terms in some other way? For example, consider the following case:

> x equals y.
> y is unequal to z : ⥽
> x is unequal to z.

The student will find it interesting to experiment with other cases, and to see if he can formulate appropriate rules.

Relational inferences in exact science

Relational inferences frequently occur in daily life. But a further reason why study of their validating forms is important to the student of reasoning is that the distinctive modes of inference employed in the exact sciences rest upon logical foundations which require for their adequate statement the principles just outlined. Now it is always a gain to render our thinking as exact as possible. The ends sought in any serious inquiry are better secured when the propositions employed are such as to permit precise comparison of subject and predicate terms than when they are not. If it is good to be able to affirm, when considering two objects, that x is longer than y, it is better to be able to affirm that it is, for example, just twice as long as y. For one may draw some important deductions from the latter proposition that cannot be drawn from the former. Hence it is important to see what such precision means

and how it is attained. The most familiar exact science, namely, arithmetic, is a good illustration of a formal science which uses relational propositions in such a way that the conclusions drawn exhibit not only logical necessity but also quantitative exactitude. We are already familiar with propositions which are quantified with respect to the subject term, but with no numerical exactitude being secured; the only quantitative distinction recognized has been that between "all" and "some" (except for the occasional appearance of more definite particular quantifiers). With arithmetic and some of the other mathematical disciplines there is a basic contrast in this respect; the relations, too, may be exactly quantified as well as the terms. And applied arithmetic secures this exactitude in dealing with factual objects and events. The following chapter will be occupied with the further discussion of this theme, so far as is necessary to clarify the main foundations of mathematical inference.

Why validating forms for inferences of class inclusion are important

But the majority of our everyday inferences deal with matters in which mathematical science has so far found little systematic application, and in which even nonquantified relations are absent. They naturally employ, for the most part, attributive propositions and propositions of class membership or inclusion. No basis for comparing one thing with another, whether inexactly or exactly, is systematically established. This is true of reasoning about most problems of daily practical experience and of attempts to establish warranted conclusions in such fields as law, morals, politics, history, and religion. In fact, the history of science seems to show that at an early stage almost all human knowledge is expressed in such propositions; only gradually does attention center upon the relations between things, and appropriate methods of analysis and measurement are found whereby more and more of the properties which underlie those relations are transformed into the kind of material in which quantitative exactitude can be realized.[1] Then propositions realizing numerical precision come to supplement, and sometimes to replace, propositions

[1] A reason for this circumstance is suggested below. See pp. 418 f., 558 f.

which lack this character. However, such progressive conquests seem to leave plenty of material as yet untransformed, and it is probable that there will always be many fields in which men need to think as clearly as they can but in which statements in mathematical form are not thus far possible, and relational comparisons have little place. Indeed, even though the contrary may be the case, hope of greater exactitude in the future can hardly take the place of power to discriminate in the present between validity and invalidity in inferences lacking such precision. When we attempt to reach conclusions on these matters, likewise when the editor, the orator, or the preacher tries to convince us of his pet ideas about them, it is important to know in just what ways valid inferences can be constructed out of class inclusion propositions and propositions which for our limited purposes may be treated as such. Accordingly, it is equally essential to analyze such propositions as they are interrelated in inference, noting and appropriately symbolizing the distinctive elements in whose terms validating forms for these inferences must be stated.

Illustrations of such inferences

It has already been observed that every proposition of class inclusion possesses five logical elements: quantifier (expressed or understood), subject term, copula, indication of quality, and predicate term. And the discussion of transitivity earlier in the chapter leads readily to the first step of the present inquiry. This concept, it will be remembered, could only be defined by reference to three distinct terms, and its use as a principle of inference contemplates a situation in which two premises are conjoined in order to yield a valid conclusion. Most class-inclusion inferences, as employed in ordinary life, likewise contemplate such a situation. Consider the following illustrations:

1. All red-haired boys are mischievous.
   John J. is a red-haired boy.
   Therefore, he is mischievous.
2. No airship is safe.
   Some of these machines are airships.
   Therefore, some of these machines are not safe.

3. Most business men are progressive.
   Most business men are Republicans.
   Therefore, some progressive people are Republicans.
4. Any animal in the barn with long ears is either a donkey or a mule.
   One with a bushy tail is either a mule or a horse.
   This animal has both long ears and a bushy tail.
   Therefore, it is a mule.
5. All the books on this shelf are books about current affairs.
   All my paper-bound books are on this shelf.
   Therefore, all my paper-bound books are books about current affairs.
6. Some true propositions are unpleasant.
   Therefore, some unpleasant propositions are true.
7. All freshmen are eligible.
   Therefore, no freshmen are ineligible.
8. No streets across the brook are paved.
   All the streets of subdivision A are streets across the brook.
   The streets intersecting at your lot are streets of subdivision A.
   Therefore, the streets intersecting at your lot are not paved.

Examination of their structure

Some of these inferences consist of two propositions, some of three, and some of four. But examination of inferences 6 and 7 will show that those involving only two propositions are inferences in a rather Pickwickian sense. Certainly, the conclusion is validly implied by the premise, but it is merely another way of saying precisely the same thing that the premise does. In 6, both premise and conclusion affirm that the class of true propositions and the class of unpleasant propositions have at least one common member. The only difference is that the class referred to as subject in the premise appears as predicate in the conclusion, and vice versa. In 7, a proposition is in the premise expressed in affirmative form; in the conclusion, the same proposition is expressed in negative form. These inferences, then, exhibit strict equivalence between implicans and implicate; each can be substituted for the other. But rarely would this occur in anyone's actual reasoning (outside of mere practice with such equivalences) except as a subsidiary process in some more complex inference, where, in order to bring out more clearly the logical interrelations of the propositions involved, it appears desirable to substitute one

way of expressing a given proposition for another. Inference 8 consists of four propositions, but it can easily be broken by analysis into a connected pair of inferences, each consisting of two premises and a conclusion.

No streets across the brook are paved.
All the streets of subdivision A are streets across the brook.
Therefore, no streets of subdivision A are paved.

No streets of subdivision A are paved.
The streets intersecting at your lot are streets of subdivision A.
Therefore, the streets intersecting at your lot are not paved.

Inference 4 is likewise reducible to a chain of inferences of not more than two premises and a conclusion each,[2] but the process is complicated, and premises not explicitly given must be assumed. It would be more circuitous to test it by such a reduction than to employ a method directly applicable to complex cases. It may, therefore, serve to illustrate the fact that inferences of the sort now considered may exhibit any degree of complexity, the premises asserting inclusions or exclusions between any number of different classes.

But 1, 2, 3, and 5 of the above list, together with the two simpler inferences into which 8 was analyzed, exemplify a common pattern, despite obvious differences in detail. We shall take this pattern as especially important, examining with care its internal structure and determining the conditions of validity that are pertinent to it.[3] In the present chapter this examination will be carried only as far as is necessary to reveal the main features of the structure involved.

Subject, predicate, and middle terms

Each of the inferences which conform to this common pattern consists of three propositions, two of which are premises

[2] As shown by my friend Professor H. H. Dubs. The chain consists of four inferences with two premises and a conclusion each, together with a substitution of one equivalent for another.

[3] Illustrations of class inclusion inferences, many of which do not conform to this pattern, are given in W. V. Quine, *Elementary Logic*, pp. 149–152. Validating forms which apply to such inferences as well as to those here considered are possible, but they require more technical preparation than will be provided in the present book. A number of such inferences, though not all, can be restated so that they exemplify the pattern here analyzed.

and the third a conclusion which they are jointly supposed to entail. The conclusion can be drawn only when the premises are brought together, and hence comes as a new idea to one who has never inferred it before. And just as in the case of the deductions employing a transitive relation that were above considered, each inference contains just three terms. Two of these terms are common to the conclusion and to one of the premises; they are technically called the *subject term* and the *predicate term* of the inference as a whole, according to their position in the conclusion. Sometimes the subject term is called the *minor term* and the predicate term the *major term;* correspondingly, the premise in which the predicate term appears is the major premise, while the premise containing the subject term becomes the minor premise. Discrimination between major and minor premise is therefore in these cases a quite arbitrary matter. Symbols are customarily assigned to the terms as follows: the subject term in any such inference is symbolized by a capital S, and the predicate term by a capital P. The third term, which appears in each of the premises but drops out in the conclusion, is called the *middle term,* and is symbolized by a capital M. For obvious reasons, it is more convenient to employ these symbols in this context than the more general symbols for classes, *a, b,* and *c.* Other elements in the three propositions being, for the moment, neglected, the pattern of these inferences, closely conformed to in every case but (3), may be briefly exhibited as follows:

M is P. S is M: therefore S is P

How the inferences are interpreted

It will be noticed that some of the propositions appearing in these inferences are attributive propositions, *e.g.*, the major premise of (1). Some are class membership propositions, as in the minor premise of the same inference. But they are now to be treated as class inclusion propositions, in the manner explained in the preceding chapter. And the only further essential point to be noted regarding the structure of such inferences is that each proposition is quantified and qualified, in

the manner likewise illustrated in that chapter; each is an A, E, I, or O proposition. Hence every premise or conclusion is to be interpreted as asserting one or another of four things: the total inclusion of the subject class in the predicate class (an A proposition); the total exclusion of the subject class from the predicate class (E); the partial inclusion of the subject class in the predicate class (I); or the partial exclusion of the subject class from the predicate class (O). In the case of the particular propositions I and O, of course, the quantifier may be vague or more definite. In the above inferences all are vague except for the two premises in (3). The following analysis will neglect instances in which more definite particular quantifiers than "some" are employed, as consideration of them would considerably complicate the study. Hence, hereafter, I and O are to be taken as symbolizing propositions of vague particular quantity only.

The inferences symbolized

It will be worth while to display more vividly these essentials in the structure of the inferences above listed, by symbolizing each of them and specifically indicating the quantity and quality of every proposition. "Therefore" will be represented by the sign $\dashv$, since it is simply a claim that strict implication obtains between premises and conclusion.

1. (A) All M are P.
   (A) All S are M: $\dashv$
   (A) All S are P

2. (E) No M are P.
   (I) Some S are M: $\dashv$
   (O) Some S are not P

3. (I_d)[4] Most M are S.
   (I_d) Most M are P: $\dashv$
   (I) Some S are P

5. (A) All M are P.
   (A) All S are M: $\dashv$
   (A) All S are P

[4] Subscript *d* indicates that the particular quantifier is more definite than the vague "some."

8.[5] (E) No M are P.
(A) All S are M: ⥽
(E) No S are P

In the last case the subject term is not explicitly quantified but it may be assumed from the context that universal rather than particular quantity is to be understood.

Any such inference is a syllogism

A technical word denoting any inference of the kind just symbolized has been familiar in Western thought since the time of the Greek philosopher Aristotle. Such a unit of deduction, consisting of two premises and a conclusion purporting to be entailed by them jointly, each of which is an A, E, I, or O proposition of class inclusion, is called a *syllogism*. The enterprise, then, which will be systematically embarked upon in Chapter 12, may be described as an attempt to answer the question: What are the necessary conditions of a valid syllogism? or: What are the validating forms applicable to syllogistic inference?

EXERCISES

1. Characterize each of the following relations as reflexive, irreflexive, or nonreflexive; as symmetrical, asymmetrical, or nonsymmetrical; also as transitive, intransitive, or nontransitive:

a. Parallel to
b. Inside of
c. Beyond
d. Near
e. Next to
f. Friend of
g. Under
h. As blue as
i. Accompanied
j. Sweeter than
k. Sister of
l. Married to
m. Shot
n. Manufacturer of
o. Searched for
p. Equivalent to
q. More expensive than
r. Shook hands with
s. Just as plausible as

2. Describe as exactly as you can the class of referents of each of these relations; the class of their relata.

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An introductory statement of the nature of the syllogism.
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These chapters give a relatively simple analysis of kinds of relations.

[5] Each of the two inferences into which (8) was analyzed exhibits this structure.

CHAPTER 11

MATHEMATICS

By way of introduction

Both the concept of class and that of relation will now be used to give a brief introduction to the nature of mathematical science. And mathematics, considered from the broad viewpoint of its bearing on correct reasoning, is a big subject. Our discussion of it must be severely limited; the most familiar branch of mathematics, arithmetic, will be used as a basic illustration of the points to be brought out.

We shall commence with an analysis of the formal nature of arithmetical numbers—of the series of "natural" numbers, to be more precise—and then it will be shown briefly how the conception of number there involved is modified and enlarged so that it may cover negative numbers, fractions, imaginary quantities, etc. Finally, we shall deal with the logical significance of mathematics when applied to the counting and measuring of physical objects.

It has been observed in the foregoing discussion, that many of the classes with which we are familiar in daily life are determined by the common possession of some attribute or group of attributes. The class of blue things is determined by the fact that all of its members agree in being blue in color; the class of beautiful objects is determined by the fact that all of its members possess beauty in common. It is possible, too, for a class to be determined by a relation rather than by an attribute. The class of states in the American Union west of the Mississippi River, for example, is so determined; each of those states stands to that river in the relation "west of." Now in all these cases every member of a class is similar to every other member in virtue of the attributive or relational property

which they share in common; any member of the class of blue things is similar to all other members in being blue.

But certain classes are determined not by any such similarity as this between the various members of the class, but rather by a similarity obtaining between the class as a whole and some other class or classes. Consider, for instance, a believer in the "cyclical" theory of history—the view that every historical epoch repeats, in the same order, the same major changes that have taken place in earlier epochs. It is evident that for him the class of "transitions in epoch B" will be determined, not by any similarity they exhibit to each other, but by a similarity which he is convinced must obtain between epoch B as a whole and some earlier epoch A; it is in virtue of such a similarity that he will decide what should be recognized as a "transition in epoch B" and what should not be recognized as a member of this class. If one is convinced (to take an illustrative case) that the history of modern Germany repeats that of ancient Greece, he will divide the former into its subsidiary periods under guidance of whatever division seems most clearly appropriate in the case of the latter. He has no other criterion for analyzing epoch B into the distinguishable briefer periods which form its constituents.

Classes determined by one-to-one correlation

The classes of this sort which are most familiar are the arithmetical natural numbers. On the top of my desk, as I write, is a heterogeneous collection of objects—books, papers, writing implements, blotter, vase, ash tray, calendar, etc. These are all material things, of course, and that fact makes them members of the class "material objects now on my desk." But the same class might be determined in another way. I count these objects, and find that there are just twelve of them. This fact constitutes them a class also—its membership now being decided not by the attribute "being material," or by the relation "being on my desk," but by the circumstance that the members are twelve in number. "Being twelve," however, is no similarity obtaining between the various things on the desk; it is a similarity obtaining between the entire class and all other classes possessing just twelve members.

Let us examine a little more fully what sort of similarity is exhibited in this latter case. On reflection, its essence is seen to be simply this: Whenever we have a class of twelve members we can pair off each member with some given member of any other class of twelve members. For instance, I notice a particular shelf in one of my bookcases on which there stand just twelve books. By counting those books, say from left to right, and counting again the objects on my desk, in any chosen order, I could pair off any particular book with some particular object on the desk, and when I am finished all the members of each class would have been exhaustively accounted for. In technical parlance, there is a "one-to-one correlation" between the members of one class and the members of the other. And this is precisely what constitutes the similarity obtaining between the two classes. Another simple example of such correlation would be found in a country practicing strict monogamy. A one-to-one correlation would obtain between the class of husbands and the class of wives, since for every husband there would be one, and only one, wife, and for every wife one, and only one, husband.

The nature of number

What, then, is meant by the arithmetical number "twelve"? As a preliminary statement, it would seem to mean what is in common between all classes similar to the class of objects on my desk and the class of books on that shelf. And what is it that they have in common? If difficult questions that lie beyond logic are to be avoided, we can hardly say with confidence more than this: They have in common just the particular sort of one-to-one correlation that requires anybody who counts them to stop at the point that he does. But the similarity exhibited in this one-to-one correlation is precisely what makes each of these classes a member in turn of a more general class—the class of all classes that have just twelve members, or, in short, the class of dozens. This class would contain the class of objects on my desk, the class of books on that shelf, and all other groups of twelve things. And this class of dozens would be, then, what is meant by the number twelve, however peculiar the definition might appear at first sight. By paral-

lel reasoning, of course, the number two becomes simply the class of all couples—that is, the class of all classes which are alike in possessing just two members—while the number three is the class of all trios; the number four, the class of all quartets, and so on.

For a formal analysis of arithmetic, however, the number zero must at once be introduced; it is the number from which, ordinarily, any process of counting implicitly begins. Now it might seem impossible to define this number by the method just explained, and if so the method fails at a crucial point. But the definition of zero proves not to be impossible. As a matter of fact, for other purposes as well as the purpose of providing an adequate definition of number, formal science finds it necessary to recognize *null* classes—namely, classes which possess no members at all. Just consider, for example, the class "inhabitants of the earth who have visited the planet Mars." This is a quite intelligible and perfectly definite class, which might sometime have members, but at present has none; it is a null class. In this respect it differs from the class "equators of the earth," which has one member, the class "poles of the earth's axis," which has two, and all classes which have a larger number of members. The number zero, then, is simply the class of all null classes, the class of all classes which lack even one member.[1]

Essential properties of a serial order

Thus far, consideration has been restricted, in the case of each of the numbers that have been mentioned, to that number viewed merely by itself. But of course none of the familiar arithmetical operations would be possible were it not for the further fact that the numbers form an orderly system, or series. In order to explain this system, and the operations it allows, we must pass from the concept of class to that of relation, and in particular to a certain kind of asymmetrical and transitive relation. And since the theme of mathematical order is a vast one, our attention will be limited to a small part of

[1] The significance of the null class in another context will be explained in the next chapter. See below, pp. 211 f. Extensionally, there is only one null class, but this point rests on considerations that need not here be introduced.

the relevant field—to the finite natural numbers (0, 1, 2, 3, 4, etc.) which constitute a serial order of a simple type. Again, a nonmathematical illustration will be of aid.

Let us note some essential characteristics of a genealogical tree in which, as we pass from generation to generation, only the relation of fatherhood is taken into account. Here is a contemporary individual of the male sex with whom we may begin; he will be referred to as X. His father, coming next in the table, will be symbolized by W; the latter's father, by V, and so on, as far back as we wish to trace the genealogy. Now with the ancestral tree thus constituted in mind, let us replace the relation of fatherhood, which is intransitive, by the relation "paternal ancestor of," which includes "father of" but is a more general relation, including also "paternal grandfather of," "paternal great-grandfather of," etc. It is evident that the individuals in the table, bound together by this relation, constitute a serial order having, among others, these three properties:

1. The relation is *asymmetrical.* Obviously, if S is paternal ancestor of W, W cannot be paternal ancestor of S.

2. The relation is *transitive.* If L is paternal ancestor of Q, and Q is paternal ancestor of T, then L must be paternal ancestor of T.

3. Of any two members of the series, one must be paternal ancestor of the other. This property of a relation generating a series, in virtue of which it must hold in one direction or the other between any two members, is called "connectedness." Such a relation, then, is not only asymmetrical and transitive, but also *connected.*

Brief consideration will show that in the case of any relation generating a serial order in this way, its converse will also possess these three properties. The converse relation of "paternal ancestor of" is "male descendant of"; it, too, is asymmetrical, transitive, and connected.[2]

The ancestral

Because this illustration brings out so simply and clearly the important properties just discussed, it has become customary,

[2] It is assumed here that the line of male descendants is unbroken.

in the theory of mathematics, to refer to the order established by any relation possessing these three properties as an *ancestral* order. And since, for purposes of such theory, there is no need to distinguish between the series and the relation uniting its members, we shall call any relation determined by the possession of these properties an "ancestral" relation. Obviously, an ancestral order is always generated by the repetition of some particular relation of a certain kind. It may be generated by the relation, "father of," in which case the ancestral will be "paternal ancestor of"; it may be generated by the converse of that relation, "son of," in which case the ancestral will be "male descendant of"; and it may be generated by many other familiar relations such as "above" or "below" (when a vertical pile of objects is contemplated), "to the right of" (in the case of a succession of objects in a straight line), "contained by" (as exemplified in a nest of boxes), etc. Each of these is capable of generating its own ancestral, and in the latter cases it happens that the ancestral is referred to by the same phrase as is used to denote the relation generating it.

The relation *successor of*

Now, with the illustration which has been employed in mind, there is no difficulty in seeing how the important concept "successor of" in such a series is to be defined. Since the ancestral "paternal ancestor of" is generated by repetitions of the relation "father of," the successor of any individual in the series, say X, will be X's father, that is, the individual who stands to him in the generating relation. As the series has here been symbolized, this successor will be the individual W. Similarly with the ancestral "male descendant of"; since it is generated by repeating the relation "son of," the successor of any individual will be that other individual in the series who is his son. In the series above considered the successor of X will be Y.

The problem of determining the order of succession must, however, be solved differently when the ancestral which constitutes the series of natural numbers is considered. It is obvious on examination that this series, too, reveals the three

properties of asymmetry, transitivity, and connectedness. The order here is generated by the relation "greater than," as applied to the classes which our analysis has identified with the various natural numbers—the class of couples, the class of trios, the class of dozens, and the rest. The relation is asymmetrical, for if one such class is greater than another the other cannot be greater than the one. It is transitive, for if one is greater than a second and the second is greater than a third, then the one must be greater than the third. And it is also connected, for it is clear that of any two of these classes one must be greater than the other. How, in this case, can it be unambiguously determined what class is the successor of any given class?

If we could presuppose the familiar concepts of arithmetic in answering this question there would be no difficulty. It would be possible to define the number three, for example, as the class of classes each of which possesses one more member than any of the class of couples possesses, or as the class of classes which results when to each couple another member is added. But such a procedure makes use of the arithmetical concepts "one" and "addition," and this cannot properly be done until the notions of number and of the operations which can be performed with it have been established. Otherwise we should be reasoning in a circle. A procedure avoiding this circularity, however, and which employs no logical concepts except those explained in the preceding chapters, is as follows:

The successor of any given class of classes determined by one-to-one correlation is the class of classes each of which has a member left over when it is similarly correlated with any of the classes belonging to that given class. Thus the successor of the class of trios, *i.e.*, of the number three, is the class of quartets (the number four), because, if any quartet were correlated with any trio in the one-to-one fashion in which the trios are correlated with each other, there will be a member of the quartet left over. This definition, too, uses the word "one" in the phrase "one-to-one correlation," but it is not the arithmetical number one, as was the case with the definition

suggested in the preceding paragraph. It stands for a purely logical operation of pairing off the members of any class with the members of another. An alternative definition is this: The successor of any given class of classes determined by one-to-one correlation is the class of classes each of which would belong to that given class if a member were dropped. That is, the number two (the class of couples) is the successor of the number one (the class of monads), because if a member were dropped from any couple it would belong to the class of monads.

The whole series of natural numbers defined

With the concepts thus established as a foundation, it is now possible to define, as far upward as we may wish to go, each of the numbers which together constitute the series of natural numbers. The number one will be defined as the successor of zero, two as the successor of one, three as the successor of two, and so on. And our familiar system of Arabic notation can readily be set up. After the numbers from zero to nine have been defined, rules will be formulated as to how the Arabic symbols for these numbers are to be combined to symbolize the successor of nine and any later member of the series. Thus, a general method becomes available for symbolizing any natural number with which we may need to deal. Since there is no end to this series, however, how can the group of natural numbers as a whole be defined? This is done by appealing again to the ancestral relation. Consider once more the ancestral constituted by repetitions of the relation "father of." What individuals will belong to such a series? Obviously, any individual who belongs to it will be either X, or the father of X, or the father of X's father, or the father of the father of X's father, etc.—more briefly, he will be either X[3] or some paternal ancestor of X. By similar considerations, the series of natural numbers will be seen to consist of zero, the successor of zero, the successor of the successor of zero, etc.—that is, it will include zero and all the

[3] I neglect here a fact which the student who pursues the subject will discover, namely, that mathematical logicians define the ancestral sufficiently broadly so that an individual is included among his own ancestors. The reader might be needlessly puzzled by this apparent subtlety.

entities which stand to zero in the ancestral relation constituted by repetitions of "successor of."

Zero is not a successor

One more fundamental principle needs to be established in order that the familiar operations with the natural numbers may be described and justified. This is, that while the series has no upper limit, it does have a lower one; it begins with zero. We must therefore prove, in the terms above employed, that while every number except zero is, by definition, the successor of some other number, zero is not the successor of any natural number. This can be demonstrated as follows: Zero is the class of null classes, that is, of classes each of which has no members. Now let us suppose that zero were the successor of some other natural number which we shall call x. Then, by our definition of "successor," if we applied to any null class the one-to-one correlation which determines the classes belonging to this supposed number x, there would be a member of the null class left over. But this is obviously inconsistent with the nature of the null class, and correspondingly with the definition of zero. Hence, by a *reductio ad absurdum* of the contradictory supposition, it must be concluded that zero is not the successor of any natural number.

Arithmetic is thus a formal science derived from logic

It is well to emphasize again that these foundations of arithmetic have introduced no concepts except, on the one hand, common-sense ideas which are already sufficiently clear (such as "left over") and, on the other hand, technical concepts derived from formal logic—in particular, concepts drawn from the theory of classes and the theory of relations. This shows that arithmetic (and the same is true of the other branches of mathematics) is a formal science whose basic concepts and operations are definable in logical terms. The natural numbers may be thought of as simply logical classes determined in a certain special way, and related in a serial order by an asymmetrical and transitive relation of a certain special kind.

The fundamental operations of arithmetic are readily definable in the context provided by the above analysis.

As for addition, where x and y are any natural numbers, their sum $x + y$ is the number obtained when, beginning with x, the number one is added y times. Or, more exactly (for the definition just given is circular), $x + y$ is the yth successor of x—that is, the successor of the successor . . . (y times) of x. Given our previous definition, this may be directly seen from an illustration of the operation of addition. Let x be any natural number, and suppose that y is the number three. The successor of x will be symbolized $S'x$. Then (by definition)

The familiar arithmetical operations

$$\begin{aligned} &x + 0 = x \\ &x + 1 = S'(x + 0) = S'x \\ &x + 2 = S'(x + 1) = S'(S'x) \\ &x + 3 = S'(x + 2) = S'[S'(S'x)] \end{aligned}$$

When one is added to x the symbol S' occurs once; when two is added it occurs twice; when three is added it occurs three times. And it is evident that any extension of the table would exhibit the same principle. The essential nature of addition may be thus apprehended. Since subtraction is merely the converse operation to that of addition, it may be defined at once without the need of illustration.[4] If x and y are any natural numbers, the remainder $x - y$ is the number obtained when, beginning with x, the number one is removed y times. Or (employing "predecessor of" as the converse relation to "successor of"), $x - y$ is the yth predecessor of x.

Multiplication may be defined by aid of the concept of addition. Where x and y are any natural numbers, the product $x \times y$ is the number obtained when, beginning from zero, x is added y times. Again, given our previous definitions, this may be directly seen by considering an illustration. Let x be any natural number, and suppose that y is the number four. Then

[4] Although the definition now given strictly applies only when y is less than x.

$$x \times 0 = 0 \text{ (if, beginning with 0, } x \text{ is added 0 times, the product is 0)}$$
$$x \times 1 = x + (x \times 0) = x + 0 = x$$
$$x \times 2 = x + (x \times 1) = x + x$$
$$x \times 3 = x + (x \times 2) = x + (x + x)$$
$$x \times 4 = x + (x \times 3) = x + [x + (x + x)]$$

That is, x multiplied by one is just x added to zero once; x multiplied by two is x added to zero twice; x multiplied by three is x added to zero three times; and x multiplied by four is x added to zero four times. Once more it is evident that the table could be extended as far as we like, and that it would continue to exhibit the same principle. Division, being the converse operation to multiplication, is readily definable in terms of subtraction.[5] If x and y are any natural numbers, the quotient $x \div y$ is the number which, when it is subtracted from x y times, will leave zero as the result.

Extension of the concept of number

By the introduction of suitable definitions, the notion of number can be extended from the positive natural numbers, alone thus far considered, to include negative numbers, fractions, irrationals, and complex numbers. In fact, the student of mathematics will have realized that such extensions are necessary if the arithmetical operations just defined are to be available without the restrictions we have had to specify. For example, the operation of subtraction, as above defined, is possible only where the minuend is greater than the subtrahend. But the introduction of negative numbers removes this limitation. Any natural number may be subtracted from any natural number.

But it is essential to observe that, with each of these extensions, the conception of number is itself enlarged so that it progressively plays a more general role in exact reasoning. The conception which is involved in the approach to the number system above followed is that number is essentially the result of a process of *counting distinguishable objects*,

[5] With the same restriction holding as in the case of subtraction; also that division by zero is to be excluded.

and this is indeed one of its most common and important roles. With the extension of the number system to include fractions, this conception is enlarged to include the notion of a *measurable magnitude*. Now measurement includes counting, but it is something more than counting, and it includes it in a special form. On the desk before me lie a number of pencils, which vary greatly in length. If I counted them to determine their number, in the fashion described at the beginning of the chapter, each of them would be regarded as a unit and no account would be taken of their differences. Suppose, now, I wish to secure an exact comparison of each with the others, that is, to express just those differences in some precise fashion. For this purpose I shall take one of the pencils as a standard for the rest, and determine the length of each of the others in relation to it. The result of such a process of exact comparison will naturally be expressed in the form of a fraction. To reveal this most simply, let us take the longest pencil as our standard; we find, then, that one of the others is $1/3$ as long, another is $3/5$ as long, another $5/7$ as long, etc. Now the process of reaching such results presupposes that each of the pencils, so far as its property of length is concerned, is conceived not merely as something that can be counted, but also as a magnitude capable of being precisely measured and, through measurement, compared exactly with other magnitudes of the same kind. Counting, however, is engaged in here too, but it assumes a special form. In order to treat a property in this way it is necessary to analyze it in terms of some appropriate unit, a certain number of repetitions of which will coincide, with such degree of approximation as is required, with any instance of that property which is being measured. Neglecting till later in the chapter the important question how such an analysis is accomplished, let us see the role played by whatever unit is selected for this purpose. In the first case above, it proved necessary merely to analyze the length of the longer pencil in terms of three equal units, one of which was approximately equal to the length of the shorter pencil; in the second case it had to be

treated as a sum of five equal units, three of which approximately equalled the length of the shorter pencil. Here, what is counted is not the two pencils, but the equal units in terms of which their length may be exactly compared; the results of the counting give the numerator and denominator of the fraction which will express the result of the comparison. To facilitate such processes of comparison in the case of any measurable property such as length, a system of standard units capable of being applied to any lengths is conventionally agreed upon, and any fraction expressing the ratio of one length to another can be worked out by the operations of arithmetical calculation that are allowed by this extended conception of number. The most serviceable system of this kind is the metric system, since it employs throughout the decimal principle, which permits any result of measurement to be readily dealt with by numerical operations using the Arabic notation. But other systems are still frequently employed, such as the system whose smaller units are the familiar yard, foot, and inch.

It becomes a principle of formal transformation

The extension of the number system to include negative numbers, irrationals, and imaginaries, involves a further enlargement of the concept of number so that it becomes, in effect, simply a principle of formal transformation. I mean by this that it becomes a method for performing any operations in which one proposition is validly substituted for another. Let us illustrate briefly how this is so. By no process of ordinary counting or measuring could one discover a negative quantity—a number of objects that is less than no object, or a magnitude that is less than no magnitude at all. But in solving many problems it is very convenient to engage in intermediary operations which assume that subtraction, the extraction of roots, and other familiar processes, can always be performed, even though the immediate result can be given no meaning in terms of actual objects or events. The mathematician would be limited intolerably if this were not allowed. When he is dealing with actual objects, a final result can always be reached which does have meaning in terms of pos-

sible processes of counting and measuring, but this result may often be reached most readily and economically by passing through intermediate steps that cannot be so interpreted. For example, I see that my book shelves are sagging noticeably in the middle, and decide to rebuild them so that each shelf will be shorter. But I want to plan the new arrangement so that all the books will be provided for and a certain space left for new purchases. To work out such a plan I would engage in algebraic calculations in which, let us say, x represents the average number of books on each of the new shelves and a the average number on each of the present shelves. In the course of the calculations I might find, at some step, that I was dealing with the quantity $x - a$. Now, in terms of the problem faced, this means less than no books at all and is therefore absurd. But if by passing through this step I can most conveniently reach a result that does give an answer to the original problem, I need not be troubled in the least by this circumstance. I know that if my original premises, interpretable in terms of counting or measurement, are true, and if I have used the rules of valid substitution consistently, the final conclusions, also thus interpretable, must be true. And it becomes evident that the function of number, as employed in the intermediate steps, is simply to serve as an instrument of valid formal transformation of the propositions with which I began. When a theory of mathematics is worked out in these terms it merges with a theory of formal logic; the two together constitute a single system, in which no line other than an arbitrary one can be drawn between logic and mathematics. From properly chosen definitions and postulates the whole body of formally valid propositions can be derived, including those which were traditionally thought to belong to the logician's field and those which constitute the theorems of the various branches of mathematics.

Applied mathematics

Now let us approach the important distinction which, in popular language, is referred to as the distinction between pure and applied mathematics. The main development of this theme properly belongs in the following Part, but cer-

tain foundation ideas may be helpfully mentioned here.

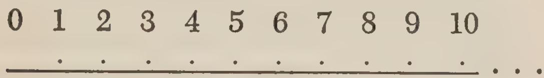
There are two kinds of mathematical problems. Some are concerned merely with the valid derivation of theorems from the definitions laid down, by the operations allowed, without specifying any set of entities to which the symbols adopted are to be applied. Those are problems in pure mathematics, or, more exactly, they are problems whose solution constitutes the development of an uninterpreted formal system. The science of arithmetic is such a system. It does not specify any classes of entities to which alone its numbers are to be applied. In the case of some problems, however, an interpretation of the symbols in terms of a certain set of entities is specified, and these are problems in applied mathematics. Various kinds of such restrictive interpretation are possible; but the most important problems of applied mathematics are those in which the symbols are interpreted in terms of measurable properties of physical objects or events. It is this sort of application to which the following discussion is devoted.

To apply mathematical reasoning to physical objects or events in any way which goes beyond mere counting, it is necessary to identify some property of those objects with a measurable magnitude. And applications become more and more widely possible, as more and more properties become analyzed in such a manner as to permit this identification.[6] But how is such identification accomplished at all?

Well, consider the very simple case which has already been used for purposes of illustration—that of the property of length. Let us establish the concept of a straight line, by defining it in terms of a geometrical point, together with our familiar notions of motion and direction. Conceive a point—that is, a mere locus in Euclidean space—to move in a constant direction, its track remaining behind as it moves. The track thus generated we shall call a straight line. Now, when a line is thought of as produced in this way, it appears, on analysis,

[6] This theme will be dealt with when we examine the methodology of exact reasoning in the factual sciences in chaps. 17 and 21. See pp. 353 ff., 460–465.

to constitute a series possessing exactly the same properties as have been described above in treating the series of natural numbers. Take any portion of the track that we please as a unit; then the track as a whole can be conceived as a series generated by successive repetitions of that unit. This series can be correlated with the series of numbers, beginning with zero as the locus where the motion originates, and numbering each unit, as it is repeated through the continued motion of the point, by the successive arithmetical digits. Such a series is evidently an asymmetrical, transitive, and connected series, just as the series of natural numbers is, and since Euclidean



space is conceived as unlimited, such a track appears capable of prolongation as far as we like, thus permitting correlation with any number, no matter how high. And by a suitably graduated measuring rod, any portion of it can be measured.

Now what specifiable property of physical objects could plausibly be identified with this straight line? If I pick up a limb that has just fallen from a tree it might be difficult to discover any such property at all. But here is the edge of the top of the table on which I am writing. I can see that this edge is not perfectly straight, but it approximates straightness much more closely than anything that I can observe in the branch of the tree; moreover, I see that for most purposes the irregularities of the edge can be neglected, and that, were it carefully shaped by precision tools, it would approximate a straight line much more closely even than it does. This gives a sufficient clue for the application of the number series to such an object. In other words, a straight line, from the point of view of such possible applications, is an ideal concept [7] with which the length of the table top, as measured along such an edge, can be identified for all practical purposes. And length can then be defined as that property of an object

[7] *I.e.*, a limit toward which we can advance by finer and finer approximations.

which is measurable by this procedure. So defined, it becomes a magnitude of a certain kind. By suitable conventions, which readily suggest themselves once we have gained familiarity with the measurement of objects having a fairly straight edge, the procedure can be extended to objects like the tree limb which have no edge and are far from straight; these too, then, have a length which is a measurable magnitude just as much as the length of objects to which a yardstick can be more easily applied.

The invaluable advantage gained by identifying properties of objects as magnitudes in this way is that when it can be successfully done the relevant operations on the number system can be assumed to apply to them as well as to the entities of pure mathematics. Once it has been determined, for example, that an iron bar is eight feet long, everything that arithmetic can deduce from the number eight, in its relations to other numbers, must also be true of the bar, so far as its length is concerned.

Main advantage of exactitude in physical science

A simple example will exhibit the gain to our powers of inference that accrues from this procedure. We have seen that arithmetical operations add numerical precision to the otherwise quite indefinite relations "greater than" and "less than." If all that is known about x, y, and z is that $x > y$ and $y > z$, then all that can be inferred about the relation between x and z is that $x > z$. To be sure, it can be inferred also that x is still greater than z, *i.e.*, that it exceeds z by more than x exceeds y or y exceeds z, but this "still" is itself merely an indefinite modifier of the relation, contributing no numerical exactitude. But if it is known that $x = 3y$ and $y = 5z$, then the same sort of precision can be realized in the conclusion; one can infer not simply that $x > z$ but that $x = 15z$. It is known exactly *how much* greater x is than z. Now, take a case where we are applying arithmetic to physical objects. The essential difference to be noted is the difference between the two following inferences:

a. This board is longer than that.
   That board is longer than the one by the door.
∴ This board is (still) longer than the one by the door.

b. This board is $2\frac{1}{2}$ inches longer than that.
That board is 4 inches longer than the one by the door.
∴ This board is $6\frac{1}{2}$ inches longer than the one by the door.

Inference (a) permits an indefinite comparison of the two boards in the conclusion, because the relation "longer than" is employed without quantitative exactitude in the premises. Inference (b) permits numerical precision in the conclusion. We can infer not merely that "this board" is longer than the one by the door, but also exactly how much longer. And this conclusion can be drawn without comparing the two boards directly, which might be inconvenient or even impossible.

There are often serious difficulties in the way of reducing properties of objects to magnitudes, which we may not at present see how to resolve, and not all magnitudes permit comparative measurement in the same way. They may possess some, but not all, of the properties of the number series. The problems arising from these circumstances will be dealt with in due time.[8]

EXERCISES

1. Explain what is meant by "one-to-one correlation."
2. Give four examples of null classes.
3. Which of the following relations can generate an ancestral order?
   - a. More expensive than
   - b. East of
   - c. Rules over
   - d. Taller than
   - e. Rests upon
   - f. Part of
4. How does the operation of multiplication involve that of addition?
5. Solve the following problems involving the concept of natural number:
   - a. If a particular natural number is definable as a certain class of classes, as 12 is defined as the class of dozens, then can you think of any mathematical concept that is definable as a certain class of classes of classes?
   - b. Using the notion of "successor of" (S'), how might you define 3 in terms of 0?
   - c. Show that $(4 - 3)$ is the third predecessor of 4.
   - d. If 4 is defined as the second successor of 2, prove, using the given definition of $x \times y$ and $x + y$, that $2 \times 2 = 2 + 2 = 4$.
   - e. Can the given definition of $x + y$ apply to the cases $1 \div 2$ or $2 \div 3$? Why?

[8] See below, pp. 353 ff., 460–465.

6. Can you suggest another way besides the one mentioned in the chapter by which the basic concepts of Euclidean plane geometry could be defined?
7. Just what is "measurement"?

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CHAPTER 12

PROPOSITIONS OF CLASS INCLUSION

Methods of representing propositions of class inclusion

From this brief inquiry into the logical nature and significance of mathematical science, our attention will now turn to inferences about the inclusions of classes with their attributes and members. As was observed in Chapter 10, much of our reasoning employs inferences of this kind. In the language of that chapter, which gave an introductory analysis of both relational and nonrelational propositions, the task with which we shall be occupied is to determine the structure of a valid syllogism, and to become familiar with the rules which define that structure.

But essential preliminaries to the study of validating forms for the syllogism are not yet completed. It will be of great help in pursuing such an enterprise to master certain algebraic and geometric methods of exhibiting the inclusions and exclusions of classes which may be involved in any syllogistic reasoning. And one must first master these methods as employed to represent a single proposition of class inclusion, before he can apply them to a syllogism as a whole. This auxiliary study may be introduced by recalling the concepts of conjunction and disjunction, which were briefly examined when molecular validating forms were discussed. These concepts apply to classes as well as to propositions, and their implications as so applied must be respected whenever an inclusion or exclusion between classes is asserted. Let us see how this is the case.

The conjunction of classes

The attribute "white" determines the class of white things, and the substantive "rose" determines the class of roses. What, now, about the class of white roses? Obviously, it consists of

all individuals that are members *both* of the class of white things *and* of the class of roses; it is thus the *conjunct* of those two classes. Just as two propositions p and q may be conjoined into $p \cdot q$, the resulting compound proposition meaning that both p and q are asserted to be true, so two classes may be conjoined, the resulting conjunct class consisting of all those members that belong both to the one class and also to the other. But the symbol for conjunction is different in the two cases. A dot performs this function when the entities conjoined are propositions, while the symbol for mathematical multiplication ($\times$) is employed for this purpose when they are classes. Thus if a represents the class of white things, and b the class of roses, the class of white roses will be represented by $a \times b$. And, following a convention suggested by the mathematical use of this symbol, this class is referred to as the *logical product* of a and b. Moreover, an abbreviation familiar in algebra is also customarily employed. There the product of a and b is ordinarily expressed by dropping the multiplication sign and simply juxtaposing the two factors; one writes ab instead of $a \times b$. This convention is adopted in the calculus of classes. Hence when symbols standing for classes are juxtaposed in this way, the operation of conjoining is understood and the conjunct class always intended.

Now consider in the light of this conception of the conjunction of classes what is meant when an I proposition of class inclusion is affirmed: Some S are P. Nothing more or less is affirmed than that there exists a conjunct class whose members are both members of the subject class and of the predicate class. When one says: Some roses are white, this is precisely what the proposition implies. And what is meant when the corresponding E proposition is asserted: No S are P? As the square of opposition indicates, this contradicts the I proposition; that is, it constitutes a denial that any such conjunct class exists. Indeed, when one makes this denial he is not necessarily asserting that anything exists. If there are any S's and P's, he is saying, no members common to the two classes are to be found. This hypothetical character of the E

proposition is quite clearly revealed when an instance like the following is examined: No mermaids are sea serpents. Obviously, there is no intention to assert that any mermaids exist, or any sea serpents; the proposition simply says that the two classes are so conceived that in case there were any members of the one they would not be members of the other. Certain bearings of this difference between the I and E propositions will be remarked upon later.

A more adequate symbolism for propositions of class inclusion

In these illustrations the mode of symbolization presented in Chapter 10 has been used: the E proposition has been represented by: No S is P, and the I proposition by: Some S is P. But certain English words remain in these formulae: they will now be eliminated. The way of doing this here adopted makes use of the algebraic method just mentioned for indicating conjunctions of classes. The great virtue of this method is that its mastery facilitates accurate analysis of inferences employing class inclusions and exclusions according to a few elementary rules. The student of algebra is familiar with the fact that mathematicians often find it convenient to express their formulae as equations in which the sum of the factors involved is equal to zero. A well known illustration is the binomial theorem: $ax^n + bx^m = 0$. Now for the same reasons the formal scientist engaged in symbolizing inferences about class inclusions finds it desirable to express his propositions in the form of such equations. But what would "equals" mean in this context, and what could zero stand for?

The null class

Passing by the first of these questions for a moment, let us consider the second. The classes of mermaids and of sea serpents were just referred to, while at the same time it was observed that these classes do not really have any members. It is evidently desirable that some symbol be available by which a class which has no members can be represented; why not the symbol 0 which, as the preceding chapter has noted, is used in mathematics to denote any class of this sort? Hence the very important technical concept in the calculus of classes—that of the null class, which is symbolized by 0. And in the light of this meaning for 0, an answer to the other

question raised is not difficult: What could be meant in any formula about classes by being "equal to zero"? It simply indicates that the class to the left of the sign of equality has no members. It is a null class. On this interpretation, $a = 0$ is a simple way of saying, in the form of an equation, that the class a is without members.

The E and I propositions as equations

Consider again, then, the E proposition in inferences of class inclusion: No S is P. The meaning of this assertion is that there are no members of the S class which are also members of the P class, or, that the conjunct class which would be symbolized by SP has no members. By the aid of the principles just discussed, this assertion can be readily expressed by an equation:

$$(\mathrm{E})\quad \mathrm{SP} = 0$$

And the I proposition: Some S is P, is the contradictory of E, affirming that there are members of S (at least one) which are also members of P, or, that the conjunct class SP does have members. How express this? Well, mathematicians sometimes have occasion to indicate the fact that a certain quantity is *not* equal to another, and this is symbolized by using the sign of equality cut by a diagonal line: $\neq$. A formula employing this symbol is called an *in*equation. Such an expression provides a convenient method by which the student of class inferences may symbolize the I proposition. Its purport is that there *are* members of SP—that is, that this is not a null class. The assertion therefore is very conveniently expressed as follows:

$$(\mathrm{I})\quad \mathrm{SP} \neq 0$$

Two of the four kinds of proposition which may function as premise or conclusion in a syllogism have thus been given algebraic symbolization. The A and O propositions remain. In order to secure analogous formulae for these it is necessary to introduce the notion of negative classes. The reader is familiar with the concept of negation (expressed by the

word "not") as used in the denial of a proposition, and our study of the factor of quality in the internal structure of propositions has further displayed its logical properties. In these contexts "not" is intended to provide a principle of exhaustive and exclusive division. Propositions are either true or false, and never both; hence "false" = "not true," and "true" = "not false." Again, considered from the atomic viewpoint, simple propositions are either affirmative or negative, and never both; so "negative" = "not affirmative," and "affirmative" = "not negative."

The O and proposition

The same considerations likewise control the application of the concept of negation to classes. What would be meant by a negative class—by, for example, a'? In the light of the above analogies, it would obviously denote all the entities in the universe which are not included in the class a. It is technically called the *complement* of a. If a is the class of trees, a' will include all the various things in the world that are not trees—a rather miscellaneous array, of course, in which no one would ever be interested except for its logical relation to the class of trees.

What, then, is meant by the O proposition: Some S is not P? Well, it indicates that some members (at least one) of the subject class are not included in—that is, are excluded from—the predicate class. But since P and P′ are mutually exclusive, and between them exhaust the universe, whatever is not included in P must be included in P′. We may, therefore, think of the O proposition as precisely similar to the I proposition except that the predicate class is replaced by its complement. Let us pursue this interpretation. The I proposition asserts that there are members common to the classes S and P—that the logical product SP is not null; hence it was symbolized by: $SP \neq 0$. The O proposition, then, asserts that there are members common to the classes S and P′, or that the logical product SP′ is not null. An appropriate symbolic expression for it is therefore:

$$(O)\ SP' \neq 0$$

The A proposition remains. Since it is the contradictory of O, as E is the contradictory of I, there is no doubt how it should be symbolized, and when it is thus symbolized an algebraic equation has been provided for each of these four kinds of proposition, expressing a single set of interpretative principles throughout. If the O proposition denies that the product SP′ is a null class, the A proposition must affirm that it is such. Its meaning must be that there are no members common to the classes S and P′, just as the E proposition means that there are no members common to S and P. Hence:

$$(A)\ SP' = 0$$

But it is desirable to see on other grounds that this is a correct interpretation of A. Consider, without any reference to the O proposition, what is said about the classes S and P when it is affirmed that all S are included in P. There are two possibilities. One is that P is a larger class than S, containing all its members and other things besides—to use technical terms, that S is a *proper subclass* of P. This is illustrated by the case: All dogs are animals. The other is that the two classes are *equivalent*, containing exactly the same members. It might be thought at first sight that "all S are included in P" does not apply to the latter situation, but this feeling will be dispelled when concrete examples are weighed. It is just as natural and correct to say "all equilateral triangles are equiangular" as to say "all dogs are animals"; yet in the former of these two A propositions the predicate class contains no more members than the subject class. The two classes are coextensive; each is included in the other. This possibility is quite consistent with the definition of class inclusion, namely, that one class is included in another whenever all members of the one are also members of the other. In short, when "all S are P" is affirmed, the meaning is that either S is a smaller class contained along with other things in P, or that the two classes contain exactly the same members. Now what do these alternatives imply in common? Clearly this—that all things lying outside of P must also lie outside of S. If any geometrical figure is not equi-

angular it is not equilateral, and if any creature is not an animal it cannot be a dog. Or, to say the same thing in the terms recently introduced, no members of the class P′ can also be members of the class S. These two classes must be mutually exclusive. But this may be most simply expressed by affirming that the conjunct class or product SP′ has no members, *i.e.*, that $SP' = 0$. This proposition, then, has precisely the same implications as the A proposition as originally expressed.

A concise summary of these results is secured by listing the algebraic expressions for the four kinds of proposition in a simple table so that their interrelations and the principles underlying their symbolization will be readily apparent.

A. $SP' = 0$
E. $SP = 0$
I. $SP \neq 0$
O. $SP' \neq 0$

Geometric representation of propositions of class inclusion

But if algebra is to be used as an aid to the formal logician, why not geometry also? Let us pursue this idea; a method by which these four kinds of proposition and their possible interrelations in inference are geometrically symbolized can be

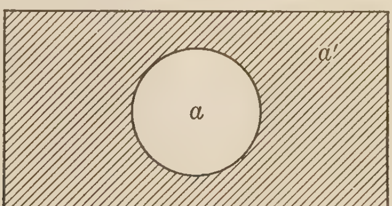

FIG. 3.

worked out very simply. When one employs A, E, I, or O propositions in syllogistic inference he is reasoning about certain classes which are positively determined and also about their complements. Now the positive classes can be represented by circles, and the negative classes by the areas outside of the circles. For example, if one is dealing with only a single class and its complement, a convenient way of exhibiting the relations involved would be that shown in Fig. 3. Here the

rectangle represents the universal class; the circle within it represents class *a* (whatever this might be); and the shaded area between the circle and the rectangle represents the complement class *a′*, containing everything in the universe that is not a member of class *a*.

Any proposition in class inclusion, however, presents us with a relation between two classes S and P; it asserts an inclusion

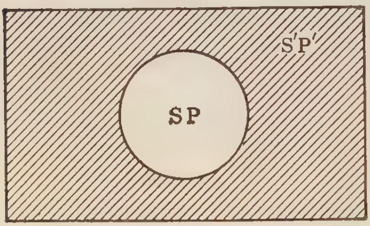

FIG. 4. S and P are equivalent classes.

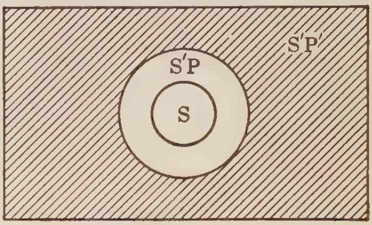

FIG. 5. S is a proper subclass of P.

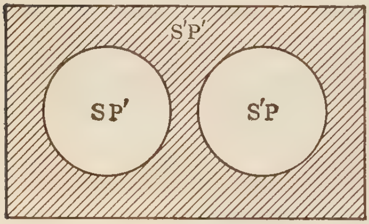

FIG. 6. S is wholly excluded from P.

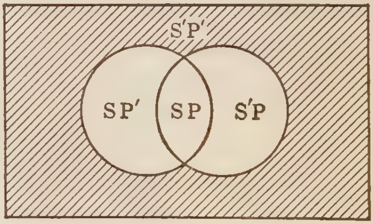

FIG. 7. S is partly included in and partly excluded from P.

or exclusion, universal or partial, between them. And there are four possibilities. Either (1) S and P are equivalent, in which case S′ and P′ are equivalent also; or (2) S is a smaller class wholly within P; or (3) S is wholly outside of P; or (4) S is partly included in and partly excluded from P. These possibilities are exhibited in order in Figs. 4–7, following the method just illustrated. Certain considerations, however, make it desirable to modify this geometrical scheme. If it were followed as it stands, proposition A would pose difficulty, since merely from its form one does not know whether it implies the first

or the second of these possibilities; much better, therefore, to find a way of representing that exclusion of S from P′ which is equivalent logically to the disjunct of these two alternatives. Also, the fourth of these diagrams would have to represent both the I and the O propositions; but they must be symbolized in such a way that their distinctive difference is visually exhibited.

The universal propositions

Accordingly, it has become customary to revise the above diagrams in two respects. For one thing, all four kinds of proposition are represented by a pair of circles intersecting in the same manner—that shown in the fourth illustration above. This has an obvious disadvantage, but ample compensation appears. For another, the differences between any two of these—A and E, A and I, etc.—are recognized by a method of shading portions of the circles and of inserting crosses in them. By virtue of this procedure the A proposition is given a single unambiguous geometrical symbol, and the difference between the I and O propositions is clearly indicated. These modifications will now be explained, and the standard diagrams for each of these kinds of proposition presented.

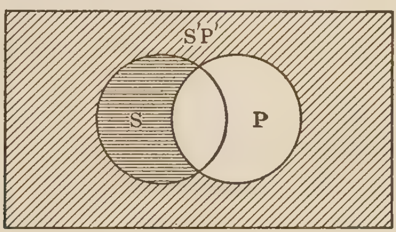

FIG. 8. All S are P, or SP′ = 0.

The essential meaning of the A proposition is that the class SP′ has no members. This is indicated in the figure by shading off that part of the S circle which is outside of the P circle and therefore included in P′, while the part of S that is included in P is left blank. Thus treated, the figure is to be interpreted as saying: If there are any members of S they are also members of P. And this is precisely what the A proposition is to be construed as implying; it contradicts the O proposition which

asserts the existence of members of S that are not members of P.

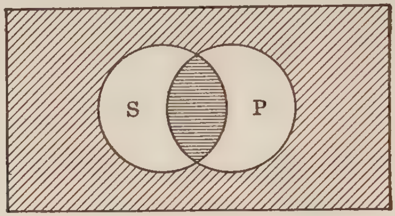

FIG. 9. E—No S are P, or SP = 0.

The E proposition means that the class SP has no members. It contradicts the I proposition which asserts the existence of members of S that are also members of P. If there are any members of S, E says, they are not members of P. This is represented in the figure by shading off the intersecting part of the two circles, while leaving the parts that lie outside of each other blank.

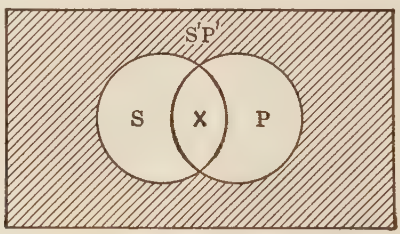

FIG. 10. I—Some S are P, or SP ≠ 0.

The particular propositions

The I proposition, contradictory of the corresponding E, asserts that there are members of the class S which are also members of P—that the product of the two classes SP ≠ 0. In order to symbolize this adequately it is not sufficient merely to remove the shading of the area SP in the E proposition. That would simply indicate that if there are members of S, some of them might be members of P. What is needed is a way of exhibiting further the contention of the I proposition that members of SP really exist. This is accomplished by placing a cross in the area in question, the presence of the cross

meaning that at least one member (and perhaps more) of the class represented by that area does exist.

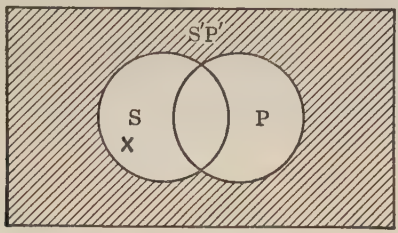

FIG. 11. O—Some S are not P, or $SP' \neq 0$.

The O proposition hardly needs specific discussion, since it involves no principle not already explained and illustrated in the case of the other three. It is the contradictory of A, and therefore maintains that there exist members of S which are outside of P—that the class SP′ is not null. Accordingly, it is represented by leaving the part of the S circle that is outside of P unshaded, and placing a cross there, to show that the class is positively occupied by at least one member.

In the next chapter a third circle will be added, intersecting both the S and the P circles, so that an entire syllogistic inference, with its three propositions and three terms, may be symbolized in a single diagram. Such a visual pattern offers genuine aid in discriminating valid from invalid reasoning in inferences about class inclusions. But certain essential points may be clarified by pondering more fully the nature of class inclusion as exhibited in any single proposition employing it, and the balance of the present chapter will be devoted to this end.

Consider first two principles which have thus far not been systematically illustrated; the algebraic and geometrical methods of symbolizing propositions of class inclusion make it easy to see their validity.

The law of commutation as applied to classes

The first of these is known technically as the *law of commutation*. Examine again the conjunct class or product *ab*, when *a* stands for the class of white things and *b* for the class of roses. The class *ab* is then the class of white roses. Are we confronted

by a class that is in any respect different if we substitute for this the class of roses that are white, symbolized by *ba*? Clearly not; this class would have precisely the same members as the class of white roses. The difference is merely one of grammatical preference or of emphasis. It is evident, then, that whenever any product *ab* is considered, it is legitimate to reverse the order of the terms and substitute *ba* for it if this is convenient. And "commutation" means just such a reversal of the order in a sequence of terms. Now since this law holds generally, it holds in any equation in which a logical product

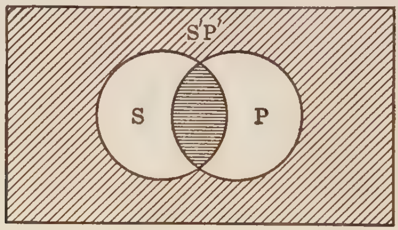

FIG. 12.

of classes appears. Given $SP = 0$ for example (the E proposition of class inclusion), it follows at once that $PS = 0$. The two equations are alternative ways of saying what is logically the same thing. And the method of geometrical representation above adopted entirely confirms the soundness of this principle. Fig. 12 shows the diagram for the E proposition. What difference, if any, would be required were $PS = 0$ to be portrayed instead? Obviously, none. All that is necessary is to reverse the order of our perceptual attention, proceeding from right to left on the rectangle instead of from left to right. Examination of the I proposition will reveal exactly the same situation; there is no logically significant change in passing from $SP \neq 0$ to $PS \neq 0$. For the same reasons the A proposition may be replaced by $P'S = 0$, and the O proposition by $P'S \neq 0$. The reader will find by testing with typical examples that the same law applies to propositions as well as to classes. If the conjunct proposition $p \cdot q$ is true, obviously the proposition $q \cdot p$ is true; from the truth of, "The clouds are dark and

it is raining," it strictly follows that "It is raining and the clouds are dark."

The law of double negation

The second principle has been already mentioned, but has not been discussed in its bearing on inclusions and exclusions of classes. It is called the *law of double negation*, and its validity is implied by the role of "not" as a guarantor of exhaustive and exclusive division. If the class a' contains everything in the universe that is not a member of the class a, what would constitute the class $(a')'$? Evidently, it can be none other than a itself. The entities not contained in the class that contains everything outside of a must be just the members of a; for example, the class of beings that are not immortal is identical with the class of mortal beings. Symbolically expressed, this law is that $a = (a')'$. Its practical virtue is that whenever one finds it convenient one may legitimately substitute in any piece of inference $(a')'$ for a, or vice versa. If $SP \neq 0$, then it follows that $S(P')' \neq 0$. The geometrical method of symbolizing class relationships, just explained, provides ready verification of the validity of this principle. Consider again the representation of a single class with its complement. Since the rectangle stands for the whole universe, outside of which there is nothing, it is clear that the class of things not in a'—the class $(a')'$—must be simply the class a.

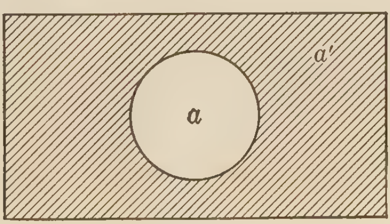

FIG. 13.

The concept of distribution

We are now going to engage in a little systematic practice with these two principles, as applied in the substitution of logical equivalents for given propositions of class inclusion. It will be necessary in doing this to make use of a new technical word which will later find important employment. This

is the word "distribution." Any subject or predicate term in a proposition of class inclusion must be either distributed or undistributed. According to the traditional definition, a term is said to be *distributed* when it is used distributively, *i.e.*, when it applies to each member of the class in the same way and not to some more than others. By this is meant that the proposition in which it occurs claims information about the entire class of objects it denotes. It is *undistributed* when this is not the case. Hence, so far as subject terms are concerned, distribution coincides with universality in the quantity of the proposition, while the subject terms of particular propositions are necessarily undistributed.

With predicate terms the matter is more complex. The law of commutation, as applied to the E and I propositions, has shown, however, that these propositions claim precisely the same kind of information about the predicate class as about the subject class. Since PS may be substituted for SP without any change in the rest of the equation, it is evident that P must have the same status as S with respect to distribution. This means that the predicate term of an E proposition is distributed; since the whole of the subject class is excluded from the predicate class, it follows that the whole of the latter must be excluded from the former. Correspondingly, the predicate term of I is undistributed; it affirms that at least one member of S is also a member of P, and therefore all that is known about P is that at least one of its members is likewise a member of S. Analysis of the A proposition shows that its predicate term is undistributed. If this proposition implied that the subject and predicate classes are equivalent, each being included in the other, the predicate term, like the subject term, would be distributed. But merely by its form, the proposition does not indicate whether this is the case or whether P is a larger class of which S is a subclass; hence no definite information is given about all the members of P. It cannot be said that they are all included in S, and obviously they are not all excluded from S. Hence the predicate term is undistributed. In the

case of the O proposition the predicate term is distributed, for it is clearly meant to apply to the whole of its class. When "Some S is not P" is asserted, the members of S referred to, although it is not specified which or how many they may be, are affirmed to be excluded from the entire predicate class. Accordingly, the whole of the predicate class is asserted to exclude the contemplated members of S.

The matter may be summarized then by saying that in the case of universal propositions the subject terms are distributed, in the case of particular ones they are undistributed; while with affirmative propositions the predicate terms are undistributed, with negative propositions they are distributed. The following table will help:

A proposition—subject distributed, predicate undistributed.
E proposition—subject distributed, predicate distributed.
I proposition—subject undistributed, predicate undistributed.
O proposition—subject undistributed, predicate distributed.

Strict equivalences of propositions of class inclusion

Now the way is open for some practice with substitutions of one proposition for another in accordance with the above principles.[1] Obviously, so far as the algebraic equations are concerned, this can be done quite mechanically; the order of terms in any logical product may be reversed, and any positive term replaced by a double negative, or vice versa. It is the interpretation of the results in nonsymbolic language that is not quite so simple. A considerable number of such substitutions are theoretically permissible; only the ones that are most frequently employed as subsidiary processes in actual inference will be explored here. These are the operations technically described as *obversion, conversion,* and *partial contraposition.*

The problem attacked when one engages systematically in these operations is this: Given a certain proposition of class inclusion, what other propositions are strictly equivalent while

[1] These substitutions are traditionally dealt with under the head of "immediate inference."

differing from it in quality or by interchanging the order of subject and predicate terms? To *obvert* a proposition is to substitute for it another which is equivalent but different in quality, an affirmative proposition being replaced by a negative and a negative by an affirmative. The original proposition is called the *obvertend,* and the one which replaces it the *obverse.* To *convert* a proposition is to substitute an equivalent of the same quality but in which the original predicate term has become subject and the original subject term predicate. The proposition operated upon is the *convertend;* the substituted proposition is its *converse.* To contrapose a proposition these two operations are combined, the obversion being performed first. The original is the *contraponend* and the proposition finally reached is its *contrapositive.* The consequence of such a complex operation is that the contrapositive has as its subject the complement of the original predicate, and as its predicate the complement of the original subject. A brief study of these operations may commence with obversion.

Obversion

The way to perform this operation is to substitute for the original predicate term its complement, introducing another negative or removing it where necessary to assure equivalence. Thus an A proposition will become an equivalent E, and an E an equivalent A; precisely the same situation appears between I and O. But the best way to accomplish these substitutions, with a minimum of awkwardness, depends upon our resources of language, which vary considerably from one case to another. Consider the following instances of the operation:

No crow is white. $SP = 0$
∴ All crows are of other colors than white. $S(P')' = 0$

All dogs bark. $SP' = 0$
∴ No dogs are creatures that do not bark. $S(P') = 0$

Some of these eggs are not edible. $SP' \neq 0$
∴ Some of these eggs are inedible. $S(P') \neq 0$

Some parents are responsible. $SP \neq 0$
∴ Some parents are not irresponsible. $S(P')' = 0$

Three points are worth noting in connection with these illustrations of obversion. One is that in some of these cases the negative class can be unambiguously denoted by a prefix such as "in," while in other cases a phrase containing the word "not" or some synonym is necessary.[2] The second is that if the operation has been performed correctly, one can return to the original proposition by obverting again, employing once more the law of double negation. If

> All crows are of other colors than white, then
> No crows are creatures of none other color than white.

But creatures of none other color than white constitute precisely the same class as white creatures, hence this last proposition is the equivalent of "No crows are white." The reader will find it interesting to verify this principle with obverses of the A, I, O propositions. The third point is that it is essential to the validity of the operation to negate the predicate term, never the subject term. Students are often tempted to violate this rule, obverting by negating the subject term and leaving the predicate term as it stands. Consideration of the following illustration will reveal that such a procedure is incorrect.

> All boys ten and over will be admitted.
> ∴ No boys under ten will be admitted.

Clearly this is an illegitimate inference. The original proposition did not affirm that only boys ten and over, and none others, would be admitted. Perhaps some boys under ten will be admitted too. At least, the obvertend does not say that they will not; hence the supposed obverse is not really equivalent to it. "No boys ten and over will be refused admittance" is, however, its equivalent.

Conversion

The operation of conversion is simple so far as concerns the E and I propositions. The law of commutation, as applied to any logical product of classes, reveals that these propositions may be converted directly.

[2] Many logicians employ the prefix, "non," as a standard device in cases where no other prefix is obviously adequate.

No conservatives are persons hospitable to new ideas. $SP = 0$
No persons hospitable to new ideas are conservatives. $PS = 0$

Some salutary things are painful. $SP \neq 0$
Some painful things are salutary. $PS \neq 0$

The O proposition cannot be converted at all, since no equivalent proposition can be found whose subject term is the original predicate and whose form is that of a proposition of class inclusion. Given "Some truths are not discoverable," the same idea may be expressed by saying: "Discoverable things exclude some truths," but this is not strictly an O proposition whose subject is "discoverable things" and whose predicate is "truths."

Principles which it must respect in the A proposition

Conversion of the A proposition must take account of two principles, both of which have been noted in other contexts. First, since it is consistent with such a proposition that the predicate class be wider than the subject class, conversion must be *by limitation* only, and the converse is a particular rather than a universal proposition.[3] Clearly, the converse of "All dogs are animals" is not "All animals are dogs"; at best it would have to be "Some animals are dogs." The only exceptions to this principle are the cases in which subject and predicate terms can be proved to be strictly equivalent in virtue of their definition. Second, even conversion by limitation is legitimate only when members of subject and predicate classes exist. It has been noted above[4] that a universal and a particular proposition differ in that the meaning of the former may be merely hypothetical, while the existence of at least one member of the subject class is part of the meaning of a particular proposition as it is ordinarily employed. It is a corollary of this difference that one may not legitimately infer an I proposition from an A unless it is understood or explicitly affirmed that members of the classes involved exist. "All trespassers will be prosecuted" may mean no more than "If any-

[3] And hence the converse is not strictly equivalent with the convertend in this case. Converting the converse will not give back the original universal proposition.

[4] See pp. 210 f.

one is a trespasser he will be prosecuted," and the latter does not imply the proposition: "Some who will be prosecuted are trespassers." The warning might prove so effective that there will not be any trespassers, and therefore that the I proposition just expressed will be false. But it is strictly implied by the following conjunct proposition: "All trespassers will be prosecuted, and there are some trespassers." This distinction between two interpretations of the A proposition is appropriately recognized by calling it a *general* proposition when it intends to claim existence for members of the classes involved, while restricting the term "universal" to apply to it when no such intention is assumed. A proposition about an individual, when treated as an A proposition, is a special case. It implies existence, in whatever universe of discourse is assumed, but is not a general proposition.[5]

If the reader should be sceptical about this difference in meaning between a universal and a particular proposition, let him consider under what circumstances a particular proposition with "some" as quantifier would be seriously asserted. A universal proposition is asserted or assumed in every inference about classes; but how about the vague particular? It says so little; when would one ever find occasion to say it at all? The answer is: When one chances to observe a case of the conjunction of one class with another, or with its complement, especially when the case is an exception to a universal proposition that has previously been taken for granted. To recur to a previous illustration, suppose that an Eskimo, hitherto familiar only with polar bears, should travel to warmer climes and meet a black bear. He would naturally say: "Some bears are not white," that is, "I see that there are exceptions to what I had previously taken to be a universal law." Again, the first scientists who studied the anatomy of whales might easily have believed earlier that all mammals live on land. They now discovered that this is not so, and a natural way of expressing the discovery would be: "Some

[5] The significance of this distinction where A propositions function in the factual sciences is dealt with below. See pp. 298 f.

mammals live in the ocean." But, for an exception to an assumed universal law to be actually noticed, it must clearly exist. Hence the vague particular proposition is existential in meaning.

Recognition of this difference, however, requires a certain modification in the results reached when the square of opposition was analyzed in Chapter 9. This modification concerns the implications of the relation of super- and subalternation. It was assumed there that an I proposition strictly follows from the corresponding A proposition, and an O from the corresponding E. Now it becomes evident that this is the case only when it is intended that members of the subject classes denoted in the A and E propositions exist. The particular is thus implied by the corresponding general proposition, but not by the corresponding universal. From "All trespassers will be prosecuted, and there are trespassers," it follows that "Some trespassers will be prosecuted," but the latter proposition cannot strictly be derived from "All trespassers will be prosecuted" alone. This simply means: "If there are trespassers they will be prosecuted." [6]

Contraposition

Contraposition combines the operations of obversion and conversion. The contraposition is partial when each of these processes is engaged in once only, that of obversion being performed first. The full contrapositive is reached by obverting a second time after the partial contrapositive has been deduced. Since the full contrapositive is rarely used except for purposes of logical discipline, while partial contraposition does sometimes appear as a subsidiary operation in ordinary inference, the latter alone will be discussed here.

The A and O propositions may be contraposed without any difficulty.

All mermaids are denizens of the sea. $SP' = 0$
∴ No creatures not denizens of the sea are mermaids $(P')S = 0$

Some insect bites are not harmful $SP' \neq 0$
∴ Some harmless things are insect bites $(P')S \neq 0$

[6] This important point is explained further in the note at the close of the present chapter. See below, pp. 231 f.

The reader should practice these operations first by inserting the obverse of the original between it and the contrapositive, in order to assure himself that logical equivalence is maintained throughout. After a while the contrapositive can be apprehended directly and the complex operation performed as though it were a simple one.

The I proposition cannot be contraposed, since when it is obverted it becomes an O proposition and the latter cannot be converted. Inasmuch as the E proposition becomes A when obverted, it can be contraposed only by limitation, and even then only when it is explicitly understood that members of the classes involved exist.

No men are immortal, and both men and mortals exist.
$SP = 0$ ($\exists$ S,P′)[7]
Some mortals are men. (P′) $S \neq 0$

In the case of the I proposition, and likewise with the others whenever it is convenient, the *obverted converse* may be substituted instead of the contrapositive. This consists in performing the same operations as those involved in partial contraposition, except that conversion is engaged in first. The obverted converse of "Some divinities are mythological" is "Some mythological beings are not beings that lack divinity."

The following summarizing table may be of service:

Original Proposition	Obverse	Converse	Partial Contrapositive	Full Contrapositive
A All S is P	No S is non-P	(by limitation) Some P is S	No non-P is S	All non-P is non-S
E No S is P	All S is non-P	No P is S	(by limitation) Some non-P is S	(by limitation) Some non-P is not non-S
I Some S is P	Some S is not non-P	Some P is S		
O Some S is not P	Some S is non-P		Some non-P is S	Some non-P is not non-S

[7] $\exists$ means "There exists at least one member of."

Meaning of *only, alone, except,* etc.

In propositions of class inclusion, certain words or phrases are often used whose implications concern the inclusions involved but are not the implications which might be supposed at first sight. What, for instance, is the meaning of "Only employees are admitted"? A little consideration will reveal that this can be expanded to "None who are not employees are admitted." But this is the partial contrapositive of "All who are admitted are employees," and when contraposed itself will yield that proposition as the result. Accordingly, the simplest way of stating its meaning is to form an A proposition whose subject is the original predicate. "Alone," "none but," and "none except" can be treated in precisely the same manner. "None but (or, except) women bathe here" is equivalent to "All who bathe here are women." "Men over thirty-five alone are eligible" means "All who are eligible are men over thirty-five." In this last case it might seem that "All men over thirty-five are eligible" is also implied, but consideration of other instances will show that it is not. "Philosophy books alone are kept on this floor" does not mean that "All philosophy books are kept on this floor," but it does mean that "All the books kept on this floor are philosophy books." A complex sentence introduced by "unless," and with a negative in its main clause, may be similarly expressed in simple form. "Unless you are over six feet tall you cannot join this regiment" is equivalent to "All who can join this regiment are persons over six feet tall."

"All but," and "all except," however, imply a universal proposition whose subject term denotes the complement of the original subject class. "All but (or, except) children may see this show" implies "All adults, *i.e.*, persons who are not children, may see this show." [8] The implication of "unless," when used without a negative in the main clause, is similar. "Unless you are truthful you will get into trouble" means "All who are not truthful will get into trouble."

Since the logical value of any of these expressions is easily

[8] It probably implies also, "No children may see this show." Logicians are not all agreed on this point.

misinterpreted, it is imperative to note carefully the equivalences listed, and to form the habit of substituting the simplest proposition of class inclusion that is identical in meaning with the proposition originally given.

Note

The principle that particular propositions imply the existence of members of the classes involved, while universal propositions do not, is so important that it is desirable to illustrate it further by the aid of a crucial case. Let us engage in a few obversions and conversions, commencing with a proposition that is obviously true, and paying no attention to the principle in question.

No explorer has scaled Mt. Everest.

Converted, this becomes:

None who have scaled Mt. Everest are explorers.

Obverted:

All who have scaled Mt. Everest are nonexplorers.

Converting by limitation:

Some nonexplorers are ones who have scaled Mt. Everest.

But this proposition is absurdly false. In the process of reaching it, therefore, some logical error must have been committed; a false proposition can never be implied by a true one. Where lies the error? Well, there seems to be no flaw in the first two operations, and hence, if the original proposition is true, the second and the third must be true. Now none of these three propositions intends to claim that the class, "those who have scaled Mt. Everest," has any members. The meaning of the original is: "No explorer belongs to that class, and I am not saying that anybody else belongs to it either"; the meaning of its converse is: "If there are any members of that class (and I am not saying that there are), they are not explorers." Accordingly, the third proposition is equally hypothetical: "If any members of that class exist (and I am not saying that they do), they are nonexplorers." But the final proposition is not hypothetical. It means, as naturally interpreted: "There exists at least one member (and maybe more) of the class of nonexplorers who is also a member of the class of those who have scaled Mt. Everest." This is clearly false, while the other propositions are true. And is it not evident that its falsity arises from the fact that it implies the existence of members of a class while the premise from which it is supposed to be derived does not imply that any members of that class exist?

The universal propositions A and E are therefore to be interpreted as hypothetical in meaning unless it is explicitly affirmed (or the context

makes it understood) that members of the classes referred to exist. Then they are also general propositions. The I and O propositions, on the contrary, are to be interpreted as existential. This becomes clearer still, and how one determines the meaning of the word "exist" in this context is revealed when it is considered how a proposition like "Some centaurs can gallop at high speed" must be taken. Since no centaurs exist, such a proposition is always false, except under one condition. This is that when one affirms it he abandons in thought our ordinary universe of discourse and places himself within the universe of a particular mythology in which centaurs appear; in that case the ordinary meaning for the "existence" of members of a class has been abandoned also, and a meaning appropriate to that mythological world has been assumed. What exists and what does not is always determined by the universe of discourse taken for granted.

One important virtue of the algebraic and geometric symbols for propositions of class inclusion, expounded in the present chapter, is that they have been devised in such a way as to take full account of this difference between universal and particular propositions.

EXERCISES

1. Convert, obvert, and partially contrapose each of the following propositions:
   a. No servants are satisfactory.
   b. Some books are popular.
   c. All great men are scrupulously honest.
   d. Some industrious students are not brilliant.
   e. All business houses are closed today.
   f. Some of these delegates are not Christians.
2. Name the operation by which one passes in inference from each of these propositions to the one which follows it:
   a. All strangers are welcome.
   b. No strangers are unwelcome.
   c. No unwelcome people are strangers.
   d. All strangers are welcome.
   e. Some welcome people are strangers.
   f. Some strangers are welcome.
   g. Some welcome people are not people who are not strangers.
   h. Some welcome people are strangers.
3. Put the following in simplest propositional form, then obvert each of them:
   a. Only reporters would have such effrontery.
   b. All except textbooks are barred from this shelf.
   c. Unless you take that corridor you will not find him.
   d. None but seniors may wear gowns.

e. Catholics alone accept the authority of the pope.
f. Except ye be born again ye cannot enter the kingdom.
g. Unless one salutes the flag he is suspected.
h. Professional men only are members of this club.

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CHAPTER 13

VALIDATING FORMS FOR THE SYLLOGISM

The preceding chapter has explained the algebraic and geometric symbols appropriate for expressing the meaning of any single proposition which may play a part in an inference concerning inclusions or exclusions of classes. When one deals with the simplest cases of such an inference as a whole[1]—that is, with cases of the syllogism—these symbols must be combined to form a larger pattern in accordance with the same principles which guided their construction.

Geometrical representation of a syllogism

A syllogism consists of two premises and a conclusion derived from them jointly, each being an A, E, I, or O proposition of class inclusion. In it there are three terms—subject term, predicate term, and middle term—the last named being the term which appears in each of the premises but not in the conclusion. Algebraically, these terms are symbolized by S, P, and M respectively, so that the major premise will assert an inclusion or exclusion between P and M, and the minor premise one between S and M. Geometrically, it is necessary to add a third circle, representing M, to the two intersecting circles with which we are now familiar for exhibiting the inclusion or exclusion obtaining between S and P. The pattern for the three circles shown in Fig. 14 will be found an appropriate one. It allots a specific area for each of the eight possible conjuncts of the classes involved in a syllogistic inference. The upper circle represents the class denoted by the middle term, while the left circle represents the subject class

[1] Leaving out of account mere substitutions of an equivalent proposition for one originally given.

in the conclusion and the right circle its predicate class. It is evident that the area in the center, which constitutes the intersection of all three circles, will represent the conjunct of all three classes; the shaded area within the rectangle but outside of the circles will represent the conjunct of the three negative classes S′, M′, and P′; while each of the other six areas will represent one of the other possible conjuncts—a positive with two negative classes, or two positives with one negative. When such a diagram is used to exhibit the logical structure of a syllogism, it is expedient to employ the same method of shading appropriate parts of any circle to indicate the implications

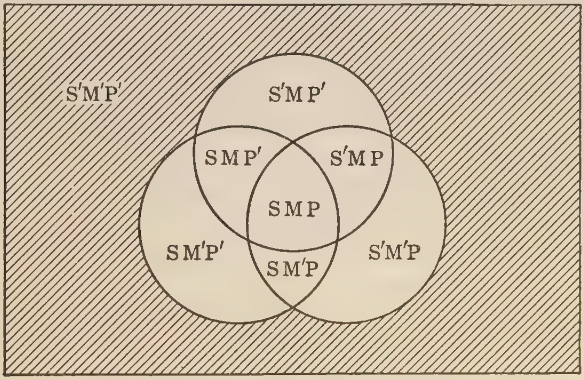


FIG. 14.

of a universal proposition, and of inserting crosses to indicate those of a particular proposition, which has already been explained. Since nothing needs to be done to the rectangle outside of the circles, it is usually omitted as superfluous.

This procedure will now be illustrated in the case of a few syllogisms. Not only is some practice in it an aid in clarifying the nature of class inclusion when three classes are involved, but also the method can display vividly the validity or invalidity of syllogisms. From the latter standpoint the general guiding principle to have in mind is this: If, when symbolizing in such a triad of circles the inclusions or exclusions asserted by the premises, one finds that he has symbolized the

Illustrated by a few valid syllogisms

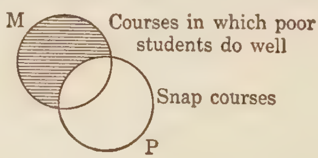
inclusion or exclusion asserted in the conclusion, the syllogism is valid; otherwise it is invalid. When one is just beginning such practice, confusion is best avoided by introducing the third circle only after the first premise has been symbolized.

1. All courses in this department must be snap courses, because poor students do well in them.

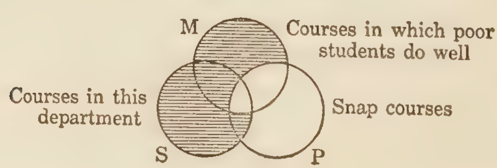
This is an elliptical inference, omitting one of the propositions really involved. Such an inference is called an "enthymeme." Hence one must first supply the assumed major premise, and then put the whole piece of deduction in such a form as will clearly exhibit the three terms and the inclusions asserted. This preliminary process may be described as "putting the inference in correct form for testing."

All courses in which poor students do well are snap courses.	$MP' = 0$
All courses in this department are courses in which poor students do well.	$SM' = 0$
∴ All courses in this department are snap courses.	$SP' = 0$

The major premise is clearly an A proposition, and accordingly will be symbolized thus:



The minor premise is also an A proposition. Adding the circle for S, and symbolizing the inclusion between S and M which this premise asserts, we have the following outcome:



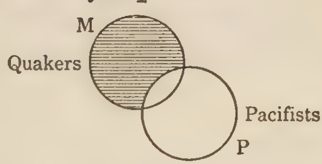
The conclusion affirms that "all courses in this department are snap courses," and it is evident that in symbolizing the premises the conclusion has also been symbolized. For the portion of the circle representing "courses in this department" that is outside of the circle for "snap courses" has been shaded off, indicating that if these premises are true, any course given in the department intended cannot be outside the class of snap courses. Positively, the same result is shown by the fact that a portion of the S circle, lying within the P circle, has not been shaded off.

2. All Quakers are pacifists. Some of the delegates at this convention, then, must be pacifists.

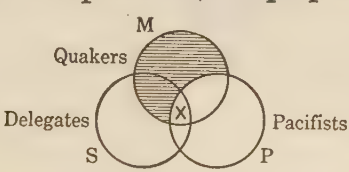
   Putting in precise and complete form:

All Quakers are pacifists.	$MP' = 0$
Some of the delegates at this convention are Quakers.	$SM \neq 0$
∴ Some of the delegates at this convention are pacifists.	$SP \neq 0$

Symbolizing the major premise:



Adding the minor premise (an I proposition):



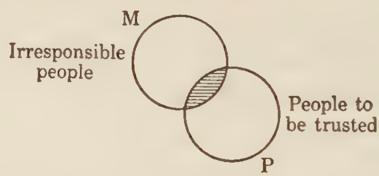
Here again, in symbolizing the premises the conclusion has been symbolized, and the syllogism is therefore valid. The circles exhibit geometrically the assumption of the inference that whatever members of a class are also members of a class that is entirely included in a third class, must themselves be members of that third class.

It should be noted that when one premise is particular the other premise should be symbolized first, irrespective of their order as given. Otherwise confusion may arise between the shading and the cross. The cross can obviously only be appropriately put in an area which has not been eliminated as necessarily null because of implications of the other premise.

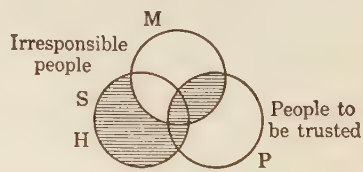
3. H. is irresponsible, and is therefore not to be trusted. Putting in complete form:

No irresponsible person is one to be trusted.	$MP = 0$
H. is an irresponsible person.	$SM' = 0$
∴ H. is not one to be trusted.	$SP = 0$

Symbolizing the major premise (E):



Adding the minor premise (A):[2]



Once more the syllogism is valid. In symbolizing the premises, that part of the "H'" circle that is inside of "people to be trusted" has been eliminated, which is what the conclusion affirms.

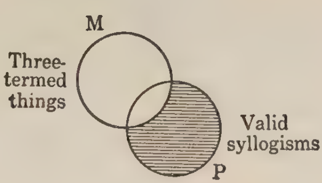
By two invalid ones

Now let us try two syllogisms that are invalid.

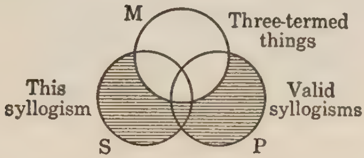
4. All valid syllogisms have three terms.	$PM' = 0$
This syllogism has three terms.	$SM' = 0$
∴ This syllogism is valid.	$SP' = 0$

[2] Remembering the principles previously explained about class membership propositions. See above, pp. 167 f.

Symbolizing the major premise:



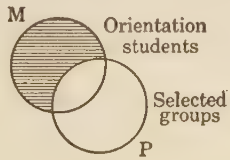
When the minor premise is added, however, all that it requires is the elimination of that portion of "this syllogism" outside of "three-termed things."



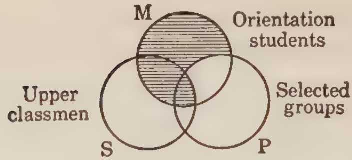
But if the conclusion is to follow from the premises, the entire area of "this syllogism" that is outside the circle for "valid syllogisms" must be eliminated. This the premises do not require, hence the inference is shown to be invalid.

5. Students in orientation courses are a selected group. $MP' = 0$
No upper classmen are students in orientation courses. $SM = 0$
∴ Upper classmen are not a selected group. $SP = 0$

Symbolizing the major premise:



Adding the minor premise, we see clearly that neither any inclusion nor exclusion is indicated between "upper classmen" and "selected groups."



But for the conclusion to follow from the premises the intersecting portion of these two circles would need to have been entirely shaded off. Accordingly the deduction is shown to be invalid.

These illustrations show how such geometrical figures, after one has gained a little familiarity with them, can exhibit in clear visual form the inclusions or exclusions between three classes which are asserted or implied by the propositions of a syllogism, and can provide a simple method for determining its validity.

The determination of validating forms

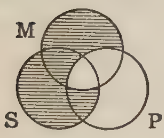
But they do not, of themselves, show the general pattern or patterns to which valid syllogistic reasoning must conform, nor do they provide a way of describing the fallacies that may be committed by an invalid syllogism. For these purposes, we must engage in further analysis, using these methods of algebraic and geometrical representation as aids. Since every syllogism has two premises, each of which is an A, E, I, or O proposition of class inclusion, a natural approach to this problem is to ask which of the various possible combinations of such premises yield a valid conclusion, and what general pattern or patterns those valid combinations disclose. In order to answer this question we shall need a simple method for describing any of the ways of combining premises that are possible. And such a description must take account of three factors.

One factor is, of course, whether a given premise is an A, an E, an I, or an O proposition, for its implications in any one of these cases are different from what they are in any of the others.

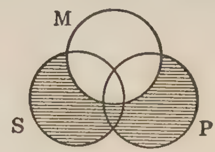
A second factor is the position of the middle term. Obviously the two following syllogisms, otherwise alike, are logi-

cally different, one permitting a valid conclusion, the other not.

All M is P
All S is M
∴ All S is P



All P is M
All S is M
No conclusion implied



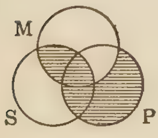
How many variations are possible with respect to this factor? Clearly four. They are called traditionally the *figures* of the syllogism, and are numbered as follows:

First figure	Second figure	Third figure	Fourth figure
M is P	P is M	M is P	P is M
S is M	S is M	M is S	M is S

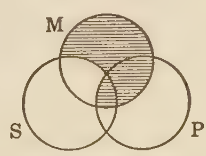
In the first figure the middle term is subject of the major premise and predicate of the minor; in the fourth figure these positions are reversed. In the second figure the middle term is predicate in both premises, while in the third it is subject in both premises.

The third factor is the order of the premises. An A proposition followed by an E proposition will not yield the same conclusion as an E proposition followed by an A, if the figure is the same.

All P is M
No M is S
∴ No S is P



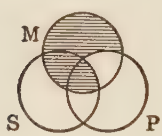
No P is M
All M is S
No conclusion implied

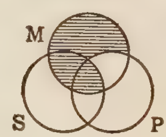


Let us call any possible way of combining two class inclusion premises, *i.e.,* any combination that is distinctive with

respect to any of these three factors, a *mood.* And a simple way of describing any mood is evident: EA2 will mean the mood formed when the first premise is an E proposition, the second an A proposition, and the middle term is in the position of the second figure; AO3 will mean the mood which results when the first premise is an A proposition, the second an O proposition, and the third figure is exemplified. Before we proceed to describe moods by this method, however, a point must be noted that permits a useful simplification. If a premise is an E or an I proposition, it is directly convertible, and this means that no difference will appear in its case whether the middle term is the subject or the predicate. The same implication is affirmed, and it is represented in the same way by the geometrical diagrams. As an illustration:

AE1	AE3
All M is P	All M is P
No S is M	No M is S
No conclusion implied	No conclusion implied





Hence there is really no need of distinguishing these moods. In such cases we shall speak of mood AE1 or 3, thus indicating that the two are logically treatable as a single mood.

There are eight such forms, or moods

Now we may proceed with our original question. Since the conclusion of a syllogism, as well as each of the premises, must be an A, an E, an I, or an O proposition, a systematic way of answering it is to find out what moods are capable of yielding as valid conclusion an A proposition, what moods an E, what moods an I, etc. And when we do this we discover that the valid moods are relatively few. An A conclusion is validly yielded only by the mood AA1; an E conclusion only by moods AE2 or 4 and EA1 or 2; an I conclusion only by moods AI1 or 3 and IA3 or 4; and an O conclusion only by moods

AO2, EI1, 2, 3, or 4, and OA3.[3] Only eight significantly different patterns of syllogistic inference are, then, valid, out of all those that are theoretically possible. That is, there are only these eight ways of asserting a class inclusion between M and P, and between S and M, as a result of which an inclusion or exclusion with S as subject and P as predicate is strictly implied.[4]

A preliminary answer has been reached to the quest for validating forms of the syllogism. Thus far there appear to be

[3] The student may check on these results in the following way. Assuming that our geometrical diagrams accurately represent the logical relations involved, it is clear that a universal conclusion can only be yielded by two universal premises, since otherwise a cross would be present and would have to be taken into account. He may list, then, all the moods containing two universal premises; with the simplification just described there are only nine of them. Scanning this list, he will quickly see that only mood AA1 shades off all of the S circle outside of the P circle and hence permits an A proposition as conclusion, and that only moods AE2 or 4 and EA1 or 2 shade off all of the intersecting area between S and P and hence permit an E conclusion. In the case of a particular conclusion, it is obvious that one of the premises will have to be particular. And by experiment, the student will find that the other premise will have to be universal; no combination of I and O premises compels the placing of a cross in either of the two areas of the S circle that it would need to be placed in if a particular conclusion were implied. Now there are only eighteen moods that meet these conditions. Among them only AI1 or 3 and IA3 or 4 dictate the placing of a cross in the intersecting area of S and P and hence yield a valid I conclusion; only AO2, EI, EI1, 2, 3, or 4, and OA3 dictate placing it in the area of S outside of P and hence yield a valid O conclusion.

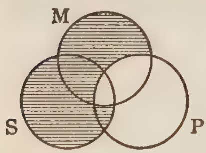
A nontechnical proof of syllogistic validating forms is given in the note at the end of the chapter.

[4] Some of the other possible combinations, it may be noticed, yield a valid conclusion with P as subject and S as predicate; the mood AO3, for example, implies the conclusion: Some P is not S. But it was decided that which term is P and which is S in the syllogism as a whole is determined by their position in the conclusion; that is how P and S are defined. Hence it might seem that in these cases P should be substituted for S and S for P throughout the syllogism. When this is done, however, one finds that the valid pattern resulting is identical with one of the valid patterns above recognized as such. Making these substitutions with mood AO3, for example, we find that it is transformed into OA3, and if the geometrical diagram for AO3 is observed against the light from the reverse side of the page, so that P takes the position of S, and S of P, the diagram coincides with that of OA3. This shows that there is really no logical difference between these two moods when considered independently of any given definition of S and P. The reader will find it interesting to examine in the same way moods AA4, IE1, 2, 3, or 4, and OA2, determining which of the recognized valid moods they are identical with.

eight such validating forms, and any syllogistic inference that is correct must exemplify one or another of those eight moods. If the conclusion is universal, the inference will necessarily conform to one of the following patterns:

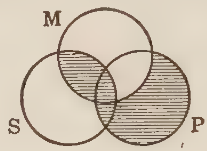
AA1

All M is P
All S is M
∴ All S is P



AE2 or 4

All P is M
No S is M, or No M is S
∴ No S is P



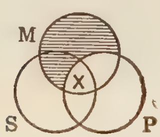
EA1 or 2

No M is P, or No P is M
All S is M
∴ No S is P

If it is particular, the inference must conform to one of the following:

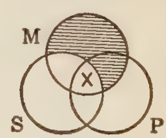
AI1 or 3

All M is P
Some S is M, or Some M is S
∴ Some S is P



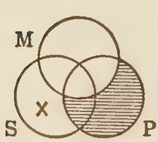
IA3 or 4

Some M is P, or Some P is M
All M is S
∴ Some S is P



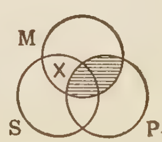
AO2

All P is M
Some S is not M
∴ Some S is not P



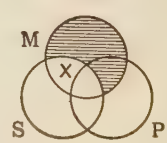
EI1, 2, 3, or 4

No M is P, or No P is M
Some S is M, or Some M is S
∴ Some S is not P



OA3

Some M is not P
All M is S
∴ Some S is not P



And one practicable way of testing any syllogism about whose validity there is doubt, is to check it against this list of valid forms.

But since there are eight valid forms, this takes time. Also, we want to be able to show, in the case of an invalid syllogism, just what is wrong with it. How can this be done?

A traditional simplification of the procedure for determining validity

Well, one way possesses the sanction of long tradition. Critical examination of these eight forms shows that they all respect certain rules which are capable of fairly simple statement, and that if any of these rules is violated the inference no longer illustrates any of the valid patterns. What rules would such a list include?

Notice in the first place that no pair of valid premises consists of two negative propositions. Evidently, the exclusion of M from P, and of S from M, whether the exclusion is universal or partial, gives no basis for inferring any inclusion of S with P, or exclusion from it.[5] This may readily be seen in the case of universal exclusions by considering the three following pairs of propositions:

1. No cats are dogs.
   No cats are sheep.
2. No cats are dogs.
   No cats are greyhounds.
3. No cats are dogs.
   No cats are sociable creatures.

These propositions are all true. But it happens to be true that no dogs are sheep, that all greyhounds are dogs, and that some dogs are sociable creatures while some are not. The reader may satisfy himself by a similar procedure in the case of partial exclusions, and in the case where one premise is a universal while the other is a partial exclusion. Notice, further, that whenever one of the premises is an exclusion the conclusion is also an exclusion. This indicates that an exclusion in a premise (combined properly with an inclusion in the other) may provide grounds for an exclusion, universal or partial, of S from P, but cannot imply that any of S is included in P.

[5] When each of these classes is positively determined. Otherwise, by obversion, what appears to be an exclusion becomes really an inclusion of positive classes.

Also, no conclusion is implied by two particular premises. If the reason for this in the case of two I premises is noted and expressed in terms of the concept of distribution, defined in the preceding chapter, another simplifying principle emerges. It is evident that in that case the middle term must be undistributed in both premises. This means that neither of the premises gives information about all members of the class denoted by the middle term, but only information about some unspecified part of the class. If such is the case, however, the members of the class referred to in one premise may be entirely different from those referred to by the same term in the other premise. When this is so, the middle term gives no common basis for comparing the other terms. It is evidently necessary, then, for the middle term to be distributed at least once in order that the class to which it refers may provide a genuine basis of comparison between S and P.[6]

Another principle whose necessity is obvious when it is stated, but which one not trained in logical discrimination may easily violate, is that neither the subject nor the predicate term may be distributed in the conclusion if it is undistributed in its premise. This would be illegitimate because then the conclusion would be affirming something about an entire class when the premises only gave information about some of its members.

Now it happens that if these principles are respected all of the possible combinations of premises that do not yield a valid conclusion will be avoided except two. Those two are AA3 and AA4. Traditionally, AA3 was regarded as a valid mood, the particular conclusion: Some S is P, being drawn from it. But this is not allowed by our geometrical diagrams, and it would violate an important principle explained in the preceding chapter, namely, that a particular conclusion is not

[6] It might be thought from this consideration that the middle term must be distributed in both premises. But this is not necessary. If something is asserted about the whole of a class in one premise, whatever is asserted in the other premise about a part of it is asserted about a part of *that* whole class. And whatever is true about the whole is true about any group of its members. So S and P can be compared through M if M is distributed once.

strictly implied by universal premises, unless a further premise is assumed to the effect that members of the classes involved exist.[7] Hence this mood must be pronounced invalid, and the same applies to AA4.

Rules of validity for syllogisms

The results reached by this analysis permit the formulation of a brief set of rules of validity for syllogistic inferences, any violation of which constitutes a fallacy that can be specifically described. Application of these rules provides a practicable procedure for determining whether a given syllogism is valid or not, and the names of the fallacies which are committed when these rules are found to be violated have been familiar to students of logic for many centuries.

1. Premises both of which are exclusions of positively determined classes imply no conclusion. The violation of this rule constitutes the fallacy of *negative premises.*
2. If one premise is an exclusion the conclusion must be an exclusion. To violate this rule is to commit the fallacy of an *affirmative conclusion from a negative premise.*
3. The middle term must be distributed in at least one of the premises. Any violation of this rule is a case of *undistributed middle.*
4. No term may be distributed in the conclusion if it is undistributed in its premise. The violation of this rule in the case of the subject or minor term is the fallacy of *illicit minor;* in the case of the predicate or major term, *illicit major.*
5. Premises which do not imply the existence of a class cannot yield a conclusion which does imply its existence. We shall call the violation of this rule *illicit assumption of existence.*

As we shall see later, these rules do not pretend to guard a thinker against all the logical errors into which one might fall in syllogistic reasoning. But they do define the conditions under which, from premises affirming an inclusion or exclusion between M and P, and between S and M, a conclusion affirming an inclusion or exclusion between S and P may be validly drawn. They also provide useful technical terms by which to describe any failure to respect those conditions.

[7] Cf. above, pp. 226 f., 231 f. A special case is constituted when both premises are class membership propositions, treated as A propositions. In this case the conclusion: Some S is P, may be validly drawn.

A further simplification pursued

Is any further simplification of the validating forms for the syllogism possible? Yes, and it can be discovered by turning for aid from the geometrical diagrams to the algebraic symbols for propositions of class inclusion. The geometrical diagrams were valuable, in part, because they vividly exhibit the fact that in a syllogism with an E or an I premise it makes no difference which term in that premise is subject and which is predicate; the structure of the inference is the same.[8]

By aid of the algebraic representation of syllogistic propositions

But the usefulness of the geometrical method in this regard is limited. If the algebraic symbols for the valid combinations are employed in a way which will now be explained, it quickly becomes evident that the syllogisms yielding a universal conclusion all conform to a certain common pattern, and likewise the syllogisms yielding a particular conclusion. Thus two forms, more general and abstract than can be directly exhibited by the geometrical diagrams, are sufficient to express the essential structure of any valid syllogism. Let us see how this is so.

The three valid universal syllogisms are as follows:

AA1

All M is P, i.e.,	$MP' = 0$
All S is M	$SM' = 0$
All S is P	$SP' = 0$

AE2 or 4

All P is M, i.e.,	$PM' = 0$
No S is M	$SM = 0$
(or, No M is S)	$(MS = 0)$
No S is P	$SP = 0$

EA1 or 2

No M is P, i.e.,	$MP = 0$
(or, No P is M)	$(PM = 0)$
All S is M	$SM' = 0$
No S is P	$SP = 0$

If the first and the third of these algebraically symbolized syllogisms are compared, it becomes evident that the only difference between them is that in the third the class P ap-

[8] If the aid thus provided had been dispensed with, and a difference of logical structure had been assumed corresponding to every difference of figure, there would have been fifteen valid combinations instead of eight. For, in that case, of the patterns yielding a universal conclusion it would have been necessary to distinguish between moods AE2 and AE4 instead of regarding them as a single mood, and likewise between EA1 and EA2;

pears where the first has P′. But if the rule regarding negative premises is obeyed it makes no further difference whether the class with which an inference is concerned happens to be positively determined or negatively determined; in fact, study of the operation of obversion shows that any proposition about a positively determined class can be transformed into one about a negatively determined class. Thus, from the standpoint of logical structure, it is unnecessary to notice whether a given proposition in a valid syllogism is about class P or about its complementary class P′; all that is essential is that if P occurs in the premise it must also occur in the conclusion, while if P′ occurs in the premise P′ must occur in the conclusion. Similarly, if the second syllogism is compared with the third, the only difference is that the positions of M and M′ in the two premises are reversed. In the second, P is conjoined with M′ and S with M, while in the third P is conjoined with M and S with M′. Now this difference is of no consequence, logically, either. The reason is that in both these inferences the conclusion is an E proposition—the exclusion of S from P. But since E is directly convertible, the same relation of the two classes can be expressed by: No P is S. This means that, throughout the inference, P might be substituted for S, and S for P, with no difference in the logical structure involved. It is at once evident that if such a substitution were made, the second and third syllogisms would become identical. In fact, this identity is capable of geometrical exhibition. Turn back to the diagrams for these two moods on page 244. Look at one of those diagrams, and then look at the other toward the light from the under side of the page, so that the P circle and the S circle will exchange places. The identity of the two structures will be evident.

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there would have been five patterns for a universal syllogism instead of three. Similarly, among the forms yielding a particular conclusion it would have been necessary to distinguish between moods AI1 and AI3, between IA3 and IA4, and between EI1, EI2, EI3, and EI4; there would have been ten forms for a particular syllogism instead of five. It was a genuine simplification to become habituated to symbols which show concretely that conversion of an E or I proposition introduces no logical difference.

The canonical universal syllogism

Thus the three moods which yield a universal conclusion reduce to a single pattern whose form, as exhibited in the algebraic equations, is determined by the following rules:

1. The middle term has a different quality in the two premises.
2. The other terms have the same quality in the conclusion as in the premises.

The pattern fixed by these rules is called the *canonical universal* syllogism, and any syllogism which purports to establish a universal conclusion may be tested by expressing it algebraically and checking it against this pair of rules. If it conforms, it is valid; if it does not conform it is invalid.

The canonical particular syllogism

Similarly, the syllogisms yielding a particular conclusion may be shown to conform to a common pattern which, when distilled and abstractly expressed, may be called the *canonical particular* syllogism. The moods which validly imply particular conclusions are the five following:

AI1 or 3	IA3 or 4
All M is P	Some M is P (or, Some P is M)
Some S is M (or, Some M is S)	All M is S
Some S is P	Some S is P
$MP' = 0$	$MP \neq 0$ $(PM \neq 0)$
$SM \neq 0$ $(MS \neq 0)$	$MS' = 0$
$SP \neq 0$	$SP \neq 0$

AO2	EI1, 2, 3, or 4	OA3
All P is M	No M is P (or, No P is M)	Some M is not P
Some S is not M	Some S is M (or, Some M is S)	All M is S
Some S is not P	Some S is not P	Some S is not P
$PM' = 0$	$MP = 0$ $(PM = 0)$	$MP' \neq 0$
$SM' \neq 0$	$SM \neq 0$ $(MS \neq 0)$	$MS' = 0$
$SP' \neq 0$	$SP' \neq 0$	$SP' \neq 0$

That the difference between the first and second of these moods is of no logical importance may be shown in the same manner as that by which the difference between AE2 or 4 and EA1 or 2 was shown to be of no consequence. It is that M is conjoined successively with P′ and S in one case; with

S′ and P in the other. But since the conclusion is an I proposition, which is directly convertible, it might have been expressed by $PS \neq 0$ just as readily as by $SP \neq 0$. That is, S and P could exchange places throughout the syllogism and no logically significant difference would appear. If this is done, however, either of the two moods becomes transformed into the other. And this can be realized visually by looking at the geometrical diagram for one of them from the under side of the page, so that the S and P circles exchange places, and comparing it with the diagram for the other viewed in the usual manner.

It is a more lengthy process to show that the three syllogisms yielding an O proposition as conclusion possess a common structure, and that it is essentially the same structure as is present in the other two. Omitting this demonstration here, we shall list at once the three rules which express this common structure, and in a note at the end of the chapter it will be shown what any inference is doing when it respects those rules and why it must be valid. The rules which are obeyed by each of these five syllogisms, and which therefore express the nature of the canonical particular syllogism as revealed in the algebraic equations, are as follows:

1. The middle term appears with the same quality in both premises.
2. The term conjoined with it in the universal premise is replaced in the conclusion by its complement.
3. The term conjoined with it in the particular premise appears unchanged in the conclusion.

Any syllogism purporting to demonstrate a particular conclusion may be tested by checking it against the canonical particular as defined by these rules. If it conforms, it is valid; if it fails to conform, it is invalid.

Acquisition of familiarity with the algebraic method of symbolizing propositions of class inclusion, and with the conjunct classes about which such propositions are concerned as that method interprets them, thus permits a further step of logical abstraction and generalization. Instead of eight

validating forms for the syllogism, or five rules which syllogistic inference must obey, just two validating forms need to be recognized—one to which every syllogism purporting to establish a universal conclusion must conform if it is valid, and another to which every syllogism claiming to yield a particular conclusion must conform.

Is there no way in which this process of generalization can be carried still further, and a single validating form applicable to every syllogistic inference be discovered?

A final simplification of syllogistic validating forms

Yes, there is, but it has the unavoidable disadvantage of requiring one who uses it to proceed counter to man's natural logical habits. Consider these two facts: (1) that for every universal proposition there is a particular which is its contradictory, and vice versa; and (2) that in any syllogism yielding a universal conclusion both premises are universal, while in any syllogism yielding a particular conclusion one premise is universal and one particular. Suppose, then, that the conclusion in all of these valid syllogisms is contradicted. The pattern of the universal and that of the particular syllogisms, otherwise different, now exhibit a common structure, which is defined by the three following rules:

1. Of the three propositions involved, one is particular while two are universal.
2. The term which is common to the two universals has opposite quality in its two occurrences, being once positive and once negative.
3. The particular conjoins the other two terms, each of which has in it the same quality that it has in the universal.

Let us see how this works out with the first of the valid universal moods and the first of the valid particular moods.

AA1	AI1 or 3
$MP' = 0$	$MP' = 0$
$SM' = 0$	$SM \neq 0$
$SP' \neq 0$	$SP = 0$
(which contradicts $SP' = 0$)	(which contradicts $SP \neq 0$)

The rules just listed are evidently exemplified in each of these cases. And if the same procedure is tried with the other valid

moods it will be found that they exemplify the rules also. But no invalid inference exemplifies them.

If a slightly more abstract symbolism than has yet been used, but which is suggested by these rules, is now adopted, a single validating form may be set up, applying to any syllogism when its conclusion has been contradicted in this manner. Let X be the term common to the two universal propositions (whether it stands in any given case for S, M, or P), while Y and Z are the terms conjoined in the particular proposition. The common pattern then will be:

$$XY = 0$$
$$X'Z = 0$$
$$YZ \neq 0$$

The inconsistent triad, or antilogism

Of course, any such triad of propositions must be internally inconsistent, since it is established by contradicting the conclusion of a valid syllogism and combining that contradictory with the two premises. Hence it is called an *inconsistent triad,* or *antilogism.* The three propositions cannot all be true together; if any two of them are true, the third must be false. It is for this reason that the use of the antilogism violates a natural habit of mind, namely, to test a piece of inference by comparing it with a standard that is necessarily right rather than with one which is necessarily wrong.[9] Its justification lies in the fact that a validating form thus set up provides a single, universally applicable pattern for revealing the structure and testing the validity of any syllogistic inference. Its three rules define the nature of every valid deduction employing propositions of class inclusion, whether the conclusion is universal or particular. It thus carries the quest for formal unity and simplicity in the analysis of such inferences to completion.

Its application to any suspected inference is quite easy. After the premises and conclusion have been put in correct form, bringing out in each the common terms and the inclu-

[9] We do, however, sometimes employ a *reductio ad absurdum, i.e.,* a method of proof which proceeds by showing that refusal to accept the conclusion would contradict the premises.

sion or exclusion asserted to obtain between M and P, S and M, and S and P, symbolize each of the three propositions algebraically. Then contradict the conclusion, *i.e.*, change the equation into an inequation, or vice versa. Note finally whether the result conforms to the three rules which define the structure of the antilogism. If it does, the inference is valid. If it does not, it is invalid.

NOTE

A brief demonstration is here offered of the validity of the canonical universal and the canonical particular syllogisms.

In the case of a syllogism with a universal conclusion, all that is necessary for such a demonstration is to consider the three conjunct classes which are affirmed to be null in the two premises and the conclusion, and to apprehend the general principle involved in the way in which these classes are interrelated. It is evident from the two rules which define the canonical universal and from the way in which they are illustrated by the three valid syllogisms falling under it, that what one is doing in any such inference is as follows: The conjunct class formed by the product of M and a second class (whether S, P, or P′) is asserted to have no members. Also the conjunct class formed by the product of M′ and a third class (which is likewise either S, P, or P′) is asserted to have no members. In the case of AE2 or 4 the second premise may be thought of as being asserted first, and the first second. The conclusion is then drawn that the conjunct class formed by the product of the second and third classes has no members.

AA1	AE2 or 4	EA1 or 2
$MP' = 0$	$PM' = 0$	$MP = 0 \ \ (PM = 0)$
$SM' = 0$	$SM = 0 \ \ (MS = 0)$	$SM' = 0$
$SP' = 0$	$SP = 0$	$SP = 0$

Now by adopting a somewhat more abstract symbolism, as we did in dealing with the antilogism, this procedure which is common to the three syllogisms may be exhibited quite vividly. Let us retain M for the middle class; M′ will therefore be its complementary class. But let us symbolize by A the class conjoined with M (whether it happens to be S, P, or P′), and by C the class conjoined with M′. The essence of the inference then is:

$$\begin{aligned} AM &= 0 \\ CM' &= 0 \\ AC &= 0 \end{aligned}$$

and this is an appropriate algebraic expression for the canonical universal syllogism.

What now, in nonsymbolic language, does this more general expression assert? It asserts that if the conjunct of a certain class with a second class is null, and likewise the conjunct of the complement of that class with a third class is null, then the conjunct of the second class and the third class must be null. One's apprehension of the correctness of this law[10] will be facilitated by considering two concrete cases. Let A be the class of gods, C the class of men, and M the class of mortal beings. Then M′, of course, will be the class of immortal beings. Well, if there are no mortal gods, and no immortal men, then no men can be gods and no gods men. For all gods will necessarily be immortal, and all men mortal, while the classes "mortals" and "immortals" exclude each other in virtue of their meaning. Again, let A be the class of inconsistent propositions, C the class of verified propositions, and M the class of true propositions. Then M′ will be the class of false propositions. So, if no inconsistent propositions are true, and no verified propositions are false, then it follows that no verified propositions are inconsistent, and no inconsistent propositions verified. For, true and false excluding each other, all inconsistent propositions will be false, and all verified propositions true. Such cases enable one to apprehend clearly that the general law which they exemplify is valid. It is the law obeyed by any syllogistic inference really yielding a universal conclusion.

Consider now the canonical particular syllogism. What is an inference essentially doing when it proceeds according to this pattern in any of the five legitimate ways? And how demonstrate that such a procedure is always valid? To answer these questions two logical principles must be noted which follow from the nature of disjunction and of conjunction as applied to classes, but which the text has not explicitly introduced. One is known as the Law of Expansion, and means that any class a is equivalent to the disjunct (technically called the "sum") of ab and ab', whatever other class b may represent. That is, a is equivalent to: either ab or ab'; symbolically expressed, $a = ab + ab'$. The validity of this law is rendered evident by reflecting upon concrete cases. The class of men is equivalent to the class of those that are either European men or men that are not European; the class of roses is equivalent to the class of those that are either white roses or roses of other colors than white. And of course it makes no difference how many classes are involved. The class ca is equivalent to the class $cab + cab'$. The other principle consists of a theorem readily demonstrated from the

[10] Assuming, of course, the definitions previously given of "conjunct," "null," "complement," etc., as applied to classes.

definitions above given, namely, that if any class is null, the conjunct class formed by its product with another class will also be null. If $a = 0$, then $ab = 0$. For illustrations: If dragons do not exist, then ferocious dragons do not exist; if the class of blonde mermaids is null, the class of beautiful blonde mermaids is also null.

These principles may now be used to demonstrate the validity of the logical structure exhibited by the five syllogisms yielding a particular conclusion. Let us begin with the particular premise, and note the conjunct class to the left of the sign of inequation. In the first of the five moods this will be SM (or MS). Now replace it, according to the Law of Expansion, by the logical sum $SMP + SMP'$, which is its equivalent. And since according to that premise $SM \neq 0$, it follows that $SMP + SMP' \neq 0$. But according to the universal premise, $MP' = 0$, that is, it is a null class. Hence, according to the second principle just mentioned, SMP′ must be a null class and be $= 0$. However, if SMP′ has no members, while $SMP + SMP'$ does have members, then the members must belong to SMP–*i.e.*, it is $\neq 0$. Now the middle term may be dropped out, since if any conjunct class has members each of the classes of which it is the product must have members. In other words, $SP \neq 0$, which is the conclusion of the syllogism. By following a similar procedure, the validity of each of the other moods yielding a particular conclusion may be demonstrated.

If the rules which express the structure of the canonical particular syllogism are carefully examined in the light of the principles just employed, what a thinker is essentially doing when he reasons in any one of these five moods will become evident. Abstractly stated, it is this: He asserts that the product of a certain class with a second class has members, and that the product of the same class with a third class has no members; then he infers that the product of the second class with the complement of the third class has members. Again, let us adopt a slightly more general symbolism which will exhibit this common procedure. Retaining M for the middle class, we shall symbolize by A the class conjoined with it in the particular premise, and by C the class conjoined with it in the universal premise; the conjunct class in the conclusion will therefore be AC′. The common pattern of these five moods may thus be symbolized as follows:

$$MA \neq 0$$
$$MC = 0$$
$$AC' \neq 0$$

Of course, when such a general symbolism as this is employed, it is necessary to remember that if the conclusion is affirmative, class C will be negatively determined and class C′ positively determined, while class M may also be negatively instead of positively determined

if the conclusion is negative. One must, in short, keep in mind that this formula takes no account of the way in which the classes happen to be determined; otherwise the same pattern for syllogisms yielding an I and those yielding an O conclusion could not be exhibited.

Apprehension of the soundness of the procedure displayed by this common pattern will be facilitated, as it was in the case of the canonical universal, by considering instances. Suppose M represents the class of cabbage palms, A the class of palm trees, and C the class of trees over one hundred feet tall. The inference then consists in saying that, if some palms are cabbage palms, and no cabbage palms are over one hundred feet tall, then some palms are not over one hundred feet tall. Another instance may be chosen in which the conclusion is affirmative, and therefore in which C′ will be a positively determined class while C is negatively determined. Suppose M stands for the class of Brazilian products, A for the class of coffee beans, and C for the class of things that are not consumed in North America. Then the reasoning is as follows: If some coffee beans are products of Brazil, and there are no products of Brazil that are not consumed in North America (or, that all products of Brazil are consumed in North America), then some coffee beans are consumed in North America.

EXERCISES

1. Put the following syllogisms in correct form for testing, supplying any missing premise or conclusion; symbolize them by the geometrical diagrams; tell whether they are valid or not by application of the traditional rules; and if there is a fallacy, describe it in technical terms:
    a. No sensible man despises friendships. He is a sensible man; therefore he does not despise friendships.
    b. All human productions are liable to change. Scientific theories are human productions, and so are liable to change.
    c. A good conscience is not wealth, but a good conscience satisfies. Hence wealth does not satisfy.
    d. Planets move round the sun. The earth moves round the sun, and therefore must be a planet.
    e. Power is desired by all men. Now money is power, and hence it must be desired by all men.
    f. The fearful are not happy. Greedy men are fearful, and therefore are not happy.
    g. Language is the communication of information by signs, hence the bristling of a cat's fur is language.
    h. Edith must be blue-eyed, for all blonds are blue-eyed.
    i. Habitual smokers are not long-lived. But you will enjoy a ripe old age, for you have avoided the weed.

j. We love God because He first loved us.
k. People who spread malicious gossip are liars, and you spread malicious gossip about me.
l. Some of the students in this class are seniors. All of the class received B or above, hence the grade of some seniors was B or above.

2. Put the following in correct form for testing, and symbolize each algebraically. If its conclusion is universal, check it by the canonical universal syllogism; if the conclusion is particular, by the canonical particular. Tell whether it is valid or not. If it is invalid, tell which rule (or rules) defining the structure of the relevant canonical syllogism is violated.
   a. I exist. Because I think, and whoever thinks exists.
   b. Some indulgences cause repentance, and nothing that causes repentance is desirable. Hence some indulgences are not desirable.
   c. No man can get rid of his worst enemy, for he cannot get rid of himself, and himself is his worst enemy.
   d. All who permit starving people to die are homicides. Now those who do not give alms are permitting starving people to die, hence they are homicides.
   e. All good men can be trusted; therefore no good man is a liar.
   f. All missionaries are morally reputable. Among the visitors at this resort are missionaries, hence some of the visitors here must be morally reputable.
   g. Those who know their subject pass examinations, and it is among those who attack an examination confidently that you will find those who know their subject. Hence those who attack an examination confidently will not fail.
   h. No student presented a class card, but all the students were admitted. Hence none who were admitted had class cards.
   i. Only upper classmen are eligible. These boys have been pronounced ineligible, hence they must be freshmen.
   j. All trespassers will be prosecuted. I shall carefully avoid trespass, and hence need not fear prosecution.
   k. Some freshmen are promoted. Some students whose grades are unsatisfactory are freshmen, hence some students whose grades are unsatisfactory must be promoted.
3. Check each of the inferences in (2) by the method of the antilogism, stating, if it is invalid, which rule of the antilogism is violated (or which rules, if more than one is violated).

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CHAPTER 14

FURTHER PROBLEMS IN THE STUDY OF FORMS

The two preceding chapters have laid the foundations for an understanding of the syllogism and its validating forms. From the establishment of these foundations one may proceed in either or both of two interesting directions. It is apparent that in the progressive reduction of the syllogistic canons which has just been completed, two motives find satisfaction. One is a theoretical delight in distilling unity and simplicity out of diversity. The other is a sense of greater ease in the practical use of the canons for testing inferences about classes when a single standard pattern is applied instead of some more complicated set of rules. Of course, this increased ease must be paid for by the greater time and effort necessary to master thoroughly such a highly abstract validating form as was finally established.

How a calculus of classes is constructed

Beyond the point which has now been reached these two motives tend to diverge rather than to remain in each other's company. The student of the syllogism whose interest is mainly theoretical—that is, who loves unity and simplicity as such, finding a sufficient reward merely in the disclosure of an order underlying and binding together the material with which he is dealing, will be apt to proceed in the direction of completing a calculus of classes. This means that he will focus his attention on the interrelations which obtain between the concepts, forms, and laws that emerge from a study of the structure of syllogistic inference, and will attempt a logical organization of them so that they will all find their place in a single formal science whose theorems are

capable of rigorous demonstration. It has been noticed above that these entities, in virtue of their meanings, are connected with each other in various ways: inclusion with implication, disjunction with negation, existence with quantity, and the like. A perennial challenge to the logician arises from this circumstance. He is eager to pursue and master these manifold interconnections until he can construct a system in which each is properly located in relation to the others.

And what will this mean? First, he must discriminate the ideas that promise to serve most successfully as the foundation of such a system from those which are readily proved or demonstrated in terms of that foundation. The former will constitute his *postulates;* they will be set forth as clearly as possible in accurate language and provided with adequate symbols for their representation. These postulates will ordinarily consist, in part, of definitions of the concepts which he proposes to employ as basic; in part, of the specification of operations which he recognizes as permissible. Undefined terms will be reduced to a minimum. All other principles, laws, and rules of valid inference will then appear as theorems, deduced step by step from the initial definitions by operations explicitly allowed. In the demonstration of each theorem, no principle is used that has not previously been established as a theorem or admitted in the list of original postulates. The postulates, of course, will be independent of each other, consistent with each other, and will be reduced to the smallest possible number that is required to prove all the theorems.[1] Thus a scientific system of validating principles for inference about class inclusions will be constructed, exhibiting the neatness, rigor, and elegance of a branch of mathematics. Several such calculi have been recently put forward by logicians; all have been influenced in many ways by the Boole-Schröder algebra of classes whose publication began in 1890, and which constitutes the first markedly successful attempt of this kind.

[1] Except when the elegance thus gained is outweighed by the inconvenience which may be caused if the number is very small.

Possibility of a more inclusive formal science

It has also been observed that certain notions that are fundamental in the calculus of classes are formally related to concepts employed in the calculus of propositions, and likewise to principles involved in the theory of relations and the structure of mathematical science which is built by its aid. This suggests a still more daring enterprise to the logician in quest of formal order and unity. Why not set up a more abstract and all-inclusive foundation, which will unite in a single system all these various branches of formal science? Those who attempt such a system will not, of course, need to deal with all the detailed theorems in the various formal disciplines; it is sufficient for them to prove rigorously that the distinctive concepts, definitions, and operations employed in each discipline, *e.g.*, arithmetic or geometry, are deducible from the more general ones which it is their responsibility to establish. Once this is accomplished, the body of theorems in any branch of formal science can be perceived to follow without more ado. Historically, the great example of a system of this kind is the *Principia Mathematica* of Whitehead and Russell, published in 1910–13. Beginning with two undefined concepts and five postulates, the authors proceed step by step to lay the foundations of all formal science, including a calculus of classes, a calculus of propositions, a calculus of relations, and a calculus of propositional functions. This work bids fair to remain for a long time the classic one in its field.

Among the postulates or the early theorems of a formal system of this kind will be included the quite general logical principles which have been introduced in the preceding chapters as we found need for them—the law of contradiction, the law of excluded middle, the law of commutation, the law of double negation, the law of expansion, and the like. These express decisions agreed upon among formal scientists as to what precise syntactic meaning should be assigned to the logical constants, in the light of the role in responsible reasoning which analysis shows them to fill.

Such a unified system of the formal sciences has an important value for the factual sciences too. Investigators in these

fields are not only eager to establish verified laws of the way in which this or that part of nature behaves; they wish also, as far as possible, to include these laws under a smaller number of more general laws from which they can all be derived. A rigorous formal system provides an ideal model which such a system of factual laws will approximate as closely as the nature of the material organized in them permits. Its set of postulates corresponds to the general laws or comprehensive theories which give unity to the factual system, while the detailed theorems in the various formal sciences that are demonstrated from those postulates correspond to the specific laws in this or that area of fact which become deducible from the general laws. The way in which such organizing systems are developed will be studied later when the methodology of factual science is under analysis.[2]

It is not the task of the present volume to guide the reader further in this direction. Our progressive advance toward unity in formalizing the structure of the syllogism in the preceding chapter may be taken as an illustration of the possibilities of systematization which the formal scientist realizes on a wider scale. Those who wish to pursue this theme more fully should consult the excellent manuals of symbolic or mathematical logic which are now available. Those who, more practical in temper, are mainly interested in the application of the forms, especially the syllogistic canons, to the inferences which they and their friends are daily drawing, learning how to distinguish fallacious from valid reasoning more readily, will find themselves confronted at this juncture with a problem which as yet has not been broached. A brief consideration of it is in order here, after which the balance of the chapter will be devoted to a summarizing review of formal fallacies and of the main lessons which reasoning in general may learn from a study of formal science.

Desirability of practice in putting inferences in logical form

The exercises and illustrative material introduced in the last chapter exemplify the simplest inferences with which one ever has to deal in the analysis of deductions employing

[2] See below, chap. 19, pp. 409–416.

propositions of class inclusion. Although even there it was necessary to warn the reader that he may need to put the syllogisms in "correct form for testing," for the most part they were already in this form—they happened to be predigested, as it were, exhibiting three unambiguous terms and an inclusion or exclusion between each pair of classes involved just as they stand. But usually this is not the case, especially in the arguments about matters of law, morals, religion, politics, and economics, which we confront in much of our most difficult thinking. Reasoning on such subjects is so intimately affected by our feelings and prejudices that it easily takes the guise of persuading ourselves or others of the truth of some favored conclusion instead of becoming an impartial search for a correct answer. Hence the inferences engaged in are likely to be expressed in a way that promises to win psychological conviction instead of the way which would aid accurate testing of their validity. In dealing with arguments of this kind the main problem is usually that of overcoming the handicap under which one has been placed by these circumstances; it is the problem of restating the propositions which constitute the inference in hand so that, without change in their essential meaning, the reasoning can be tested by the validating forms presented above. When this has been done, determining its validity or invalidity ordinarily poses no difficulty.

But putting such inferences in correct form is an affair of art rather than of science. No infallible rules for guidance can be laid down; skill and facility in making the transformations needed can be won only by persistent practice with material of gradually increasing difficulty. Of complex inferences, none is closely similar to any other; each must be analyzed and reduced to correct form in the light of its individual peculiarities.

Two general problems: 1. Disentangling the common terms in a syllogism

The two general problems in dealing with class inclusion inferences which most beginners find insistently demanding this laborious acquisition of artistic skill for their mastery, are: first, the problem of disentangling the common terms in a

syllogism where they are not immediately obvious, and second, the problem of analyzing a lengthy argument into a series of syllogisms which constitute its formal equivalent. It will be well to consider each of these.

In the exercises above selected, the common terms are pretty clearly indicated by the form in which the reasoning is presented. But suppose one were to meet an inference like the following: "Drugs are of no use in cases of sickness, because we must depend on nature to effect a cure." Here there is no obviously common term between the given premise and the conclusion drawn. Yet if the reasoning is valid there must be some common idea between them, and if it is not valid the inference must be stated in such a form as will drag to light precisely what confusion is leading the author astray. To discover the common idea if there is one, and if not, to decide just what logical misbehavior is involved, is a matter of art; it means casting about among the various possible ways of completing the argument, and selecting the one most plausibly operating in the author's mind. In this particular case the missing idea is probably something like this: "Drugs are not nature"—or more exactly, perhaps: "The action of drugs is not a natural process." Once this has formulated itself in our minds there is no difficulty in restating the other premise so as to provide a middle term: "The processes which we must depend upon to cure cases of sickness are natural processes." The entire argument may then be put in syllogistic form:

> The processes which must be depended upon to cure cases of sickness are natural processes.
> The action of drugs is not a natural process.
> ∴ The action of drugs is of no use in cases of sickness.

Here the predicate term in the conclusion is not the same as the predicate term in the major premise, which shows that the author has been guilty of a fallacy. He is assuming that two classes are identical which, as analysis would show, do not necessarily have the same members—the class "processes

to be depended on for curing sickness" and the class "processes of use in cases of sickness." This is usually called the fallacy of *four terms*. But the inference has been restated in such a way as to make apparent the common terms where they exist, and to disclose the precise illogicality of which the author is most probably guilty.

Special difficulties that may be encountered

In this case the problem of disentangling the common terms is complicated by the fact that, as presented, the inference is an enthymeme. Even when that complication is not faced, however, it may take some experimental reflection to determine just how an argument should be restated so as to bring out the common terms and class inclusions clearly. Not only may one or more of the propositions need to be converted or contraposed, or transformed so as to eliminate such words as "only," "unless," "except," etc., but an idea expressed by a noun in one proposition may have to be expressed by a clause or modifying phrase in another. Most readers, no doubt, found some puzzled consideration necessary when dealing with Exercise 9(i) on page 257 before they realized that the phrase "will enjoy a ripe old age" may be replaced by "are long-lived," and likewise the phrase "have avoided the weed" by "are not a habitual smoker." These substitutions, when made, gave the inference the form of a syllogism. But the difficulty is naturally apt to be greater when one premise is missing and must be supplied.

2. Analysis of a train of argument

When one encounters trains of reasoning, such as long arguments used by others in the endeavor to convince him of the soundness of their beliefs, an added difficulty arises besides those just examined. He finds in their speeches and writings not only the rhetorical device of playing upon his prejudices and interests in clever ways, but also the skillful interweaving of argumentative portions with those of description, exhortation, or the citing of concrete examples in support of the implied premises. All this is apt to carry a hearer or reader along rather seductively, and puts systematic logical criticism of the reasoning under great handicaps. Accordingly, one needs some practice in the dissection of

lengthy arguments, in order to acquire facility in the elimination of the parts that are formally irrelevant and in the reduction of the rest to a form suitable for logical testing. The three main steps to be taken in subjecting material of this sort to such a preliminary analysis may be described as follows:

1. Discriminate the main skeleton of the argument, as a guide to more detailed analysis.
2. Set aside all formally irrelevant portions, such as merely descriptive matter, exhortation, or flights of fancy.
3. Pick out and reduce to separate syllogisms the inferences drawn from the assumed premises to the conclusions one is asked to accept, placing them in their proper order.

These steps may be exemplified by studying a portion of a typical advertisement.

Around each city is a metropolitan district of a few square miles, the natural trading area of that market. You can prove this to yourself. As you drive in your automobile toward any large city, you know immediately where the metropolitan district begins. At a certain point, you pass from open country, where there are ten to fifteen families per square mile, into suburban and city districts, with a thousand or three or four thousand families per square mile. The transition is not gradual—it is sudden. No one could go far wrong in marking the actual market boundary lines of any metropolitan city.

Within the metropolitan area, concentration of families per square mile is 145 times greater than in the small town and rural communities outside of that area. The average production per square mile is 500 times greater. The average purchasing power per family is 3½ times greater. Within the metropolitan area, you can get effective newspaper coverage—outside of it you cannot.

It follows that where people and dollars and circulation are concentrated, there is the greatest opportunity for volume selling at low cost.

These self-evident facts have developed the P + D + C principle of selecting markets and advertising media.

P is population, measured in families, because these are the buying units.

D is dollars or annual wealth production, because this is the measure of purchasing power.

C is circulation coverage as represented by the percentage of

families reached, because this measures your ability to influence the buying habit of the entire market.

The truth will out when P + D + C is applied.

When this advertisement is read over a few times, it becomes clear that its main argument is reducible to two syllogisms, and as soon as the general tenor of the argument is grasped it can be disentangled from the logically inessential phrases. The first syllogism is embedded in the first paragraph, and attempts to establish the conclusion that around each city there is a definite metropolitan district of limited area. The overt premise is stated in the fourth and fifth sentences of the paragraph. It is that as one passes from the country to the city district there is a sudden transition from an area of ten or fifteen families to the square mile to one of a thousand or more families per square mile. The implied further premise is, of course, that wherever this is the case there exists a definite metropolitan district. The other sentences simply introduce this overt premise to the reader, or add the further assumption that such a metropolitan district constitutes the natural trading area of the city market.

The second paragraph states the minor premise of the second syllogism. The third paragraph, in form, appears to give the conclusion of this syllogism, but when carefully examined it is seen to be really the major premise, and the conclusion is not specifically stated. Summarized briefly, the syllogism is as follows:

> An area where people and dollars and circulation are concentrated furnishes the greatest opportunity for volume selling at low cost.
>
> The metropolitan area is an area where people and dollars and circulation are concentrated (omitting the percentages).
>
> ∴ The metropolitan area furnishes the greatest opportunity for volume selling at low cost.

The fourth paragraph restates the major premise in the form of an advertising policy, while the three following statements present the assumptions made with respect to the

methods of measurement involved. The final statement is a quite irrelevant exclamation. It is added because it had been used in headline form as a bid for the reader's attention.

Summary of previous references to formal fallacies

Let us now give fuller attention than has yet been given to the subject of formal fallacies. It has been noted that a fallacy is committed whenever a piece of inference fails to exemplify the proper validating form, or—and this comes to the same thing—whenever it violates one of the rules which determine the structure of that form.

The most common fallacies that one may fall into have occupied our attention briefly when we were engaged in analysis of the validating forms that, in each case, are involved. With molecular forms, for example, the mistake may be made of assuming contrary propositions to be contradictories; and one may treat a weak disjunction as though it were strong. When using material implications as major premises, he may assume that a conclusion follows from a minor premise which denies the antecedent or affirms the consequent, or he may treat the material implication as though it were strict. With atomic forms, of course, there are the fallacies which result from misinterpretation of the elements involved in these forms, such as are revealed in an analysis of the square of opposition. Beyond these, one meets distinctive fallacies according to whether he is dealing with relational inferences or inferences of class inclusion. In the former case the main fallacy is that of treating a nontransitive relation as though it were transitive. In the latter case there are the various possible errors in conversion and obversion, and violations of the syllogistic rules, such as negative premises, undistributed middle, illicit major or minor, and illicit assumption of existence. There may also be the fallacy of four terms.[3]

These references to fallacies are too scattered if left as they stand, and in any case a few further considerations need to be mentioned.

[3] The first- and last-named fallacies may, of course, occur in relational inferences too. But they occur there more rarely.

A specific consideration important enough to be given a name in traditional logic will be introduced first. It requires a more detailed analysis of the last-mentioned fallacy—that of four terms. This fallacy may appear in any kind of deductive reasoning but is more likely to appear in inferences about class inclusions than elsewhere.

The fallacy of an ambiguous term

It is easy for the beginner to forget that matters of validity or invalidity are not determined by any word or words as such, but by their meaning. Hence the fallacy of four terms may be present even when the same words are repeated in an inference, if the meaning in the two cases is different so that two different classes are referred to rather than a single class. These situations are frequent enough to justify a distinctive way of describing them; they exemplify the fallacy of an *ambiguous* term. Consider the following syllogism:

> Whoever obeys laws submits himself to a governing will.
> Nature obeys laws.
> ∴ Nature submits herself to a governing will.

Here is ostensibly a valid syllogism, consisting of three universal affirmative propositions in the first figure. But it is advisable to scrutinize it carefully to see whether any of the terms, although verbally unchanged, may not vary in meaning in the course of the argument. There is ground for suspecting this in the case of the phrase "obeys laws." In the major premise the phrase refers to the situation of a human being in relation to a political superior. In the minor premise the phrase is perhaps used only metaphorically, for the laws of nature from the standpoint of science are simply generalized descriptions of what happens. They explain rather than prescribe, and if they are broken they are shown to be incorrect. Where such differences of meaning obtain, the fallacy of four terms is present even if verbally only three terms appear in the syllogism.

The student will find the following syllogism an interesting theme for study on this matter of ambiguity:

Who calls you a man speaks truly.
Who calls you a fool calls you a man.
∴ Who calls you a fool speaks truly.

Cases of an ambiguous term should be carefully distinguished from the fallacies of distribution and the fallacy of passing from hypothetical premises to a conclusion which assumes existence. In these latter situations the offending term refers to the same class in both its occurrences; only, in the fallacies of distribution, the same members of that class may not be involved, and in the other fallacy the class is taken to have members in the conclusion whereas the premises did not imply that it does. But in the fallacy of an ambiguous term two different classes are referred to, the difference being hidden by the identity of the words used to denote them.

Fallacies of division and composition

Two other four-term fallacies are sufficiently common to merit distinct names. In them, no complete verbal identity obtains, but a partial identity does, and it is of a sort which makes it easy to confuse the two classes needing to be distinguished. One of these has been traditionally called the *fallacy of division* and the other the *fallacy of composition.* Each will be briefly described and exemplified.

The fallacy of division consists in substituting for a term denoting a collection, during a piece of reasoning, a term denoting members of the collection. It thus assumes that what is true of the collections is true of their members.

Fraternities do not maintain a high scholastic standard.
This boy is a fraternity member.
∴ He does not maintain a high scholastic standard.

Clearly, it might be true that the average grade in any fraternity of a college is lower than that of other groups of students, without the consequence that the grade of every individual fraternity member must be low.

The fallacy of composition is the converse of the fallacy of division. It assumes that what is true of individuals will hold

true of the collections which they happen to form taken as collections. A classic instance of this is John Stuart Mill's attempt to demonstrate from psychological assumptions the truth of the Utilitarian principle of morality. Mill implies first these premises:

> The good is what is desired.
> Each person desires his own happiness.

He then proceeds:

> This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good, that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons.[4]

Essentially the same kind of error is revealed in reasoning that because something has a certain property in small quantities it will have more of that property in larger quantities. A child, at least, will be tempted to conclude that since a tablespoonful of a certain medicine is beneficial, a cup of it will be still more salutary!

Such a collection must, of course, be clearly distinguished from the class of its members. All valid reasoning about classes proceeds on the assumption, deducible from the definition of a class, that what is true of any class is true of each of its members and what is true of every member is true of the class. Thus, what holds of the class of fraternity members must hold of fraternity members individually; what can be affirmed of the class of soldiers can be affirmed of soldiers individually. But a fraternity is not the class of fraternity members but a collection of them, and is itself a member of a different class; hence what is true of fraternities may not be true of fraternity members, just as what is true of armies may not be true of soldiers.[5]

Fallacy of an overt fourth term

This description of the errors of ambiguity, division, and

[4] *Utilitarianism,* chap. 4.

[5] It might be true of an army, for instance, that it is composed of regiments from Maine, Iowa, and Texas. This could hardly be true of any soldier.

composition permits their discrimination from cases in which the fallacy of four terms is obvious and unabashed. "George Washington told us to avoid entangling alliances; we certainly ought not, then, join the League of Nations." About the best one can do with this by way of putting it into precise syllogistic form is as follows:

> Entangling alliances are things George Washington told us to avoid.
> The League of Nations is an entangling alliance.
> ∴ The League of Nations is something we ought not to join.

By restating the premise and the conclusion this material can be worked into a syllogism whose middle and minor terms conform to the rules, but whose major term in the conclusion is very different from the corresponding term in the major premise. Hence in order to deduce such a conclusion there is need of another premise not provided, namely, "We ought to do nothing that George Washington told us to avoid." When such illogicalities are referred to in connection with cases of ambiguous term, division, or composition, they may be distinguished as exemplifying the fallacy of an *overt* fourth term.

Classified list of formal fallacies

By the way of summary, it may be helpful to add the following classified list of the formal fallacies which have been mentioned:

A. Fallacies of Molecular Inference
   1. Mistaking contrariety for contradiction
   2. Fallacies of disjunctive reasoning
      a. Mistaking weak for strong disjunction
      b. Mistaking nonexhaustive for exhaustive disjunction (traditionally called "imperfect disjunction")
   3. Fallacies of hypothetical inference
      a. Denying the antecedent
      b. Affirming the consequent
      c. Mistaking material for strict implication
   4. Imperfect definition or division, *i.e.*, inequivalence between definiens and definiendum, or between the genus divided and the species into which it is divided

B. Fallacies of Atomic Inference
    1. Misinterpretation of quantity or quality
    2. Fallacy of four terms
        a. Ambiguous term
        b. Division
        c. Composition
        d. Overt fourth term
    3. Fallacies of relational inference, *e.g.*, mistaking a non-transitive relation for a transitive one
    4. Fallacies of inference concerning class inclusions
        a. Improper substitutions, in obversion or conversion
        b. Syllogistic fallacies (as determined by the traditional rules)
            (1) Negative premises
            (2) Affirmative conclusion from a negative premise
            (3) Undistributed middle
            (4) Illicit major
            (5) Illicit minor
            (6) Illicit assumption of existence

Are there other fallacies than these?

On many accounts it would be desirable to terminate the discussion of fallacies at this point. But to protect the reader against unnecessary puzzlement one more consideration must be introduced. If he has explored or comes to explore the literature of logic, he will notice references to a number of other "fallacies" besides the ones above mentioned which are nonetheless discussed in the same context—errors called "begging the question," "*non sequitur,*" "*argumentum ad hominem,*" or "*argumentum ad—,*" with various other words concluding the phrase. When he struggles to understand these kinds of mistaken reasoning, and in particular to see how they are related to the fallacies just listed, he will most likely come to the conclusion that logicians themselves have fallen into some serious fallacy in their treatment of fallacies.

The confusion may, however, be cleared up quite simply. The fallacy that many logic manuals have committed in treating of this subject is that of cross division, which is one form of imperfect division. They have mixed three different ways of dividing fallacies; once these are carefully distinguished no difficulty need arise. Let us see how this is so.

The principle of division which has alone concerned us

in the foregoing chapters, and which is followed in the list of fallacies just offered, yields a purely *logical* classification of fallacies. That is, they are classified simply in accordance with the nature of the logical form which they violate; no other consideration plays any part in the matter. But for centuries logic was viewed and studied in intimate connection with rhetoric—in fact, for many ancient and medieval writers the two subjects were hardly separated at all. Accordingly, in addition to considering fallacies from the strictly logical standpoint, it was natural and often very convenient to refer to them as appearing in the context of public debate between two or more disputants. This led to certain modes of description which, when systematized, constitute a *rhetorical* division of fallacies.

Three ways of dividing fallacies: 1. Logical

From the standpoint of this rhetorical approach, any of the fallacies previously discussed would be called a *non sequitur*—that is, a case in which the conclusion does not really follow from the premises which purport to yield it. A debater will charge his opponent with a *non sequitur* when he is confident that by analysis he can show this to be the situation.

2. Rhetorical

But also, in debate, each contestant is supposed to be demonstrating a certain definite answer to the question which they are formally discussing. In such a context two important errors are possible which would not be included in a purely logical classification. Suppose, for example, instead of proving the answer to a given question that I am supposed to be defending, I prove something else which I hope the audience will uncritically accept as its equivalent. Then my opponent, if he is alert, will accuse me of the fallacy of *irrelevant conclusion.* My conclusion may follow from the premises offered, but does not really deal with the precise point at issue. To prove, for example, that the home is the bulwark of national life, and then assume this to be the same as to prove that women should not enter industry, would be an instance of this sort of error.

Another rhetorical fallacy frequently met is that of *petitio principii,* or *begging the question.* This fallacy is committed

when the author of an argument assumes somewhere in his premises the conclusion which the argument purports to establish. If the assumption is expressed in somewhat different language in the two cases, hearers may be deceived into supposing that the reasoning has really accomplished something. Occasionally this fallacy appears in a single syllogism. Here the author assumes what is substantially identical with the conclusion in one of the premises, with the result that no real inference has taken place. The following will illustrate this way of begging the question: "Veal from young calves should not be eaten by people, for it is not fit for human consumption." Here the major premise when formulated is no more than a tautology—that is, the ideas of subject and predicate differ verbally only. Hence there is no actual reasoning involved; the conclusion is essentially the same as the minor premise. The other, and more common, way of begging the question, is to construct a series of two or more syllogisms, some early premise being identical in meaning with the conclusion of the last. Such an argument is called *reasoning in a circle.* When other matter, descriptive or hortatory, is injected between the segments of the circle, and the conclusion is supported by emotional needs, this quite irrational procedure easily deceives. The following case may suffice:

Whatever the Bible says is true.
The Bible says there is a God.
∴ That there is a God is true.

Whatever God reveals is true.
The Bible is God's revelation.
∴ The Bible is true.

It will be observed that in instances of *petitio*, as in those of irrelevant conclusion, no formal fallacy is necessarily committed in any of the syllogisms when they are separately considered. But when viewed in the rhetorical context involved, it is quite evident that the inference is inadequate. *Petitio* assumes a difference between two propositions when they are

the same; irrelevant conclusion supposes identity where there is a significant difference.

Now there is a third way of considering fallacies which, when worked out in some orderly fashion, yields a *psychological* division of them. It might take either of two different forms, but the guiding principle in both consists in bringing to light the human motives responsible for the error. In one form, fallacies would be divided according to the motive in the author which explains why he fell into them. Were a new psychological classification to be developed in contemporary times it would be likely to take this form; such irrational forces as were discussed in Chapters 4 and 5 would play a prominent part. Historically, however, the psychological division which has left its mark on the inherited terminology of logicians was strongly influenced by the rhetorical context just described. Accordingly, it took the other form; it divided fallacies according to the motive in the audience which the author was counting on to assure acceptance of his conclusion despite its failure to follow logically from his premises. 3. Psychological

For example, consider again the illustrative piece of reasoning used earlier in the present chapter, about George Washington and the League of Nations. It was there observed that in order to establish the proffered conclusion a premise is needed that is not overtly introduced at all, namely, "We ought to do nothing that George Washington told us to avoid." Why is it not introduced? Because if it were openly stated, few would believe it without qualification, while if it is not openly stated the admiration which Americans feel for George Washington may entice the mind of a hearer or reader to the conclusion he is asked to adopt without his being conscious that a fallacy is being committed. Under sway of the sentiment of loyalty to the father of our country, one may thus be induced to pass readily and smoothly from the thought of George Washington's warning to the thought "avoid the League of Nations." There are, of course, various other ways of playing upon the irrational tendencies of people for the purpose of hiding logical inadequacies, ways which can be

distinguished in detail by noting the various common motives that are characteristic of human nature.

Playing upon reverence for the great men of the past, just exemplified, or upon accepted authority, is one of these; others consist in playing upon prevalent superstitions, upon sympathy for the unfortunate, upon ignorance, upon fear of evil consequences, or upon known specific biases of the individual or group immediately addressed. The scholastic logicians, in their meticulous way, gave technical names to each of these illegitimate appeals: *argumentum ad verecundiam, argumentum ad populum, argumentum ad misericordiam, argumentum ad ignorantiam, argumentum ad baculum,* and *argumentum ad hominem.* The list could, if desired, be expanded considerably by the aid of modern psychological analysis.

It is essential not to confuse them

It is thus clear that the fallacies listed in any one of these three modes of division are not comparable with those listed in the other two; the same fallacy may be differently described according as it is considered from each of these three standpoints. What from the rhetorical point of view is a *non sequitur* may also, from the strictly logical point of view, be a case of undistributed middle or of an ambiguous term, while from the psychological point of view it may be an instance of *argumentum ad hominem.* Three distinct principles of division are being used; and it is not surprising, therefore, that each of the resulting classifications cuts across each of the others.

At the same time, it is no accident that the fallacies which have attracted most attention from the rhetorical or psychological approach are more likely to be four-term fallacies from the viewpoint of pure logic than any other kind. Why is this? The answer is that people often fall into such fallacies as those of distribution, negative premises, etc., through nothing more than carelessness or inadvertence. Where this is the case, ordinarily no one is interested in a psychological or rhetorical description of the situation; it is evident that the remedy is simply systematic care and practice in dissecting

fallacies of these various types, so that one may be on better guard against deception by them. Where some persuasive psychological motive is appealed to which is capable of a distinctive description, the logical error committed is usually more open and glaring when brought to the light of critical attention. And the most unabashed cases are the four-term fallacies.

Accent and amphiboly

Two other forms of misinterpretation of meaning that may lead to invalid reasoning should be noted. One is due simply to accent. Read, for example, the prohibition "Thou shalt not bear false witness against thy neighbor," accenting the last word, and the commandment may carry an unintended implication. This has been sometimes called the fallacy of *accent*. The other results from an ambiguous grammatical structure and is termed *amphiboly*. "I know that you the enemy will slay." Many of the revelations of the ancient oracles were of this sort. Of course no proposition can be used with assurance as a premise until ambiguities of meaning have been removed.

Main lessons enforced by the formal sciences: 1. Clarity

Before the present chapter ends we must do our best to answer one more question. What shall we select as the major lessons which reasoning in general, if it is wise, may learn from the formal sciences? Part II, though concentrating upon the central theme of implication, has dealt with varied topics and has presented many validating forms; what ideas stand out as embodying the main deposit that reason can appropriately use for its guidance wherever it functions? I find four such lessons.

First, the lesson that reasoning cannot even get under way in dealing with any subject until *clarity* of meaning has been secured in the terms which refer to that subject. This is essential because otherwise one does not know what it is that he is trying to think about. For this reason, as has been noted more than once in the preceding chapters, it is impossible to tell what the implications of any proposition are if the meaning of that proposition is not clear. Hence, argument about it

under such conditions is entirely futile. How, for example, can the proposition often asserted by religious people, that our knowledge needs to be supplemented by faith, be profitably considered until it has been made clear just what is meant by the terms "knowledge" and "faith"? One understands, then, why the man whose habits of thinking have been influenced by the formal sciences often makes himself a bore to others at the commencement of any serious discussion by insisting on a careful definition of the vital words to be employed. His motto is: If we talk, let us know what we are talking about.

This demand for clarity of meaning is a twofold demand. On the one hand it insists on the elimination of vagueness, and its replacement by definiteness of meaning. The words employed in daily life cover an area that is bounded by an uncertain penumbra instead of a sharp outline; it would be difficult, for example, to tell precisely the characteristics in virtue of which an article of furniture ought to be called a table rather than a desk. But as long as such vagueness obtains one does not know how to reason accurately about the subject in question; there is a marginal area where he cannot tell what implications may be properly drawn. Hence the primary insistence, in scientific thinking, on accuracy in the definitions explicitly or implicitly employed, and a reduction to the minimum of this margin of vagueness. Such a demand is not appropriate everywhere and always, of course. There are situations in life in which precision is irrelevant or even quite absurd; when a lover tells his beloved "I adore you more than life itself," this would hardly be the place to demand careful definition of the vague terms employed. The poet has his place as well as the careful analytic thinker, and no formal scientist would wish to deny him license to use loose and ambiguous metaphors; on occasion, language may properly aim at evoking images and emotions instead of giving an accurate denotation of facts or logical relations. But when the quest for truth is concerned, vagueness is never a virtue. Then, nothing is ever gained by inaccuracy.

On the other hand, precision of meaning requires the elimination of ambiguity. Even if no penumbra of vagueness distresses us when using a word, it may be ambiguous—that is, it may have more than one meaning. In this case a thinker is responsible to choose and specify which of the alternative meanings he intends, so that the right set of implications will be indicated for subsequent inferences employing the word. This is relatively easy when the different meanings involve obviously different universes of discourse; the word "spring," discussed in Chapter 2, is a good illustration of this circumstance. When there are differences of meaning within realms of discourse that are closely related it is harder. Here we meet one main reason [6] why the words in terms of which people discuss social problems are almost incurably ambiguous. "Democracy," for example, means one thing as a political structure, another as a general social ideal; but because these realms of discourse are so intimately connected it is exceedingly difficult to keep the two meanings, with the appropriate implications of each, properly distinguished. Yet for accurate thinking about them to proceed, the distinction must be kept clear.

Thinkers have sometimes supposed that the meaning of a word must be vague or ambiguous when the thing it denotes is blurry or fluctuating by nature. Such a notion is a mistake. And it becomes a serious mistake if it encourages lack of responsibility to render one's thinking as clear as possible when dealing with these as well as with other matters. Unclarity of meaning must not be confused with elusiveness in the object meant. This becomes evident as soon as one reflects that the meaning of the word "distinct" is no clearer than that of "blur"; of "order" than that of "chaos"; of "clarity" itself than that of "vagueness" or "ambiguity." We can talk with precision about each of these things. Were it not so, even a clarifying discussion of the theme now occupying us would be impossible.

2. Consistency

A second major lesson is that of *consistency* in the employ-

[6] Other reasons will be noted later. See below, pp. 512 ff., 596 ff.

ment of any meaning that has been adopted. Here, too, popular ways of thinking frequently offend; under the stress of impulse and emotion people are very apt to say at one moment something that is inconsistent with what they have previously said in the same context of discussion. In fact, even in systematic reflection the common desire to have our cake and eat it too may lead us into inconsistencies of which we are unaware until our attention is specifically called to them. In theories about morals, for example, more than one thinker has set forth the doctrine that pleasure is a universal criterion of good, while admitting somewhere in the course of his discussion that under some circumstances certain pleasures are morally bad. No one can think to any purpose, or make any progress toward a rational conclusion, except when his thinking is self-consistent; in the quest for truth nothing is ever gained by contradicting at one moment what has been affirmed at another. A careful thinker may change his mind, of course, when he sees good reason for doing so; but then he consciously sets aside all previous affirmations which depend on the rejected belief, and accepts the responsibility to work out anew a coherent elaboration of his present conviction.

The fundamental character of the virtue of consistency has, however, been sufficiently emphasized in the detailed analysis of formal science in which the preceding chapters have engaged; there is no need to enlarge upon it further.

3. Rigor and 4. system

A third lesson is that reasoning must be *rigorous*. Whenever one draws conclusions from premises, he is responsible for making sure that they strictly follow from those premises. This is especially important in lengthy chains of inference. It may be intriguing, at times, to fool other people into accepting as valid a conclusion that is not so, but nothing is ever gained by deceiving oneself in this regard. When we reason, we want to reach a satisfactory solution to our problem, but the only way in which we can be confident that the verified consequents reached at the end prove the antecedent

hypotheses to be true, lies in the circumstance that those consequents strictly follow from the antecedents. Otherwise, they lead us astray, except by accident. Hence it is important to see to it that there is no logical gap at any point between the premises with which one begins and the conclusions finally drawn.

Finally, a fourth lesson is that knowledge must be as *systematic* as possible. It is, of course, better to know something than not to know it, even if it cannot be bound together with other items of knowledge in a comprehensive system. But there is tremendous economy in achieving such a system wherever it proves possible. A few general principles, clearly mastered, can hold together in unity a vast mass of detail when its varied items can all be shown to follow logically from those general principles as postulates. Then, in deduction, one can pass at will from any part of the system to any other, seeing exactly how each proposition affects the truth of others, instead of being confined to the implicates of that proposition as an isolated affair.

Summary of Part II

A brief summary of Part II is now in order. After explaining the nature of formal science and its relation to the principles of correctness at the fourth step of a piece of reasoning, Chapter 7 attempted a preliminary clarification of certain basic concepts such as "inference," "implication," "truth and falsity," "validating form," "entailment." The following chapter was occupied with the study of molecular validating forms, and of the logical constants which such forms employ. With Chapter 9, the atomic study of validating forms began Its successor explained the difference between forms for relational inferences and forms for inferences concerning class inclusions. Chapter 11 gave a brief introduction to the foundations of mathematics, as illustrated chiefly in the branch of arithmetic. In Chapters 12 and 13 the forms for inference concerning class inclusions were studied in some detail, special attention being given to the theory of the syllogism. A brief discussion of the nature of formal systems and an analysis of

formal fallacies has, in the present chapter, brought the section to a close.

EXERCISES

Put the following arguments in correct syllogistic form, and locate fallacies where they are present. Give the technical name for each of the fallacies.

1. Man is a rational being, hence a lover's acts are rational.
2. Let us eat, drink, and be merry, for tomorrow we die.
3. Veal from very young calves should not be sold, for it is not fit for human consumption.
4. All generous men provide amply for their families. Mr. L. provides amply for his family, therefore he is a generous man.
5. The bill before the House is well calculated to elevate the character of education in this country, for the general type of instruction in all our schools will be raised by it.
6. All great business men began with small salaries. I began with a small salary, so I shall become a great business man.
7. Whatever is right ought to be done. It is my right to build on this property. Therefore I ought to build on it.
8. This doctrine is not true, for Scripture condemns those who hold it.
9. I have made this offer attractive to your pocketbook. You must accept it.
10. You ate raw meat today, for what you bought yesterday you ate today.
11. Abraham Lincoln, whom you greatly admire, was the chief founder of the Republican Party; why then do you consider voting for a Democrat?
12. The schools deserve to be well administered, for they ought to have the best government we can give them.
13. Opium puts people to sleep, for it has a sleep-producing effect.
14. He that is of God heareth God's words; ye therefore hear them not, because ye are not of God.—John VIII, 47.
15. This policy must be adopted; otherwise the masses are in danger of being misled by radicals.
16. There is no need in the world for people to be without money. The people are the government, and the government can make money.
17. Great men have been born in the slums. I was born in the slums. Therefore I ought to be elected Senator.

18. All candid men acknowledge merit in a rival, hence not every learned man is candid.
19. The ancient historians were very credulous in accepting testimony, and so they ought never to be believed.
20. Nonsectarian schools are irreligious, inasmuch as their policy is not to permit the teaching of religious creeds.
21. Suicide is not to be condemned, for voluntary death was embraced by many great heroes of antiquity.
22. Riches are for honor and good spending; therefore unusual expense must be limited by the worth of the occasion.
23. Since the various species of brutes were created to prey upon one another, it follows that man was intended to prey upon other animals.
24. Soldiers are the best peacemakers, for the object of war is durable peace.

The three following will require two syllogisms each:

25. Military training should be universally required, for we are sure to have a war some time and everyone should be prepared to do his duty. Of course pacifists decry such arguments, but pacifists are notoriously weak-minded people. We may be sure that anybody who opposes such a conclusion is quite illogical.
26. The present Supreme Court is the most reactionary we have ever had, hence you will not find Mr. Justice X, a member of the court, a very progressive man. But you think I ought not to publish such opinions. Have I not the right to publish my opinions about the court? And ought I not to do what it is right for me to do?
27. "Every injustice is the taking away of a good. There is no other good than virtue; but virtue cannot be taken away: thus it is not possible that the virtuous can suffer injustice from the wicked. It now remains either that no injustice can be suffered, or that it is suffered by the wicked from the wicked. But the wicked man possesses no good at all, for only virtue is a good; therefore none can be taken from him. Thus he also can suffer no injustice. Thus injustice is an impossible thing." (From Maximus of Tyre, condensed by Arthur Schopenhauer.)

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PART III

REASONING AS DESCRIBING AND EXPLAINING FACTS

CHAPTER 15

THE ENTERPRISE OF FACTUAL SCIENCE

From formal sciences we turn to the sciences dealing with facts. And facts constitute a kind of reality different from the kind exemplified by a formal relation. The problems of the sciences which deal with them are not completely solved merely by disclosing the formal structure which tests the validity of an inference from premises to the conclusions drawn. Factual science requires further processes—especially, observations designed to tell whether the conclusions implied by premises that have been tentatively adopted agree with what is actually the case or not, and which can test, when properly used, the truth of the premises. To secure, not merely validity, but also truth, is the aim of factual science.

Distinction between formal and factual science

The distinction between these two kinds of science is one thrust upon all thinkers by many experiences of daily life. And we found it necessary to face it before completing our analysis of a piece of reasoning into its temporal steps. If the problem is raised, whether seven is one-sixth of forty-two or not, even the amateur is well aware that the question is to be answered by a different procedure than if it asked whether or not there are seven books by T. H. Huxley on the upper right-hand shelf of my library. The former is a problem in formal science, and is to be solved by purely deductive inference from the definitions and other postulates which determine the system of numbers; no appeal to any fact lying beyond this system is necessary. The latter is a question of what is true or false about certain facts, to which deductive inference alone is unable to provide an answer. The relevant facts must be observed; that is, somebody must go to the

specified shelf and count the Huxley books there before an appropriate solution has been reached.

Now factual science is a careful and meticulous development of the principles exhibited in such a simple everyday procedure as this, in the way in which they supplement the principles with which the analysis of the preceding section has been concerned. And the process by which many of its important results were initially established is almost as homely as the illustration just given. One of Galileo's early scientific discoveries was made when he was standing in the cathedral of Pisa as a lad of eighteen. While he watched the great lamp swinging from the roof, the hypothesis was suggested that the length of time occupied by each oscillation was the same, irrespective of the width of the swing. We have learned that some questions about equality would be capable of solution merely by formal inference from the mathematical definition of the word, such as the question whether equality is a transitive and symmetrical relation or not. But this question is of another sort. Galileo wanted to know whether, with respect to time, equality obtained between certain facts, and the only way to tell was to observe those facts, measuring and comparing them. This he did by counting the beat of his pulse—the only timepiece he happened to have at hand—while contemplating oscillations of considerably varying breadth. The result thus reached was rough and approximate only, but so far as could be told it verified his hypothesis; an important law of the pendulum was provisionally established.

Verification the distinctive feature of factual science

These simple illustrations make it evident at once that the distinctive aspect of factual science is that which, on the one hand, is common to all its investigations, however varied they may be in detail, and, on the other hand, is absent from the formal patterns providing the subject matter of the preceding chapters. This aspect presents itself most clearly and challengingly in the process denoted by the word "verification"—the process by which a proposition is determined to be true or false. To be sure, it makes a difference in each of the other steps of a piece of reasoning which is concerned

not merely with formal relations but with questions of fact. The problem will be clarified differently, the hypotheses suggested will be of another sort, and the distinction between implication and entailment will be fundamental. But the solution of a formal problem demands no observational verification, while the solution of any problem about facts does demand this latter step. Just what is this process? It might seem to be a very simple affair, but it is not. How does one go about it, so to perceive facts as to establish, whenever possible, one hypothesis about them as true while showing that others are false? By what ideas is it guided, and how are they justified? Do special difficulties appear according as the facts with which an inquirer is dealing exhibit certain special features? If so, what are these difficulties and how are they met? Such are the fundamental questions on which the discussion of the ensuing chapters will be focused.

Clarification of the present study in relation to Parts I and II

First, however, the present enterprise will be more clearly understood if certain threads are picked up from our preceding studies. The various introductory inquiries of Part I culminated at the point where the essential difference between superstition and science came vividly to the fore. To clarify this difference the motives which lead to the former were examined, and with them, the social interests which perpetuate it and the disappointing consequences which it brings. Attention was also given, though more briefly, to the gradual emergence and historical expansion of the latter, and its distinctive nature as the systematic quest for dependable truths about the world and for constant improvement in the methods by which such truths are discovered. As the use of the phrase "truths about the world" indicates, the kind of science which one naturally has in mind in this contrast is factual science. Not that superstition (or, at least, blundering irrationality) is absent from the field dealt with in formal analysis; it appears there in the tempting fallacies that have been noted above. But in factual investigations it is still more dangerous; it displays itself in popularly influential and seriously confusing habits of mind. From this stand-

point the present enterprise will be a continuation of that early study, but the emphasis will now turn from the psychological and historical considerations suggested by the difference between superstition and factual science to the detailed examination of the latter's procedure, bringing to light the logical basis on which its enormous and growing success rests, and the lessons which it can teach our reflective habits in general.

The main connecting thread with the theme of Part II should also be briefly stated. Except when one assumes the role of formal scientist, aiming to master or expand knowledge in his chosen field, formal relationships are always instrumental to factual understanding; validity of inference is entirely subservient to the avoidance of falsity and the discovery of truth.[1] It is always important that inferences be valid, for only thus can the conclusions reached partake of whatever confidence a thinker has in his premises. But are the premises true? Do they agree with the facts to which they refer? Formal science provides no answer to these questions, but unless one knows how to answer them with some assurance, every bit of reasoning that is concerned with matters of fact is left hanging in the air; one cannot tell whether it is rationally grounded or not. In particular, it is clear that we need to know how to establish universal propositions about factual matters. The preceding Part has shown that in any inference about classes one of the premises must be universal; even when no such premise is specifically formulated, the inference can only be justified if it is implicitly assumed.[2] How are universal propositions dealing with facts verified? A quite adequate foundation must be discovered on which answers to these questions can be worked out.

The relation between pure and applied factual science

To clarify the nature of this foundation with respect to another important point, let us remind ourselves of a distinction drawn in the opening chapter and occasionally mentioned

[1] Or to the solution of problems of evaluation which are to be considered in Part IV.

[2] With such specific exceptions as were illustrated on p. 184.

since. This is the distinction between practical and theoretical reasoning. In the former, it will be remembered, one's problem is to determine what to do in some situation where alternative lines of action present themselves, each as possibly desirable. In such situations one makes use of the best knowledge that is available about the facts involved to guide his selection between these alternatives, but his immediate purpose is neither to correct this knowledge nor to add to it. It is to accomplish some wished-for result by its aid. In the latter, the problem is rather to understand some puzzling object or occurrence. Action of an appropriate sort is always involved in the attempt to gain such understanding, but the primary purpose, on which all else depends, is to gain an item of true knowledge which had previously been lacking. Now this difference is reflected in the presence of two kinds of factual science. The aim of one kind is to apply, in the most successful way possible, a certain branch of knowledge to practical results; for this reason the sciences it comprises are called *applied* sciences. Engineering is a typical example. The aim of the other kind is to extend knowledge about a certain realm of fact, and the sciences that compose it are usually called *pure* sciences. The applied sciences thus exhibit the principles and methods appropriate for practical reasoning, in the form which they have assumed as a result of long centuries of self-conscious effort to improve them. Similarly, in the pure sciences, one can find the best available principles for the guidance of reasoning in theoretical problems, since they follow the procedures which long experience has shown to be most successful in gaining true understanding of elusive facts.

But before any given item of knowledge can be applied to practical ends it must be discovered. The steam engine could obviously not be invented, for example, until knowledge about the expansion of steam had been gained. In this important and definite sense, pure science underlies and precedes applied science. Thus, from the point of view of the course of inquiry on which we have at present embarked, the pure factual sciences occupy a peculiarly fundamental position. Without clear

understanding of the lessons which they can teach we may perform very inadequately the processes connected with verification when we are reasoning, either about everyday problems or about more technical ones. They show us how most wisely to proceed whenever we seek to enlarge our understanding of the world about us, and it is their business to provide the specific items of knowledge which we may need to apply whenever we are faced with practical problems. They furnish the support to which, directly or indirectly, appeal must always be made when one wishes to reason soundly about matters of fact. They do not, of course, constitute an external authority to be accepted on blind faith. Their inquiries are open to critical inspection by all who sincerely seek truth, and the justification of their methods is not in the least esoteric.

Pure natural science will be examined first

Pure factual science, as a going intellectual concern, consists therefore in those systematic investigations whose purpose is to establish by well-warranted methods the true propositions which any subsequent thinking about the matters investigated will wish to use.

How does it set about this responsible task? By what essential ideas is it guided? What major lessons, slowly learned in the long history of man's attempt to discover truth about facts, are now funded in its procedures and instruments? In plunging into these questions, let us begin with those branches of pure science which have proved historically most successful, and which seem, therefore, capable of exhibiting the principles involved in all competent factual inquiry with greatest clarity and distinctness. This means beginning with the methodological structure of the so-called *natural* sciences, *e.g.*, physics, chemistry, biology, leaving aside for the time being the *social* sciences—those concerned with human behavior in the various associations which men and women form. There are complications in the latter for which we are not yet prepared.

Two basic considerations

Well, let us open this theme with two basic considerations, and see whither they lead.

Our aim is to approach facts in such a way that we can

determine which propositions that might be suggested about them are true, which false. Now, the first consideration is that the outcome of any such confrontation of fact can always be expressed in the form of a proposition. This means that it can be conveyed in an assertion that a certain subject term is connected in a certain way with a predicate term. And the second consideration comes to light when we notice that among the myriad of connections that may be observed and recorded as holding between one object or event and others, a special sort of connection is given peculiar emphasis by factual science. That an indefinite number of connections may be asserted about any bit of subject matter is evident on the briefest consideration. The object or event may be qualified by some attribute; its temporal relation to preceding and subsequent events may be affirmed; that it is composed of certain elements may be noted. It may be compared with some similar object—either one remote in distance or time or one immediately at hand, so that the comparison can be rendered more detailed and exact. It will not take us long to observe, however, that peculiar importance is attached to the connection which in Part I, when discussing the association of ideas, we called "contiguity in space or time." Here is little Johnny, who has just come down with a cold. The doctor who examines him will not be much concerned about the fact that it is more annoying than the one he had two months ago, or about the fact that his temperature is about the same as it was then. He will give more attention to the circumstance that Johnny came down with it the day after his birthday, and that he had just returned by a crowded bus from a visit to a cousin some miles away; he will give still more to the circumstance that a severe inflammation of the throat is found to accompany Johnny's cough, fever, and other distressing symptoms. Now the latter group of connections illustrates the fact that events which happen in close temporal and spatial contiguity to the event which one is trying to understand are of great scientific importance. Let us replace the rather vague word "contiguity" by the more precise one introduced in Part II for denoting the

togetherness of two or more entities undergoing logical examination, and call these connections instances of *spatio-temporal conjunction.*

The difference between description and explanation

What is the reason for special emphasis by factual science on this sort of connection as over against others? We shall need to explore this question rather fully. And in stating the answer it will be helpful to draw a distinction and use certain concepts that have not yet been introduced. The reason is that concentration on the spatio-temporal conjunctions of events marks the circumstance that one is trying to pass from mere *description* to satisfactory *explanation* of them. Let us consider briefly this distinction. What is meant by "description"? Well, any sort of connection whatever between an object and other things may be used, on occasion, to help describe that object, for in some context or other it may contribute toward the unambiguous identification or clear analysis that description is usually intended to serve. Hence a description may consist in listing the properties of the object involved, measuring those features that are capable of measurement, comparing it with similar objects in other ways, recording its spatial and temporal locus, or stating any further connections that may be observed. Accurate description is important especially in the case of objects whose behavior is not yet well understood, because it is the necessary preliminary to the attainment of more significant knowledge about them.[3] But whenever the step proves possible we wish to pass from description to explanation, and only when this has been done do we feel that the kind of understanding that is most valuable has been secured.

Clarification of this difference

What then is meant by "explanation," and just how does it differ from description? For many centuries, and in the popular mind still, the answer would naturally be expressed in terms of the ideas of *cause* and *effect;* to explain anything is to trace its occurrence to some cause, of which it is the inevitable effect. And before leaving them with a somewhat limited role to play, we shall work for a while with these ideas

[3] Chapter 21, below, develops this point in fuller detail.

too. But since in the course of history different conceptions of causality have dominated human thought, this answer by itself is insufficient. What sort of affair is the causal relation, as it is exhibited in the explanations provided by mature science? [4]

Three kinds of spatio-temporal conjunction

The key to the answer here is found by concentrating further on these connections of spatio-temporal conjunction. Let us survey a random list of such conjunctions as experience presents us with them. We shall find that they can readily be divided into three groups: there are accidental, probable, and universal conjunctions.

By an "accidental" conjunction is meant a case of spatio-temporal contiguity between two events, each of which seems to occur just as readily in the absence of the other as in its presence. When this is the situation it is, of course, impossible to predict with any assurance that when one of them occurs in the future the other either will or will not occur. If we state our experience of them in terms of the propositional forms which are now familiar, we can obviously only say that "some cases of the one have been conjoined with cases of the other." For example, as my gaze falls on the slope beside my window, a squirrel is running on the branch of an oak tree, while at the same time a leaf lying on the ground a few yards away stirs in the breeze. Now this conjunction between the squirrel and the motion of the leaf is purely accidental, so far as my present knowledge about them indicates. Hence I could not sanely assume from it that in any future instance of a squirrel running on a limb there will probably be a leaf stirring on the ground not far away. To guide anticipations of the future, a conjunction affirmed by a proposition must be more than accidental, so that we can pass with a greater or less degree of assurance from the occurrence of one event to an expectation of the occurrence of another. Were all conjunctions in

[4] Some philosophers would draw the line between description and explanation, not at this point, but at the point where comprehensive theories binding together various conjunctions are introduced. See chap. 19, below. Provided that the importance of such theories is recognized, and the desirability of making them as comprehensive as possible, this is simply an issue as to how "explanation" is best defined.

nature merely accidental, it would obviously be impossible to forecast the occurrence of anything on the strength of the occurrence of something else; no traces of predictive order could be discovered. Such a world would be what is meant by the word "chaos." So far as our present knowledge goes, many events in nature are connected in this quite accidental and unpredictable fashion, but not all of them are.

By a "probable" conjunction is meant a case of contiguity between a pair of events which seem connected more often than would be plausible if their association were merely accidental, but to which there are exceptions. Such is the connection, for example, between a high pressure area and clear weather, between unusual height and unusually heavy weight in people, between blond hair and blue eyes. Expressing this situation in propositional form, we would say that "most cases of the one have been found conjoined with a case of the other." When the connection is of this kind it is still impossible to predict with confidence that when one of these events occurs the other either will or will not occur. But some rational anticipation can be exercised now, as it could not before. If event B has on the average in our experience attended event A three times out of four, then when A happens there is a certain probability, supplying a degree of intelligent guidance, that B will follow, even though the prediction of B will obviously not be unqualified. In proportion to the frequency with which A and B have been observed together, a likelihood appears that B will happen when A happens, and this probability rises toward practical certainty as the percentage of exceptions to their conjunction falls toward zero.

By a "universal" conjunction is meant a case of spatio-temporal contiguity between two events which, so far as experience to date testifies, are connected in a quite regular and dependable way. Every instance of one has been followed by an instance of the other, hence we expect that its occurrence in the future will always be conjoined with an occurrence of the other in the same way. Such are the conjunction between the eating of food and the cessation of hunger, between the

tap on a billiard ball and its motion across the table, between the arrival of spring and the appearance of new vegetation, and innumerable other familiar contiguities. Our experience of any case of this kind would be summarized in a universal proposition: "All cases of the one have been found conjoined with a case of the other." We thus formulate, in other words, a relation of spatio-temporal conjunction between each member of an entire class of events and some member of another class. And with regard to the predictive use of this experience we would express the relation in the "if . . . then" form by which a material implication is naturally stated: "If a case of the one takes place, then a case of the other will take place." [5]

Explanation is in terms of universal conjunctions

Now, although further analysis will introduce needed qualifications, it can be said in a preliminary way that when it proves possible to verify a conjunction of this kind between one object or event and another, we feel that, so far as the latter is concerned, we have passed beyond mere description and have reached a genuine explanation of its occurrence. We know why it takes place when we can appeal confidently to a spatio-temporal concomitant which regularly precedes its happening. In popular parlance, it is the effect of that concomitant as its cause. This is the conception of explanation on which modern science, in all its various branches, rests. Anything is explained when it is viewed as one member of a class of similar events, and when another class of events has been found to be regularly conjoined with it in this way. We know then under what specific conditions the class has been exemplified in the past, and can anticipate under what specific conditions it will be exemplified in the future.

If it is asked why this conception of explanation has come to prevail, a fairly plausible answer appears from careful analysis of the motives which typically underlie both our practical and our theoretical reasoning.

[5] *I.e.*, what has apparently been confirmed as a general proposition is employed in the role of a universal proposition. Its right to perform that role is part of its general meaning (cf. above, pp. 226 ff.) in virtue of the postulate of predictive uniformity soon to be explained.

Practical and theoretical reasons for this circumstance

Whenever a practical problem is faced, the aim of the reasoning being to find out what to do in order to attain some desired end, one is controlled by a general need which may be described as follows: Things are transpiring in their own natural course and order, but apart from our interference the end desired will probably not be attained; how can our powers be inserted in such a way that it will be attained? Knowledge of the course of events must be predictive knowledge if it is to help us here. That is, it must take such a form as will enable us, given a certain event as happening now, to tell what is likely to happen in its vicinity next. By such knowledge alone may we forecast from moment to moment what is going to occur without our interference, and also what is going to happen when by our interference an additional event is introduced which otherwise would not have occurred. Since most human reasoning is practical, we find ourselves under a perennial and insistent need to master the dependable spatio-temporal conjunctions between events. Letting A refer to any event which is of concern to us, our demand is to understand it in such a way that we can affirm, when it happens or is brought about, what event B will follow it in regular sequence. When B happens, since we may wish it to occur again or to prevent its occurrence, our need is to know what preceding event A in the same region is an unfailing condition of its happening. In sum, to accomplish any practical result—that is, to control nature in the various ways that seem possible—our knowledge about whatever takes place must be predictive knowledge, expressed in laws affirming a universal spatio-temporal conjunction between any event of one class and an event of another class.

Nor, of course, is theoretical reasoning satisfied with merely accidental or even with probable conjunctions. It must provide the verified propositions which practical reasoning will be able to use when seeking confidently to anticipate the future, and hence it must establish universal propositions whenever it can. Moreover, curiosity, anyway, seeks an unfailing order in nature, a constant pattern underlying the di-

versity of events as they unfold. It is true that to satisfy this quest it is not intrinsically necessary that spatio-temporal regularities be emphasized as fundamental. For causal order is not the only regular order in nature; there are esthetic order, order of economic value, order of size, order of weight, and the like. To view objects in terms of any of these comparative series is to satisfy to some extent the need now considered. But the cause-effect order satisfies it too, especially when, in the development of causal knowledge, it becomes possible to discover inclusive laws binding together in a systematic unity large numbers of diverse facts. In the pursuit of such knowledge, thinkers appease at a single stroke the insistent practical need for prediction and control of future events and this intellectual need for systematic understanding in terms of an orderly pattern. Small wonder then that when one talks about explaining an event, or a group of events, he means primarily revealing its regular connections with the other events that can be predicted to happen on account of its occurrence or must be assumed to have happened if its appearance is to be understood. Explanatory truth is attested by dependable prediction.

Even description is affected by the interest in securing explanations

Returning from this examination of what explanation means in mature science, let us watch the scientist when he is supposedly confining himself to mere description. It is evident that his interest in attaining explanations wherever he can influences at every moment the way in which he describes objects too. Any object or event stands in an indefinite number of connections with other things, and is a member of an indefinite number of classes. But the scientist emphasizes those classes which give promise of showing regular spatio-temporal conjunctions with other classes, and looks everywhere and always for hopeful clues in this regard. Experience and training teach him the art of picking out these clues in dealing with various sorts of fact, so that even when at the moment he is concerned with mere description, his description will lay stress on those attributes and relations that he believes will be fertile in leading toward the discovery of an explana-

tory order in the future. It would have been possible for Galileo to describe a swinging pendulum for the rest of his life without discovering the law of its isochronous motion, for a vast number of connections might have been noted between the pendulum and other things which have nothing whatever to do with this law. His genius lay in the fact that he knew how to guide his attention to those features of its motion which suggested the possibility of a universal conjunction. Hope for subsequent explanations thus makes a profound difference in present descriptions. When the scientist counts, weighs, measures, and analyzes, his activities are consciously or unconsciously preparatory to the possible establishment of a system of dependable spatio-temporal conjunctions in which the thing he now patiently describes will be securely accounted for.

One of the most interesting aspects of science which reflect this circumstance will be studied in some detail in Chapters 20 to 22. We shall see there that when the scientist is dealing with material in which causal explanations seem at present beyond our attainment, he develops a method which enables him to measure as accurately as possible the degree to which any spatio-temporal conjunction falls short of the perfect regularity of the causal relation. This constitutes a more general method than that of causal explanation, applicable to all sorts of factual conjunctions whether accidental, probable, or universal, and guiding such rational predictions as are possible in any given circumstances. He knows, thus, how far he now fails of realizing his scientific ideal, and is organizing his facts in such a way as to permit their most effective use pending the discovery of some way in which the ideal can be more closely approximated.

The basic faith of factual science—that everything can be explained

From these introductory comments it will appear that anyone who participates in the enterprise of factual science must, by that very fact, entertain an important and radical hope. Indeed, it is more than a hope—it is a profound faith about the world of facts which, in his various inquiries, he confronts. This faith, expressed in terms of the preceding discussion, is

that every object which exists and every event that happens is capable of explanation. Otherwise, obviously, its investigation would be futile, or at least discouraging. On what does such a conviction rest, and how can it be justified against the sceptical doubts which might be raised as to its truth?

The answer seems to be that it rests on an experienced fact, whose claim to that title no one seriously doubts, and on two general assumptions which, following established custom, we shall call "postulates." As has been seen, a postulate of formal science is any undemonstrated proposition that is used in the demonstration of other theorems. Similarly, a postulate of factual science is any conviction about factual matters which is not, itself, subject to verification but is presupposed by every attempt at explanatory verification. What is the fact referred to, and what are the two postulates?

Its foundations: 1. the world is found not to be a chaos

The fact is that our world is found not to be a mere chaos. That is, not all the spatio-temporal conjunctions which it exhibits are accidental; at least some pairs of events are so connected that given the occurrence of one it is possible to draw some guiding inference about the occurrence of others. There is, in short, some predictive order in the realm of fact. To dwell further on this point, except by way of brief illustration, would probably be superfluous. While we were yet in our cradles, before any conscious attempt was made to analyze nature and pick out ordered sequences of facts, such sequences began to force themselves upon our attention. When we sucked the bottle the milk came; when we cried mother appeared; when the rattle fell it made a bang. Our world gradually proved replete with many frequently repeated conjunctions of events, so that, given one, our expectation of another was shown well founded by the outcome. With growing experience this ordered system of conjunctions expands. Distinctions are made between those which seem to be universally dependable and those that have varying degrees of probability. And since awareness of these connections makes possible rational anticipation of future occurrences, on which all successful living depends, we not only pay special atten-

tion to those with which we are already familiar but look, in any new situation which confronts us, for others.

2. The postulate of predictive uniformity

But this fact alone would not help the scientist very much. When he explains an event, as has been seen, he means not merely that a certain order has been exhibited in its conjunctions with other events in his past experience, but also that one may rightfully expect a similar order in any of its occurrences that he may later meet. And to believe this is to take one's stand on what is clearly a postulate rather than a fact. The faith which it embodies has been traditionally referred to as belief in the "uniformity of nature"; its significance is more accurately disclosed, however, if it is called the postulate of *predictive uniformity.* This phrase aptly expresses what it essentially is—the assumption that our world is such that a given group of events will show in subsequent experience the same kind and degree of interconnection that they have shown already. If event B has always followed event A in my past experience, I naturally assume that in any later experience of A, B will also be found to follow it. If in the cases previously noted B has accompanied A five times out of six on the average, I assume that approximately this percentage will continue to hold under the same conditions. If the connection has been entirely irregular, B proving just as likely absent as present when A occurs, I expect that this quite accidental concomitance will continue, and that the occurrence of A in any new situation will justify no inference about the occurrence or nonoccurrence of B.

The most obvious and frequent use of the postulate of predictive uniformity is in problems where the conjunctions between A and B, which one anticipates on the basis of what has been previously observed, have not yet taken place, *i.e.*, lie in future time. But the scope of the postulate is not limited to such cases. We often form anticipations about present events which are future *to us*—take place, that is, at some point in space sufficiently distant so that our knowledge about them can only come in later experience. At a meeting of party leaders on election night, for example, confident predictions

are made as to what is happening in neighboring states on the basis of the local returns just coming in; the postulate of predictive uniformity is applied here to events that are simultaneous with those on which the prediction rests. Similarly, the historian often makes predictions about what he will find to have taken place in the remote past, being guided by what he has now found about events of the same sort in the more recent past. Prediction concerns what is future to the predicter, and this includes more than what is objectively future.

Since any such subsequently observed appearances of A and B are not yet fact to the one who predicts them, it is obvious that the world's order, when projected over a wider area in this fashion, can hardly be called a fact. What justifies us then in assuming any such postulate?

Experience confirms this postulate

Well, it is supported by the testimony of experience as to how a vital human need is best satisfied, and the use of reasoning as the way of solving our problems implies it. Consider our inescapable living need to make some anticipations about the future, coupled with the fact that experience supports us in making this kind of anticipation and discourages us from making any other. To be alive means that the future is not indifferent; so far as possible, as has been noted above, one needs to meet its vicissitudes in ways that will secure desirable ends and avoid undesirable ones. Now when the record of experience is honestly surveyed, the only kind of anticipation it justifies is the very accommodating one that assertions based on the conjunctions of events already observed will hold true of events belonging to the same class that have not yet been observed. Of course, in any given case the anticipation may prove mistaken. Even a connection between events that is apparently quite invariable, so far as our previous acquaintance with it goes, may show exceptions. Nonetheless, no other predictive postulate than this would have any justification whatever. Our lack of absolute certainty that what we find in the future will be thus like what we have found in the past, gives no warrant for adopting any different assumption. Suppose that B disappoints us on some occurrence of A, although

we had previously found them regularly conjoined—can this justify the subsequent expectation that instead of B, A will be followed by some other event C which has not heretofore been found with it at all? Anyone who had followed this shifty procedure would have run against many more disappointments than if he had been guided by the postulate of predictive uniformity, as a little reflection will quickly show. If we are to anticipate any subsequent experience, the only supposition about its nature for which we have any warrant is the supposition that it will be continuous with previous experience in exhibiting the same general pattern of interconnections that has been already exemplified.[6] To dream some other future is clearly irrational. When, then, we are disappointed in any anticipation of one event on the basis of another, we shall, if we are wise, think of the conjunction between that pair of events as less than universal, but we shall continue to be guided by the fact that they have been conjoined as frequently as the testimony of experience indicates that they have.

But this postulate is grounded in another basic consideration. Reasoning, at each of its steps, takes for granted that the assumption expressed in the postulate is a sound one; hence, so far as we find it desirable to employ reasoning rather than any of its alternatives, we find it desirable to adopt this postulate. No one could clarify a problem at step two unless he assumed that the features discriminated in it would show the same properties and modes of behavior that they have shown in his past experience; otherwise their discrimination would be of no present value. Suggested solutions at the third step consist of ways that have successfully solved earlier problems similar in this or that respect to the present problem. But to consider them would not help us if we did not assume that what has been successful in the past will be successful in dealing with similar situations in the present and the future. And

[6] The significant relation between this principle and the laws of similarity, frequency, and means-end contiguity in the association of ideas is obvious. In this form those laws are found to support the scientific quest for truth.

the employment of material implications at the fourth step still more explicitly assumes this postulate. To anticipate that if one thing is the case something else will be the case, can rest only on uniform experience that this conjunction has regularly obtained in the past, together with the faith that what has thus regularly obtained will dependably continue in the same way in the future. Should one, then, abandon this postulate as unjustified, he would have to abandon reasoning as a way of solving his problems. The two stand or fall together, and the success of reasoning in comparison with its alternatives is ample testimony to the rational justifiability of the postulate.

3. The postulate of causality

But we believe more than is implied merely by such uniformity as has been described, and the added factor in our convictions will be clarified by a fuller examination of what is assumed at this fourth step of any piece of reasoning. It can be stated most simply in terms of the concept of causality. As scientists we nourish the dream that all events with which we need to deal in our commerce with nature can be fully explained, and this means not only that we assume it proper to project the order of previous into subsequent experience, but also that we dare to believe there must be a definite determining condition or set of conditions for everything that happens. This assumption has usually been expressed in the following form: Every event has a cause, and the same cause always produces the same effect. If a leaf now falls from the tree beside me, I assume that there must have been some preceding event such as a flurry of the breeze, given which, together with the then existing state of the leaf, the fall was bound to occur. If a happening B is conjoined on two-thirds of its occurrences with A and on the other third with C, I believe that there must be some as yet undiscovered difference between these two kinds of situation; that, if I could drag it to light, there must be some determinate factor present in the occurrence of B which is attended by A and absent from the occurrence of B which is not attended by A. If not, how could the appearance of B be rendered intelligible? It would be

inexcusable in an orderly nature; it would have to be viewed as a sheer miracle. This may be called the postulate of *causality;* it is involved in all the experiments in which scientists engage, and it is a constant encouragement to the endeavor to replace less dependable correlations by unexceptional laws.

The justification of this postulate would follow lines similar to those used with the other. First, if man has a living need to anticipate his future at all, he also has a need to render that anticipation as definite and certain as possible. This means that he will not be wholly satisfied with spatio-temporal conjunctions of greater or less probability merely, but will always hope to find some way of replacing them by universal laws. Then he can say not simply that, given A, it is very likely that B will occur, but, given A, there is every reason to expect B and no reason to expect its failure to appear. Second, if experience encourages thinkers to continue assuming the postulate of predictive uniformity, once they have begun to use it as a guide, so does it give them sufficient encouragement to continue to hold the postulate of causality, once a system of explanations reached by its aid has commenced to grow. Time and again in the history of science, irregular, merely probable conjunctions have led by patient study to the discovery of unexceptional laws underlying them. Eclipses and comets were first studied in terms of charted observations revealing no regular law, but in scientific astronomy their behavior is shown to depend on factors which, when located, permit quite confident prediction. Malaria was once explained by its connection with swampy or night air, and of course only irregular correlations were discoverable; when knowledge of the *Anopheles* mosquito which breeds in the swamps was gained, a causal law was gradually established. Why not then have faith that this process can continue without meeting any check intrinsically involved in the nature of the facts which challenge understanding? Indeed, even in the baffling social problems which at present seem to defy the effort to discover quite dependable conjunctions, the scientist is persuaded to assume that this is not because they are not there,

but because so many conditioning factors are operating in any given situation that his powers are as yet inadequate to isolate any of them and to discover the precise effect which is dependent on it.[7]

Reflection on this postulate of causality and its application in problems of explanation brings into clear relief two further considerations which will need to be kept in mind in subsequent chapters.

Causal laws presuppose an unchanged context

The first is that every causal conjunction presents itself in a certain context of surrounding objects and events which is assumed to continue unchanged during the occurrence of cause and effect. A boy, for example, pushes a rock off the end of a pier and watches for its splash in the water. We would think of this as a quite dependable conjunction; the fall of the rock is the cause A which will produce as its effect the splash B. But suppose that while the rock is falling a boat drifts out from under the pier so that the rock falls in it instead of in the water. Here a different effect C is produced. Or a bowler has just started a ball down the alley, and anticipates the strike which his expert aim and spin should produce. But suppose that the timbers under the middle of the alley, having become rotten, should take that moment to give way. Here again the momentum of the ball will produce a different effect than the one expected. These illustrations show that every assertion of causal conjunction implies a certain context of attendant circumstances, and that only when that context remains unchanged is it expected to hold. For completeness' sake this should be recognized in our symbolism. We shall say then in referring to a causal law not simply that, given A, then B, but that, in context D, given A, then B. If something happens to D the occurrence of A may bring about C instead of B, the nature of C depending on just what change has

[7] In connection with this postulate there are, to be sure, certain sobering considerations that have become increasingly prominent as a result of scientific work itself. For the present it may be said merely that these indicate certain limitations under which science stands in its attempt to apply this postulate in the investigation of some types of fact, but that they do not require abandonment of the causal postulate. The main limitations which apply in social science will be noted later; see below, pp. 512–519, 526 ff.

taken place in D. We can usually state in general what context is assumed by a given law, but are often unable to describe it fully.

Should D, then, be regarded as part of the complete cause of B, since it must be present for B to happen; and should we call A not *the* cause of B as we have been doing, but merely a part of the cause? This is a question of what terminology is most convenient in analyzing the situation. Both D and A are clearly requisite for the occurrence of B. The difference is that D refers to a set of passive conditions which merely need to remain unchanged, while A refers to an active factor which, taking place under those conditions, produces B. This difference is appropriately recognized by restricting the term "cause" to this active factor and calling the passive conditions the "context" rather than a part of the cause. Another helpful way of clarifying the difference is to say: The context comprises those passive conditions which are *necessary* to the occurrence of the effect but which by themselves are insufficient to produce it, while the cause consists of the active factor or factors which, occurring in that context, render the conditions not only necessary but also *sufficient*.

Causality does not imply that nature is a single causal system

The fact that such an assumed context may unexpectedly change leads to the other consideration. It is that the postulate of causality does not imply that all events in nature are bound together in a single causal system. This may be the case, and in Chapter 19 we shall see that the idea of such a system operates as a guide in the formation of comprehensive scientific theories, but it is not required by the present postulate. Every event may have a definite cause, through which it can be explained, and yet it may be that there are independent chains of causation which affect each other only when they chance to intersect. The illustrations just employed clarify this point. Think of the boy pushing the rock off the pier. So far as our evidence at present indicates, the succession of causes which led to this event has nothing whatever to do with the causal chain which issues in the drifting of the boat. The former sequence concerns motives in the boy's mind; pre-

sumably he did not even know that the boat was there. The latter sequence concerns the direction and velocity of the wind, the play of the ripples under the pier, the size and position of the boat; presumably these had nothing to do with the boy's wish to see a splash. The two lines of causation meet only when the rock falls in the boat, and an effect is produced which neither, apart from its intersection with the other, would have brought about. One who guided his anticipations merely by the knowledge that rocks falling in water cause splashes could not have predicted this effect; similarly with one who has in mind only what happens when boats are blown about by the wind. It could be predicted solely by one who, in addition to knowing such causal relations as these and reasoning under their guidance, chanced to know, in advance of its occurrence, that both these causal sequences were unfolding at this particular time and place.

One passes, then, from mere description to a search for explanation when he concentrates on the spatio-temporal conjunctions of a given event with others, and systematically follows up those conjunctions which apparently are especially dependable. In this way is gained knowledge that can guide whatever confident prediction of events in subsequent experience is possible. Such knowledge is of the greatest value from a practical standpoint, while satisfying at the same time the theoretical need for a constant order underlying the diversity of happenings which the world exhibits. The quest for explanations of this sort rests ultimately on a universally familiar fact and two postulates. The fact is that the world is not a chaos; in every moment of our experience some strands of constancy are evident in it. The postulates are those of predictive uniformity and causality. And from these considerations it is clearly very important that factual science pursues the right method in carrying out its aim to correct and extend our knowledge of these regular conjunctions in nature.

Six important concepts employed in factual science

Now the establishment and affirmation of factual knowledge under the guidance of these postulates is a process that radically differs from the drawing of conclusions on the warrant

of given premises, with the aim merely of securing formal validity. It involves inference, of course, but the inference serves the process of passing from a limited number of given facts to general laws. No purely formal structure is adequate for testing the success of this enterprise; its warrant is of another sort. This difference is registered in certain concepts which are fundamental in the logic of factual science, with whose meaning the reader will hence need to be familiar.

Six of these concepts will be mentioned here; the first is that of *generalization*. This is a broad concept, meaning any process of thought whereby conclusions suggested or verified by a particular group of facts are assumed to be applicable to all other facts that are members of the same class. Thus, one is generalizing when he believes that certain traits exhibited by a few politicians whom he has met are traits that any politician would possess, or that a law verified by the moon's orbit will hold of the orbit of any planetary satellite.

The second concept is that of *induction*. This is narrower in meaning; it is generalization systematically employed for scientific purposes, and guided by whatever canons and cautions the previous development of science has shown to be pertinent. Thus the above piece of reasoning about politicians would be a case of induction as well as one of generalization, if the general law affirmed as the conclusion were established after careful examination of particular politicians under the guidance of approved scientific methods. Hence it is evident that any scrupulous and responsible attempt to explain a group of events in terms of some universal law is an illustration of induction.

The third concept is that of *phenomenon* (plural, *phenomena*). We have noted that generalization and induction deal with events as instances of classes; it is our wish to draw conclusions about a whole class on the basis of what we find to be the case about certain of its members. The present concept provides a technical term for briefly denoting this circumstance. A phenomenon is simply a class of facts, considered

as forming the subject matter of inductive inquiry.[8] That is, the class is taken as presenting itself in certain particular instances, whose spatio-temporal conjunctions with instances of other phenomena are being given systematic attention. Thus the swing of a pendulum, the revolution of the moon, being a politician, are all phenomena; anything is a phenomenon which is capable of repetition under various circumstances and is the subject of scientific study as such.

The fourth concept is that of *datum* (plural, *data*). The result of any careful and responsible observation of a phenomenon, recorded so as to be available for subsequent use, constitutes a datum.

The fifth concept is that of *sample*. In occasional situations one can engage in what is known as a "perfect" induction, by which is meant that before affirming a conclusion about the phenomenon involved he investigates every member of the class which is concerned. Thus I should have performed a perfect induction if, after examining every car stored in a certain garage, I announce that all of them are Chevrolets. Such a conclusion simply summarizes the individual data which lead to it; no predictive function is performed. But in most problems of inductive inquiry this is impossible; either not all members of the class are now available (as is the case, for example, if some of them lie in the future) or, if they are available, too much time and expense would be required to examine every one. Under such conditions the scientist perforce employs a sample—that is, a selected group of instances of the phenomenon to which the conclusion is expected to apply. It is important to have a word by which to register the status of this group and the relation which obtains between them and the whole class. Thus, in the above illustrations, the particular politicians whom I had observed to exhibit certain traits were the sample used in my reaching the affirmation that all politicians possess those traits. Similarly, the orbit of the moon is the sample which, in the other case, was held to

[8] It thus has no connection with the adjective "phenomenal" as popularly used.

justify the conclusion that the orbits of other planetary satellites conform to the same pattern. A special reason of importance for having a technical term of this kind is that it has been discovered in the history of scientific method that the selection of samples may be done wisely or very poorly; this difference cannot be accurately described, nor its grounds clearly explained, without a word in terms of which the difference can be stated.

The sixth concept is that of *evidence.* We have already employed this word; let us now assign it technical meaning. Given any sample, for what inductive conclusions is it adequate evidence—that is, what conclusions are properly warranted by it? Or, given any generalization which we might be interested in drawing, what sample would supply sufficient evidence for it—that is, what data, presented in what way, would justify our drawing it? [9] The investigator in any field of factual science differs from the untrained thinker in being acutely conscious of these questions and of the need of guiding his work by a rational answer to them.

These concepts and terms will now be used as current coin in the treatment of problems to which they are relevant.

EXERCISES

1. Define: science, law, explanation, cause, order, chaos, prediction, accidental, induction, sample, evidence.
2. Does the advance of knowledge involve the discovery of exceptions to laws previously assumed to be universal, as much as it involves the reduction to order of events previously appearing accidental?
3. After reading Joseph, *An Introduction to Logic,* Chapter 15, write an essay on "The Postulate of Causality."
4. Suppose we were not at all interested in predicting events, but tried to explain events in the most dramatic manner possible. What kind of explanation would result?
5. Discuss the distinction between pure and applied science, illustrating the dependence of the latter on the former.

[9] Some writers use the word "evidence" more broadly than this. They talk about the premises which imply a certain proposition as constituting its evidence. I think it preferable to treat the concept as one distinctly belonging to factual science.

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CHAPTER 16

CAUSAL LAWS AND THE PRINCIPLES GUIDING THEIR VERIFICATION

Distinctive problem of factual science

Having made these postulates and concepts his own, the natural scientist faces his distinctive problem—that of reaching a true explanation of the phenomena that challenge his attention, and of formulating such descriptions as will contribute toward the possible later establishment of an explanation. And his way of dealing with this problem is the way of reasoning as analyzed in Chapter 2, not any of its alternatives. That is, he proceeds by clarifying a perplexing phenomenon through the accumulation of relevant data, by formulating hypotheses of possible explanations or significant probabilities, by deducing such implications from them as will guide verifying observations or experiments, and by carrying out in whatever way is most promising the process of verification. As in the case of reasoning about other problems, he often finds it necessary to retrace his steps in the course of any inquiry, supplementing his data, revising his hypotheses and rendering them more precise, drawing further implications, and engaging in new experiments. Often a verifying observation that eliminates certain hypotheses from further consideration suggests a reformulation of some remaining hypothesis in such a way that it may be fully confirmed by the available evidence.

Three corollaries from these considerations deserve attention before we become absorbed in necessary details. First, the process of seeking explanations and accurate descriptions must always be guided by some hypothesis which in whole or in part anticipates the result sought. Second, it is very rare for any important hypothesis to be confirmed exactly as it was

initially formulated. Usually the verified solution will take its definitive form only when the inquiry is nearing its end. Third, when we speak of the hypothesis as "confirmed" or "verified" we do not mean that it has taken a final form, beyond the reach of further revision. All that can be meant is that it has shown itself to be superior to other hypotheses which had been considered as possibly true; it remains after they have been eliminated. But, like any factual truth, it is still to be held tentatively; when new data appear, or better methods of analysis and measurement are discovered, new hypotheses may be suggested, one of which will replace it in its turn. Illustrations subsequently employed will testify to all of these circumstances.

Principles guiding the quest for true explanations

The ensuing three chapters will concentrate on those procedures of factual science that are directly relevant to the quest for explanations. That is, consideration will be restricted to problems in which, on the basis of the available data, the possibility of a universal law of spatio-temporal conjunction is definitely suggested. Our question is: How are hypotheses of such laws followed up to determine what the true explanation of the phenomena in question is? Other phases of science, indirectly relevant of course to this enterprise, but immediately concerned with the less ambitious aim of careful description or the measurement of probable conjunctions, will be left aside till the present analysis is completed. We begin with the verification of causal laws because, on the whole, the procedures involved are the simplest to comprehend.

As the emphasis of the previous chapter fell on the clarification of certain ideas which are basic to the enterprise of pure factual science, the study now embarked upon will emphasize the *principles* that are fundamental in guiding the process of causal induction—the process, that is, which issues if successful in the establishment of universal laws on the basis of a sample composed of selected instances of the conjunctions in question. There are three such principles which, since the time of John Stuart Mill (1843), have been generally known in Western thought as the Methods of Agreement, of Differ-

ence, and of Concomitant Variations. The second includes and supplements the first, while the third likewise includes and supplements the second. In strict accuracy, none of these is a "method"; their role is rather to direct one's choice of appropriate methods when investigating evidence for inductive conclusions. Hence they will be referred to here as principles, not as methods. As we shall see, their task is essentially to serve as major premises for the "if . . . then" step of a scientist's reasoning when he passes from the formulation of his hypotheses to the observations or experiments which he believes will demonstrate their error or establish their truth. The present chapter will analyze the establishment of causal laws under the guidance of the principles of agreement and difference, while the chapter which follows will explain and illustrate the distinctive virtues of the principle of concomitant variations.[1]

Relation of these principles to the factors of advantage in association of ideas

But the logical significance of these principles will be best revealed if we begin their exposition by returning to the theme of Chapter 4—the nature of superstition and its difference from science. It was there pointed out, in discussing the factors of advantage in the connection of ideas, that most superstitions arise through some influence on our generalizing inferences of the factors of recency and vividness, whereas, when the play of these factors is insistently controlled by the factors of frequency and means-end contiguity, the ideas generated are much more likely to lead to a verifiable conclusion. Now to have learned this lesson is highly important, but it precedes rather than constitutes the first step in conscious mastery of the methodology of factual science. One who has reached this stage is on his guard against superstitions, at least in their cruder forms—he will not adopt generalizations which rest merely upon emotional congruity or upon some vivid memory—but he may still fall into the errors of popular generalization which factual science, as a result of long effort to improve its procedure, has learned how to reduce to a minimum. It is

[1] In formulating these principles we shall depart considerably from Mill's statement of them, which at several points is far from satisfactory.

the major further lessons acquired through this long effort that are funded in the three principles now to be explained and justified.

The principles of agreement and difference

The principles of agreement and difference embody the two primary lessons which the scientist will bear in mind in any induction but which popular thought, even when it has gained some intelligent self-protection against irrational associations, may blithely ignore. And the first of these lessons, which becomes articulate in the principle of agreement, will be described at once. To draw conclusions, as suggested at the close of the last chapter, about all politicians merely on the basis of observing a few politicians that have chanced to come under one's notice, would exemplify blindness to this lesson. It is better to draw them thus than to do so on the basis of some single recent or distressing experience of a politician, but the result affirmed may still be considerably in error. For what ground of confidence does the author of such reasoning have that the politicians he has not observed are like the few whom he has observed in respect of the traits which he ventures to attribute to them? To use the technical term which is now available and appropriate, what assurance has he that his sample is adequate, *i.e.*, truly representative of the whole class to which his conclusion is supposed to apply?

Most of the saws and proverbs handed down from ancient times, which summarize the popular knowledge of prescientific tradition, clearly betray a failure of those who trust them to become critically conscious of this question and of the need of possessing a clear answer to it. Such proverbial knowledge tells us that a hearty meal makes one drowsy, that a soft answer turns away wrath while grievous words stir up anger, and the like. These are not superstitions, being based on frequent observation of situations in which they hold good, but they do not constitute science. Only a little reflection is needed to show that there are many exceptions to these generalizations, and when the attitude of responsible science has appeared exceptions cannot be brooked. One cannot honestly affirm a universal law if they are present. Mastery of the dis-

tinctive features of scientific method begins when it is consciously realized that these proverbial generalizations have not taken into account a consideration of basic importance, and systematic effort is devoted to its clear apprehension and accurate formulation.

Their historical significance illustrated

The essence of this first lesson is quite simple. Consider briefly one of the items of popular knowledge just used for illustration. That a hearty meal makes one drowsy cannot stand as a scientific law, despite the measure of truth which it contains, because, as has been noted, there are exceptions to it. What exceptions? Well, if a man has become accustomed to eat a hearty meal at a certain time in the day he will not be so likely to feel drowsiness as one unaccustomed to it; the former will reveal exceptions to the law even though the latter may not. Again, if the person who has partaken of a hearty meal is emotionally aroused he will not be drowsy; the generalization will not apply to cases of this kind. In brief, it is evident that a law which has been suggested by a number of observed instances may hold only under certain circumstances; and perhaps under no set of specifiable circumstances does it always hold. Now unless our generalization has been guided by conscious recognition of this fact it may run seriously astray, because it might be that all the instances we happened to observe agree in the conjunction they exhibit merely by chance, or that they took place under the special conditions in which the law does hold good. Unmindful of this possibility, we blithely affirm a universal law, with a very strong likelihood that it will be quickly overthrown by more extensive observation of the same phenomenon.

How the procedure of simple enumeration is corrected

Logicians have sometimes referred to this highly inadequate procedure of popular generalization as the method of *simple enumeration.* That is, an inquirer makes a cursory canvass of his experience of the phenomenon in question, simply enumerating instance after instance as it happens to present itself, and if some other phenomenon seems regularly associated with it he pronounces them causally connected without more ado. Such generalizations are, of course, valuable prac-

tical guides as long as nobody has taken the responsibility to replace them by more carefully verified laws—as long, that is, as science has not yet come upon the scene to fill its distinctive role in the presence of the relevant facts. They are obviously superior to superstitious associations.

How is the inadequacy of generalization by simple enumeration to be corrected? In general, of course, by recognizing the nature of its essential weakness, and governing our inductive procedure accordingly. But what does this mean in practice? The answer is: It means an active and patient search for possible exceptions to the hypothesis which is undergoing verification—for cases where the suggested induction fails. We do our best to puncture its pretensions. And this is usually done by widening the range of our observation, taking into consideration as many as possible of the varied circumstances under which the phenomenon studied can be found to occur. Only a hypothesis which has shown itself able to stand the test of such a systematic investigation can be affirmed as a verified scientific law. We can hardly say that an event A is regularly conjoined in its appropriate context with another event B if one can find cases of the former in which the latter is absent, and the conjunction is the more certain in proportion as no such case has been found in a larger variety of instances of A.

Instances of such correction

Consider an illustration already employed. An Eskimo who had never wandered far from his native habitat might be pardoned for believing the generalization that "All bears are white." All those that he has observed or heard about are of this color, and it has not occurred to him that a bear's color might depend on variable climatic or other conditions, so that elsewhere the animal may exist with different colors than the one he and his acquaintances have always noticed. Now a trained scientist will take account of this possibility from the very beginning of his investigation; as a result his generalization will be vastly more dependable. He will not only investigate whether bears can be black, brown, or gray, as well as white, but he will also find out if each of these colors is regu-

larly conjoined with certain other significant features of the animal possessing it or with his environment. The knowledge gained will thus be accurately verifiable and a much more adequate guide in the anticipation of subsequent experiences about bears.

Another illustration will reveal more fully the nature and value of the principle now being clarified. Suppose that, having noted the phenomenon of the skin becoming tanned, I wish to inquire whether it is always caused by exposure to the sun. Clearly, merely casual observation of the cases noticed at a beach resort in the summer would be far from adequate if I really want to establish verifiable knowledge about the matter. That would be to trust in the method of simple enumeration. It might be possible that only under certain limited conditions does this conjunction take place, and that a universal law explaining the effect that perplexed me must be stated in terms of other factors than those thus taken into account. Accordingly, I must investigate the tanning of the skin amid all the variety of circumstances under which it takes place. This means that I must examine cases of exposure among people of various races and complexions, at various seasons of the year, and under varied atmospheric conditions; and only when, after such an examination, a definite conjunction is brought to light which is unexceptional, have I the right to affirm it as a verified law.

What the correction accomplishes

If we consider what happens as a result of this sort of examination, in the light of the distinctions drawn in the last chapter, it is evident that two discriminations are systematically made so far as the procedure described can make them. First, a cause A of the effect B is discriminated from factors which we might have thought to be a cause but which are not. Mistaken hypotheses are eliminated. We might, for example, have thought that cold wind causes tanning as well as exposure to sun; we should discover that this produces an effect similar to tanning in certain respects but not identical with it. Second, among the factors which are not causal the context D is discriminated, which is equally necessary with A but plays

the role of merely passive attendant. Thus we will note whatever atmospheric conditions and whatever state of the skin are necessary to the occurrence of tanning but which, by themselves, are obviously insufficient to produce it. Briefly, the examination places us in a position, so far as it can, to affirm that a certain definite cause, in a specified context, dependably brings about a certain definite effect.

Brief formulation of the principle of agreement

It is now time to formulate carefully, in its role as guide to scientific induction and critical reasoning in general

I. THE PRINCIPLE OF AGREEMENT

The conjunction of two phenomena may be affirmed as a causal law if every observed instance of one is followed or preceded in its proper context by an instance of the other, no exception having appeared in any of the varied circumstances under which it occurs in that context. This principle is termed the principle of "agreement," because all the instances of the one phenomenon are found, when it is successfully applied, to agree in being conjoined with an instance of the other. Expressed briefly in negative terms, as a principle guiding the elimination of mistaken hypotheses, it amounts to this: Nothing is a cause or an effect of a phenomenon if when it occurs in the proper context that phenomenon does not accompany it.

"The sun warms an object exposed to its rays, in the context where no cause of increased cold is present." Here is a causal law which presumably can be readily established merely by application of the principle of agreement. Observation of such cases under the most varied circumstances would be found to verify its truth. Similarly with the law "An object heavier than air above the surface of the earth falls, in the context where no support is present."

In just what way does this principle act as guide in the scientist's verification of a hypothesis? It does so by filling the role of implicit major premise in the process of inference by which he passes from the formulation of his hypothesis to the detailed observations by which he proposes to test it. Pre-

liminary data suggest that phenomenon A, in a certain context, is regularly followed by phenomenon B. If he seeks merely to find out whether this hypothesis can be confirmed, the principle of agreement is the appropriate one to apply. He will then reason: "If I can find that every instance of A in the assumed context, no matter how varied other circumstances may be, is followed by an instance of B, then A may be pronounced a cause of B." Obviously, this is a valid inference only if the principle as above stated is assumed as its major premise—only, that is, if whenever such a dependable conjunction is established between a pair of phenomena a causal conjunction may be legitimately asserted. And it becomes evident also that the relation between antecedent and consequent in the formulation of the principle is to be taken as an instance of equivalence rather than merely of superimplication. It is a definition of the conditions under which a causal law may be properly affirmed. Having reached this point, the scientist will select a group of instances of A, sufficiently varied and numerous to provide an adequate sample of the differing circumstances under which A occurs in the proper context. By observing then what happens in these instances he carries out the verification.

A defect which it exhibits when applied alone

Why does this principle need to be supplemented by that of difference? Before the latter principle is formulated it will be helpful to answer this question. And in doing so we shall describe the second important lesson not recognized in the method of simple enumeration but which the scientist has been forced to learn. The answer is that, when applied alone, the principle of agreement may lead to the verification of *many-one* laws of conjunction between phenomena. Now what is meant by a "many-one" law, and why is it less than fully satisfactory? Well, we are confronted by a many-one law when we find effects that are produced by more than one cause, or causes that produce different effects. Examining various instances of phenomenon A, we find it always followed by B, but it also appears that phenomenon C is also always followed by B. Or, examining various instances of

phenomenon B, we find that it is always preceded by A, but that there are situations in which A is followed by something else instead of B. Examples of each of these cases are very frequent. To take the illustration recently used, exposure to the sun will tan the skin, but so will exposure to a mercury-arc lamp. Again, strangling will lead to a victim's death; but so will other causes, such as shooting him through the heart. Still again, a tap with a cue will result in a definite motion of the billiard ball, but if that motion has taken place one cannot be sure that the tap in question preceded. For other causes might have brought it about in the same context, such as a sudden brush with the elbow. Here, more than one cause dependably brings about the same effect. Or a small boy, watching a carpenter at work, might generalize to the conclusion that a hammer applied to a nail always leaves it embedded in a piece of wood. But when he experiments with hammer and nails himself, he will find that under some circumstances the cause is followed by a different effect, *e.g.*, when the wood is too hard or so short that it splits. Here more than one effect is produced by the same cause. Now when an inquiry leaves us with such a result as this we do not have the causal understanding that for both theoretical and practical purposes is desired. Consider the former of these two kinds of situation. We may establish with considerable assurance that, given A in context D, B may be expected to appear, but we do not know at all that, given B, A must have occurred. C might have occurred instead. And of course, where this is the case, our understanding and control of the conjunction is a merely one-directional understanding and control. To produce B, A may be produced, but if we wish to avoid B we may not know what to do, for there are the other events which, if given, will bring about the appearance of B also.

Accordingly, science seeks where possible to discover *one-one* laws of conjunction between events—that is to say, laws expressing such a connection between two phenomena that each universally occurs when the other occurs: given A in

context D, then B; given B in that context, then A. The kind of law thus established is far more valuable than the kind which merely affirms a many-one connection. And the fundamental difference is that when a one-one law is verified we can say not only that if A happens then B will occur, or that if B happens A has taken place, but also that *if and only if* A happens does B happen. A is not only *a* cause of B; it is *the* cause of B. That is, we know that given A, then B, and also that given the absence of A, B can be counted on to be absent likewise. In terms of the formal relations studied in the preceding Part, we have now established a relation of material equivalence between A and B, whereas a many-one law is only an instance of material implication with one of these phenomena as superimplicate and the other as subimplicate.

An illustrative case in which the defect is corrected

Take one of the numerous cases in the history of science where this has been accomplished. Before Pasteur's investigations on hydrophobia, this dread disease was known to be caused by the bite of a mad dog. But there were many instances in which such a bite was not followed by hydrophobia. In other words, whenever B occurred, the evidence seemed to indicate that A had preceded, but there were cases of A which were not followed by B. Accordingly, it was exceedingly desirable to analyze A in such a way that a uniform cause of B might be discovered. Such an analysis Pasteur secured. By a series of experiments, he finally discovered that when tissue from the medulla of a mad dog was injected directly into the brain of another animal, hydrophobia appeared in every case and the period of incubation (which had varied greatly in his earlier experiments) became constant. Thus a one-one law was established between hydrophobia as the effect and poison from rabid medulla tissue affecting the brain as the cause. As a result of thus identifying the precise, universal cause, Pasteur was able to develop a dependable antidote for the poison, which otherwise would hardly have been possible.[2]

[2] Another case of a less dramatic sort is the following from the history of optics. The law had been verified that all solids exhibiting periodical colors

What is the key by whose use we may hope to pass from many-one laws to the establishment of one-one laws? Reflection on the illustrations just used will show that the answer is this: more thorough analysis of the complex situation involved in the occurrence of the causal relation concerned. It was only, for example, by analyzing the condition present in a mad dog that it proved possible to replace a many-one law about hydrophobia by a one-one law. Similarly, apart from analysis of the phenomenon of death there is no likelihood that many-one laws in which death appears as effect would ever be replaced by one-one laws. But when such analysis is made, revealing the different physiological processes which take place between death by strangulation and death by shooting, such laws become possible. Instead of just death as terminal phenomenon of the conjunction, different kinds of death are now discriminated, each universally correlated in the assumed context with its appropriate cause.

To correct this defect analysis is needed

But just what form does analysis take when it is employed for this purpose? Stated in general terms, it consists in a systematic comparison of instances in which any phenomenon undergoing analysis occurs, with instances like them in other respects in which it does not occur. Such comparison guides an investigator's attention so that he can, as far as possible, do one or both of two things: (1) distinguish within B an effect which is universally conjoined with A from an effect universally conjoined with C, or within A a cause universally conjoined with B from other factors; (2) distinguish some common property of A and C which is then recognized as the universal cause of B, or some common property in the various effects of A. The first clause of (1) is illustrated by the two ways of producing death just mentioned. As a result of de-

This consists in comparing positive with negative instances

by exposure to polarized light are transparent. But the converse proposition did not hold; there were transparent solids revealing no such colors under these conditions. However, by analysis and systematic comparison it was discovered that all transparent solids disclosing this phenomenon agreed in the single common property of double refraction. It was then possible to announce a one-one law between the exposure to polarized light of doubly refracting substances and the appearance in them of periodical colors.

tailed comparison of these ways, B (death) is analyzed into two different phenomena: the process of dying because of inability to breathe, which is universally conjoined with strangulation; and the process of dying because of a heart lesion, which is universally conjoined with shooting through the heart. The second clause of (1) is illustrated by Pasteur's hydrophobia inquiry. The first clause of (2) is illustrated by the motion of the billiard ball. By comparing a tap from a cue, A, with a brush by someone's elbow, C, it becomes evident that they share the common property of being moving physical objects; the universal cause of B (the motion of the ball) is then the impact of such an object.

The principle of difference formulated

In the light of this description of comparative analysis as a scientific procedure, the principle of difference which embodies the lesson thus clarified may be stated. It includes the principle of agreement but makes a significant addition to it. And it, too, functions as major premise when a scientist passes in inference from his hypothesis to its detailed verification,[3] in those situations in which he is ready to apply

II. The Principle of Difference

The conjunction of two phenomena may be affirmed as a one-one causal law if the occurrence of one, in its proper context, is always followed or preceded by the appearance of the other, and its nonoccurrence, under circumstances otherwise similar to those in which it occurs, by the nonappearance of the other. This is called the principle of "difference" because, when it is successfully applied, the occurrence of either phenomenon is found to differ from its nonoccurrence in that, in the former cases the other phenomenon always accompanies it, while in the latter cases it does not. Briefly expressed in negative form, the principle added here to that of agreement is: Nothing is the universal cause of an effect or the universal effect of a cause if in the absence of either in the proper context the other occurs.

[3] As a definition of the conditions under which one phenomenon may be pronounced not merely *a* cause, but *the* cause, of another, or not merely *an* effect, but *the* effect.

Controlled experiment the best means of such comparison

Now it so happens that there is a special way of applying this principle in inductive inquiries which is so very valuable that the scientist employs it as a matter of course wherever it is feasible. I mean *experiment under controlled conditions.* Experimentation may be engaged in when one is being guided merely by the principle of agreement, but it can be of quite distinctive value when the principle of difference is being employed. And what is the essence of scientific experimentation? The answer is: It consists in artificial manipulation of phenomena, so that they or the parts into which they can be analyzed may be added or removed one by one, enabling the experimenter to observe precisely what happens in the presence or absence of each, and with much less likelihood that unnoticed factors are betraying his calculations. With experiment, a thinker definitely passes from a mode of verification which does not involve more than the systematic observation, with no special tools to aid him, of what is happening in the world apart from his interference, to modes which do depend upon his interference. And there is a significant difference between a situation in which one is simply watching carefully what Nature is doing already, and a situation in which he actively manipulates objects so as to force her to engage in performances which, apart from such manipulation, would rarely occur, if ever, or to omit performances that would otherwise take place.

The primitive thinker who first experimented in this proper sense of the term when attempting to verify a hypothesized law, was taking an epochal step in the history of thought. Every branch of knowledge moves by leaps and bounds to hitherto undreamed achievement as soon as inquirers learn how to introduce active experimentation in a systematic way. And the distinctive virtues of such artificial manipulation can be best displayed by looking at some simple illustrations of its use.

A simple illustrative experiment

Let us begin with a very elementary experiment, requiring no special apparatus, that you or I might perform in daily life. Suppose that, after you have begun to walk with your shoes on some morning, you feel the pressure of a sharp substance

against one of your feet. Is it a pebble, or a protruding nail? (The reason you limit your consideration to these two hypotheses, at first, is, of course, that previous unpleasant experiences of the same sort have been found due to one or the other of these causes.) Accordingly, you devise a simple experiment to decide between these alternatives. B might be caused by either A or C; which is it in this case? You take the shoe off, turn it upside down, and shake it. If a pebble rolls out, and when the shoe is slipped on again the painful pressure has vanished, you brand the pebble as the cause of the discomfort. So far as you can see, all other relevant conditions have remained the same except the presence of the pebble; hence, since the pressure is now gone it must have been due to the one attendant condition now gone. Should, on the other hand, the result of the experiment be negative—that is, should no pebble roll out of the shoe—you would proceed with another experiment designed to test the other alternative. You would feel the inside of the shoe carefully to locate a possible nail. If you find one, and on removing it the pressure ceases, again you are confident that the true explanation has been reached. Should it prove impossible to verify either of these two hypotheses, you would clarify the problem further by trial and error manipulation until some other plausible suggestion appeared. This means, in the terms used above, that you would analyze the interior of the shoe more thoroughly than before, in the effort to discover some previously unsuspected factor that might be the cause of the pressure. In this case a thick joint in the seam worn through the lining, or a warp in the leather, might prove the offender. But in any case your manipulations would have involved an analysis of the structure of the shoe into parts not previously noticed, and would have led to the forming and testing of hypotheses about the relations of those parts which would never have been considered apart from such experimentation. In this simple form drawn from daily experience, experiment reveals all the important logical qualities evident in the more refined procedures of the laboratory.

Consider now a relatively simple scientific experiment. In 1668, Francesco Redi arranged an experiment to test the common belief of biologists that in some forms life is spontaneously generated, the appearance of maggots in decaying meat being often cited in evidence. The experiment consisted in exposing meat in several jars, all being prepared under the same conditions except that some of the jars were left uncovered, some were covered with parchment, and others with wire gauze. The meat in all the vessels became spoiled and flies were attracted by the smell. In the uncovered jars a large crop of maggots appeared, while none at all appeared in the others. In the case of the jars covered with wire netting, however, flies had laid eggs on the netting, and when they hatched maggots swarmed on the surface of the wire gauze. Redi's conclusion from the experiment was that maggots arise in decaying meat, not spontaneously, but from the hatching of the eggs of insects. What had previously been thought to be the cause of B, as a result of observation guided merely by the principle of agreement, or perhaps even by simple enumeration, was now shown not to be a cause at all. And the real cause, in this situation at least, was clearly disclosed.

A case from scientific history

In the light of these illustrations let us note more fully in what definite ways this procedure affords an advance over mere observation with no manipulation of the causal factors involved. On the one hand, the method of controlled experiment, by introducing or removing this or that specific factor while holding other conditions constant, provides a degree of assurance otherwise unjustifiable that no hidden accompaniment of an apparently verified cause is really responsible for the occurrence of the event to be explained. It thus gives a better warrant than mere observation guided by the principle of agreement can provide, that a supposed causal explanation is sound. So far as experiment can be carried out systematically, it compels any exception to a hypothesized law to present itself if it exists. Observation alone can only afford the assurance that no such exception has appeared in the natural occurrence of the conjunction in a wide variety of cases. On

Experiment thus gives greater assurance and encourages analysis

the other hand, the method of experiment greatly aids the analysis by which one-one laws are established. An inquirer introduces artificially this or that factor into a field otherwise held without change, and sees what happens that did not happen before the factor had been introduced; he then removes it and notes what happens solely because of its disappearance. And if early experiments on any problem are ambiguous, he proceeds to analyze the situation still more minutely, and to rearrange the experiment so as to test part by part of the ambiguous factor, till some part is reached which proves regularly conjoined with the event to be explained. Thus less certain many-one laws can be replaced wherever possible by more certain one-one laws.

Because of these characteristics of the experimental method scientists can speak, as they frequently do, about a "decisive" experiment. As a result of preliminary observation, let us say, the theory has been strongly indicated that A in context D is the universal cause of B, so that given A, B will follow, and on the elimination of A, B will disappear. If conditions are sufficiently within one's control, a single experiment may be set up which can, in essence, complete the verification. Arrange a situation such that, while so far as can be told every other factor remains as it was, A, and only A, is introduced, and see if B follows. Then arrange it so as to eliminate A, while every other factor remains the same, and see if B disappears. Or arrange a situation in which two processes go on at the same time: one the occurrence of A, and the other its nonoccurrence, but with attendant conditions the same in both cases. If such an experiment clearly confirms the theory, it may be affirmed with high confidence as a universal law. Repetitions would have the value merely of correcting accidental mistakes. Because such mistakes are always possible, however, some repetitions are desirable, and use of the word "decisive" should not be taken to imply that they are unnecessary. Since each experimenter has his own characteristic pronenesses to error—his personal equation—it is desirable that the experiment be checked by other competent investigators.

Further advantages of controlled experiment

But experiment has further scientific advantages that are very important. They may be briefly noted. It forces Nature to reveal her secrets without our having to wait till she spontaneously produces the condition we seek to study, and they can be examined at the most convenient times and places. Thus it permits fuller cooperation between interested inquirers. Many such conditions occur very rarely without our interference; some would not occur at all within an ordinary investigator's lifetime; and many, though occurring spontaneously, would not do so in ways permitting detailed analysis and measurement. It is, of course, part of the task of experimental science to invent and improve instruments which make it possible to study and measure details otherwise inaccessible to accurate examination. It would be possible, as earlier suggested, to write a history of factual science in terms of the invention and development of instruments to supplement the senses in observations and controlled experiments. Such a history should begin early enough to include the telescope of Galileo and his contemporaries, which powerfully stimulated the imagination to glimpse the possibilities of such inventions. It would follow the story down through the various -scopes, -meters, and -stats, to the most complex and delicate machines that a well-equipped scientific laboratory at the present time will possess.

The function of scientific laboratories, such as now form part of the equipment of every important institution of learning, as well as of many industrial concerns, is to house these expensive and delicate instruments, and to provide controlled conditions of their employment where the problem is such that no considerable space is required for the indicated experiments. In the present half-century, for the first time, the psychological and social sciences are establishing laboratories, in the attempt to furnish, in their fields, conditions favoring systematic experimental research.

Mention of these sciences, however, requires us to note that experimentation may often be of great value even when conditions of laboratory control are far from realizable. They

constitute the ideal situation, and in the physical sciences it is frequently realized. But in the biological, and especially in the social sciences, it is often impossible of attainment. Here, effects often take time to reveal themselves, and extraneous factors have a large opportunity to affect them. Often the factors whose causal influence we are eager to study interact in complicated ways: introduction of A, besides affecting B directly, brings about changes in the context D which may affect B indirectly. For example, use of a certain fertilizer in a vegetable plot not only may affect the growth of the vegetables but may stimulate the appearance of certain weeds which in turn affect the vegetable crop. Under such circumstances the experimenter has to plan a sufficiently varied series of experiments so that he can tell, by comparison and calculation, what part of the effect is due to each of these indirect influences and what part is due to the direct influence of A. How to do this to best advantage in detail depends very greatly on the specific nature of the problem whose solution is sought.[4]

In discussing experimentation, it is well to note also that experiments may be performed merely for the purpose of eliciting a more promising hypothesis than had hitherto appeared, or to be an aid to observation in securing preliminary evidence, instead of in the systematic way above described, aiming at a conclusive result. Under these circumstances it is usually the principle of agreement that is guiding the experiment, not the principle of difference.

Problem arising from our inability always to apply this principle in the most satisfactory way

Is use of the principle of difference impossible apart from experimental procedure? One often seeks to understand phenomena which are not subject to controlled experimentation. The astronomer cannot experiment on his materials. Usually the geologist cannot upon his, because they occur too slowly, and on too vast a scale. Except in countries dictatorially administered, the material of the social sciences is only in small degree subject to experimentation, since free human

[4] Cf. R. A. Fisher, *The Design of Experiments*, chap. 6.

beings may not be thus exploited without their consent.[5] And in any science, specific problems may at any time appear, of such a sort that observation instead of experiment must be depended on to establish whatever conclusions can be reached or in which the kind of experimentation that is possible cannot hope to exclude all factors that might be causally relevant. What does a trained inquirer do when confronted by these situations? Is he condemned to do the best that he can under guidance of the principle of agreement alone?

No. There is, happily, a way of applying the principle of difference to situations of this kind so that, if sufficient and appropriate data are attainable, one-one causal laws can be established in almost as dependable a form as may be secured under conditions of laboratory control. Traditionally, this procedure has been called the Joint Method of Agreement and Difference. Some detailed instances of it will now be given.

Investigations on dew

During the years 1811 to 1814, Dr. W. C. Wells engaged in intensive investigations of the phenomenon of dew. The traditional theory (still reflected in our popular speech about the "fall" of dew) was that dew descends like rain upon the objects and surfaces which become covered with it. This theory had already become undermined before Wells' inquiries began, and several investigators had suggested that there might be a connection between dew and the low temperature of the objects on which it appears. Their explanations of this connection, however, did not satisfy him, and he investigated the whole subject anew.

At an early stage in his study he endeavored to find out whether dew appeared on an object whenever and only whenever the latter's temperature was considerably colder than the air above it, *i.e.*, whether a universal one-one law could be verified as obtaining between these two phenomena.[6]

First, he studied a variety of situations in which dew appeared. It appeared on grass; the grass was from 7° to 12° colder than the air several feet above. It appeared on a board

[5] See below, pp. 517 f.

[6] W. C. Wells, *An Essay on Dew*, Part I.

raised four feet from the ground; the board was 4° colder than the air around it and colder still than the air above it. It appeared on the roof of a house; that, too, was somewhat colder than the air. It appeared on the leaves of bushes and the upper leaves of trees; these were found to be colder than the surrounding air. By thus applying the principle of agreement, Dr. Wells was able to show a causal relation between the lower temperature of these surfaces and the appearance of dew.

But in order to show that dew did not form under any other causal condition, he had to investigate cases where dew did not appear. And these had to be similar in other respects to the cases in which it did appear, in order to demonstrate clearly that no other factor than a lowered temperature in relation to the surrounding air was responsible for the dew.[7] This he did in the following way, applying now the principle of difference. He observed that no dew appeared on cloudy nights, and found that the temperature of the ground, as also that of a roof or raised board, was the same as the temperature of the air. He noted that no dew, or very little dew, appeared during a windy night; then, he found, the temperature of the ground was only slightly lower than that of the air. He found by experiment that dew ordinarily did not appear on bright metallic surfaces placed on the ground, even when the grass around them was covered with dew; the metals were of the same temperature as the air above, while the grass was colder. He covered small areas of ground with hollow cylinders a foot in diameter and two and a half feet high; the grass inside the cylinders was 10° warmer than the grass outside, and showed only a small deposit of dew in comparison with that on the grass outside. And of course he noted that no dew appears during the middle of the day, when by common observation the surface of the ground is warmer than the air. The only relevant common factor in all these situations seemed to be the absence of a temperature differential between the objects

[7] He was thus securing, as best he could, a substitute for the maintenance, in an experiment, of other conditions unchanged except for the occurrence of the supposed cause.

concerned and the surrounding air; in all other significant respects each of them was similar to at least one of the situations in which dew appeared.

In brief, A (lowered temperature of an object in the appropriate context) was regularly conjoined with B (the formation of dew) irrespective of a wide variety in other circumstances E, F, G, H, etc., *i.e.*, the different substances involved, their different position in relation to the surface of the ground, their different size, shape, mechanical condition, and the like. Likewise, absence of A was regularly conjoined with the absence of B, even though each of the other circumstances E, F, G, H, was sometimes present. It seemed clear, then, that none of these other factors could be causally related to dew, and that A was the universal cause.

Here the experimenter's inability to produce or remove the hypothesized cause at will was compensated for by systematically comparing a variety of cases in which the effect took place with an equally varied group, similar in other respects, in which it did not take place.[8] The two groups of instances, together, gave him a sample that was adequate to reveal a one-one causal law.

Liebig's studies in organic chemistry

Another illustration which brings out the same considerations is found in Liebig's investigations, during the 1830's, of the manner in which certain acids and salts, such as those of lead, bismuth, copper, and mercury, destroy life when introduced into an animal organism.[9] Liebig first placed solutions of these substances in contact with various types of animal tissue (muscular fiber, membranes, etc.) and also animal products such as albumen and milk. He observed that in each case, whatever the variation of other factors, the acid or salt left the water in which it had been dissolved and entered with the animal substance into a chemical compound which resisted the action of the ordinary causes of decomposition. Since organic life consists in a continual decomposition and recom-

[8] The latter group of cases constitutes what is called in statistical investigations a "control group." See below, chap. 21, pp. 476 f.

[9] J. S. Mill, *A System of Logic*, Part III, chap. 4.

position of the organs and tissues, anything that incapacitates them for such processes destroys life. Thus Liebig was able to establish on strong evidence, guided by the principle of agreement, the theory that the proximate cause of death, as produced by such poisons, is the union of the salt with the animal substance to form compounds undecomposable by organic action.

But it was very desirable to supplement this inquiry by a study of cases as similar to these as possible which do not result in death, in order to show that anything which prevents the cause from taking place prevents the effect from occurring. Liebig found such cases by testing the so-called antidotes to these poisons. He observed that when these were present the acid or salt of the poison formed an insoluble compound with the antidote, which, because it is insoluble, could not act at all on animal tissues. Peroxide of iron thus combined with arsenic acid, sugar with salts of copper, and sulphuric acid with salts of lead. Moreover, as still further confirmation of his theory, Liebig was able to show by additional experiments that the reason nitrate of silver is harmless when taken internally is that there is always in the digestive tract a certain quantity of common salt and free muriatic acid, both of which substances combine with the nitrate, and, if its quantity is not too great, convert it into chloride of silver, which is incapable of combining with the organic tissues. In thus extending his investigation under guidance of the principle of difference, Liebig established a one-one causal law between the union of mineral salts with animal tissue and death as occurring in this context.

Historical significance of this procedure

Many scientific investigations exhibit, at some stage of their development, this method of applying the principle of difference, even though they may subsequently find a way of setting up some more decisive experiment. In such situations the procedure just described is significant as giving preliminary confirmation to the hypothesis investigated. Thus, in the pre-experimental stage of the scientific study of yellow fever, Carlos Juan Finlay reasoned that the mosquito *Aëdes calopus*

was the carrier of this disease, as a result of observations guided by the principle of difference in this manner. He noted that yellow fever zones and the zones of distribution of this mosquito corresponded very closely, a zone of the one being present wherever there was a zone of the other, and being absent wherever the other was absent.

In fact, how do the procedures of critical observation and of systematic experimentation cooperate in scientific inquiry when conditions are such that the latter can be used? Usually in the fashion intimated by this illustration. In the beginning, observation is employed in the manner above described; this usually constitutes the second step of some scientist's attack upon his problem. It clarifies the data, shows the presence of some causal relation, and suggests how that relation might be successfully conceived in one-one form. The fifth step will usually consist in some decisive experiment or set of experiments, culminating either in the definite verification of this hypothesis or in the demonstration of its inadequacy. In the latter case it will usually also provide clues as to how the hypothesis should be revised in order to conform to the evidence which has now come to light. Active experiment, where available, is *par excellence* the method of verifying one-one causal laws; mere observation, or observation aided by a few simple manipulations of the relevant materials, plays, in the main, the subsidiary role of clarification, giving rise to probable explanations which need more conclusive testing in order to become verified laws.

Experiment dispenses with conscious reference to the principle of agreement

It is worth noting that when experiment under conditions of laboratory control is practicable, the specific cautions implied by the principle of agreement do not need to be consciously held in mind, whereas, when this is not the case, the first step in employing the principle of difference is identical with the employment of the principle of agreement and must be guided accordingly. The reason is that experimentation in that manner, by its very nature, accomplishes the end which obedience to these cautions is expected to secure, and more besides. This end, it will be remembered, is to reduce to a

minimum the possibility that under certain circumstances A in its proper context might not be attended by B (or B by A); hence the conjunction must be examined under a wide variety of conditions. But if A can be artificially introduced or removed, B always appearing in the former case and disappearing in the latter, while other circumstances remain the same, it is evident at once that A produces B independently of any other factors except the necessary context. Apart from controlled experiment there is a worrisome possibility that some unnoticed factor hovered at hand—a constant attendant of A when it is present—which a more adequate investigation would prove to be the real cause instead of the one supported by the inquiry to date. The principle of agreement has to be guided in such a way as will give as much assurance as is possible under its use that any such factor which really makes a difference in the effect will not escape notice but will be brought to light. Experimental control affords this assurance by revealing in artificially secured purity the conjunction between A and B. When, other circumstances remaining the same, A is introduced, there is a minimum of likelihood that any additional factor surreptitiously appeared with it without attracting the investigator's attention. Hence whatever else happens as a result of its introduction may with high confidence be regarded as the effect of that phenomenon and of nothing else.

Joint causes

Sometimes inductive investigation guided by the principle of difference discovers a situation which deserves special mention. It may be that an effect in which we are interested requires for its occurrence in the appropriate context two or more phenomena that are capable of happening independently of each other. For example, to fry an egg while stirring it produces a different effect than that which is produced by frying it alone, or stirring it alone, and it is an effect which is frequently wanted. Such situations are usually referred to as cases of *joint* causation; two or more causal factors must act jointly in order for the effect to take place. Illustrations of joint causation appear in all branches of science. In economics,

a prominent instance is found in the fact that for an object to possess commercial value two causal factors must be present—there must be a demand for it, and it must be sufficiently scarce so that the demand cannot be supplied without special effort. Neither of these factors alone produces such an effect.

EXERCISES

Exercises covering the material of this and the following chapter will be found in Appendix B, pp. 748 ff.

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CHAPTER 17

EXPLANATION BY FUNCTIONAL LAWS

Chief deficiencies of the methods thus far discussed

Two basic principles which guide verification of scientific laws have been discussed, illustrated, and concisely formulated. But there is a third principle, at least equally important. To introduce its consideration, let us ask what we want scientific explanation to do that even the procedure of controlled experiment guided by the principle of difference does not always do. And the answer is that in the gradual advance of scientific knowledge, chiefly two things become desirable that will not be secured by this technique when it is employed merely in the manner above described.

In the first place, there are many factors whose regular conjunctions it is highly important to study, which can never be banished entirely. The gravitational attraction of the earth, the light and heat of the sun, and the presence of the investigator himself will illustrate these. Some of them may, of course, be reduced so that their causal potency is less than it usually is, but they cannot be completely removed. We need a way of establishing dependable laws about the behavior of these things, even if we can neither introduce nor eliminate them and learn what would happen in their complete absence.

The second consideration is still more significant. We need where possible, both in the interest of exact understanding and that of practical control, to express the relation between two phenomena conjoined by a causal law in such a way as to indicate what degree or other quantitative value of the one is conjoined with what quantitative value of the other. When this can be done we know how to predict and control not only

a phenomenon's occurrence, but also all the measurable variations in which it can occur. This is the manner in which the third principle—known traditionally as that of concomitant variations—is put to use in the exact sciences with the most distinctive success, and although it implies a causal relation between the facts thus explained, full justice is not done the result when it is described in merely causal terms.

Given a tap from a cue (in an appropriate context), and the billiard ball tapped will move across the table—here is a causal law. Given a tap, the ball will move with a velocity varying proportionately with the force of the tap—here is a statement of the same facts, employing the principle of concomitant variations in addition to that of causal explanation. It tells us not only that the tap is the cause of the motion, but also that any quantitative rate of velocity in the motion is conjoined with some corresponding degree of force in the tap. To broach at all adequately this phase of scientific explanation, one must pass from the conception of cause to that of *function*. The latter is employed in a more inclusive and satisfactory method, in which all the other scientific procedures thus far described merge, and in which they find, judged by our knowledge at present, their culmination, so far as concerns the dependability and exactitude of the results reached. In fact, even the phrase "concomitant variations" fails sufficiently to imply all that is significant in this method, as we shall see. Let us begin with the concept just introduced, and see what it promises to accomplish in the factual sciences. What, then, is a function?

The conception of function

In the sense in which science uses this term, and in which we may appropriately speak of a functional explanation, it has two meanings. For one thing, it means the normal or characteristic activity of any entity, as when we speak of the function of a lever, of the lungs, of a public official. With this meaning one is not especially concerned in inductive logic. The other meaning, which is of direct concern, has both a broader and a more precise connotation, the common idea in both being that of a regular relation between two or more

processes of change. We must examine this idea rather carefully.

In the broader sense, a function may be defined as such a relation between one phenomenon and another that the one varies in some determinate way with the other. The former may then be called a function of the latter, which means that for every distinguishable change in the latter there corresponds a distinguishable change in the former. Thus it may be said that the general health of a person is a function of his digestion, that the crops of a given region are a function of its soil and climate, or that the duties of Congress are a function of the Constitution.

This conception is very important. The major implication for inductive reasoning is, of course, that whenever two varying phenomena are related in this way, both an economy and a comprehensiveness of explanatory power are attainable that cannot be expressed in a merely causal statement. In a single formula it is possible to express not only the fact that A and B are regularly conjoined, one being present in the appropriate context whenever the other is present or absent whenever it is absent, but also the fact that this conjunction obtains in detail as well as in general, a particular state of A being always conjoined with some particular state of B.

Consideration soon renders it evident that in order to realize the advantages which the functional method of explanation promises, it must develop in a definite direction, assuming, wherever possible, the more precise form mentioned above in anticipation. We have just seen that when one affirms that B is a function of A he wishes to be able to predict not only that, given A in the proper context, then B; but also that, given a particular state of A, then such and such a particular state of B. Now to accomplish this with full success, both A and B must have previously been analyzed in a particular way. What is that way? Well, to answer this question we must recall what was said in Chapter 11 about the application of the series of numbers to properties of physical facts. Let us examine an appropriate example. How could one make sys-

tematic use, for purposes of prediction, of the idea that a man's general health is a function of his digestion? Evidently, only if each of these two phenomena has been so analyzed that a particular digestive state, occupying a certain position in the series including all possible states of digestion, can be seen thereby to be conjoined with some particular state of health, which occupies a corresponding position in the series of all possible states of health. That is, a serial order having at least certain of the properties of the number series must have been discovered in the distinguishable states of each phenomenon. Otherwise, the notion that a functional relation obtains between the two consists merely in the claim that they stand in detailed correlation, without providing any way in which the correlation can be systematically used for purposes of prediction.

Mathematical functions

Realization of this need has led to the concentration of scientific attention on the discovery in factual relations, wherever feasible, of *mathematical functions,* because these exhibit precisely the kind of serial order which makes possible the detailed prediction just described. Functional explanations of this kind have, at any given time, a more limited field of application than the general concept of function does, but their value is far greater. They may be said to mark the maturity of scientific method so far as concerns its quest for exactitude and system in the laws which it establishes. We shall now explore their use from this point of view. And we shall hereafter use the traditional phrase "concomitant variations" to refer to functional conjunctions of the broader and less precise kind, restricting the concept "function" to those in which numerical exactitude is realized.

Essential definitions

First, a few basic definitions are needed. The reader will remember from Chapter 11 the essential properties of a serial order in general and those of the number series in particular; he will recall also that when such an order is discovered in physical objects or events this means that the property of those objects with which he is dealing has been found capable of being treated as a variable magnitude, whose every dis-

tinguishable state differs from other possible states by its position on some continuous numerical scale. This was illustrated by the property of length, which everyone is accustomed to treat as a variable magnitude in this way. Now any particular state of a phenomenon with respect to such a property, occupying a determinate position on the scale—in this instance some specified length, say five feet—is called a *value* of the magnitude concerned. Let us restate our definition of "function" in terms of these concepts; it is such a relation between two (or more) variable magnitudes that for every value of one there is conjoined some definite value of the other (or others).

The simplest and clearest illustrations of such functions are, of course, to be found in the sciences of pure mathematics. Any algebraic equation—such as: $x = \frac{b}{2}$—is a function. For whatever value is assigned to b, there will be a corresponding value that must then be assigned to x. If b is 6, x is 3; if b is 10, x is 5, etc. And two further technical distinctions must be made. For one, any statement of a functional relation implies that one of the magnitudes is to be taken as given, while the other is functionally dependent on it. Thus in the illustration just used, b is assumed as given, while x is asserted to be a function of it. Now the magnitude which in any functional formula is taken as given is called the *independent variable*, and the other magnitude, asserted to be a function of it, is the *dependent variable*. More than one dependent variable is often functionally related to the same independent variable. For example, the surface and volume of a sphere are both functions of its radius. This means that for any value of the radius, say nine inches, there correspond definite values for the surface and volume, respectively, expressed in square or cubic inches. When stated in this form the radius is the independent variable, since it is taken as given, and the surface and volume, affirmed to be functions of the radius, are dependent variables. In this case it would be correct to convert the law and say also that the radius is a function of the surface or of the volume. Then the radius would be the dependent

variable, and whichever of the other two were taken as given would become the independent variable. There are functions which are not thus convertible, A being a function of B while B is not a function of A. The postage of a letter is a function of its weight, since for any weight that might be specified a definite postage is determined. But the weight is not a function of the postage, for no definite value of the former is determined by any given value of the latter. Let the postage be six cents; we know that the weight is more than one ounce and not more than two ounces, but the exact weight between these limits is not determined.

In such functions as have thus far been mentioned there is only one independent variable, since a determinate value for it alone is needed to determine completely the values of the others. Sometimes there must be more than one independent variable in order that definite values for the dependent variables may be prescribed. Thus the density of a gas is a function of its volume and its pressure; definite values must be assigned to both the latter quantities in order to determine a corresponding value of the former. Here the density is the dependent variable, and the volume and pressure are both independent variables.

The other distinction is that between direct and inverse functional correspondence. The correspondence between the radius and the volume of a sphere is *direct*—that is, as the radius increases the volume increases and as the radius decreases the volume decreases. Given a certain quantity of gas, however, the correspondence between volume and pressure is *inverse*. As the volume decreases the pressure will increase, and as it increases the pressure will decrease. But in both cases a definite value for the dependent variable is determined by any given value of the independent variable.[1]

Illustrations of concomitant variation and functional correspondence

Let us consider a few illustrations of explanation through the use of these concepts, advancing from cases in which

[1] There are functions, too, which involve both direct and inverse correspondences. Consider the equation $x = \frac{a}{c}$. Here, x varies directly with a, and inversely with c.

only a concomitant variation is exhibited, to cases in which both the phenomena involved have been systematically analyzed into variable magnitudes, so that a mathematical function may be sought.

Suppose that a group of friends fall into a discussion of the varying standards of living of different families. What is the explanation? The suggestion would not long be lacking that the size of the income enjoyed by a family is the main determining factor—in other words, that a family's standard of living is a function of its income. This would constitute an explanation in terms of concomitant variations, affirming that for every difference of income there corresponds a difference in the standard of living. In order to verify the affirmation, of course, it would be necessary to analyze what is meant by standard of living in terms of some serial order. This might be done in rough fashion by drawing up a list of the main commodities enjoyed by people, proceeding from bare necessities at one end to rare luxuries at the other. If all the available data indicate that for every advance in family income there is an advance in the position of the family's habits of consumption with respect to this order, and for every decline in income a decline in such position, the hypothesis could be pronounced, so far as these data go, a verified law.

Now in this case no mathematical function has been discovered, since our analysis of one of the factors involved, that of the standard of living, is in terms of differences which, though serial, have not been reduced to values of a numerical magnitude. If the units in which that factor is analyzed could be transformed into values of such a magnitude, it would be possible to attempt an exact functional explanation in problems like this.

An illustration from Pasteur's investigations

Examine next one of Pasteur's experiments on the source of the microorganisms appearing in sterilized liquids. Having formed, by other investigations, the hypothesis that the source of these organisms was in the atmosphere, he prepared an experiment designed to test whether their appearance was dependent on the presence of atmospheric dusts. He

. . . took a series of bulbs of about a quarter of a litre in capacity, and after having half filled them with a putrescible liquid, he drew out the necks by means of the blowpipe, then caused the liquid to boil for some minutes, and during the ebullition, while the steam issued from the tapering ends of the bulbs, he sealed them with the lamp. Thus prepared, the bulbs were easily transported. As they were empty of air . . . when the sealed end was broken off, the air rushed into the tube, carrying with it all the germs it held in suspension. Closing the tube immediately afterwards with a flame, and leaving the vessels to themselves, it was easy to recognize those in which a change occurred. Pasteur opened and resealed twenty of these bulbs in the country, far from all habitations; twenty more in the Jura, at 850 meters above sea-level; and twenty more in the Montanvert, at 2,000 meters. In the first twenty there were eight bulbs in which organisms appeared; in the second twenty there were five; in the third twenty, only one.[2]

Pasteur had previously shown that when air was admitted to sterilized liquids near the city streets, microorganisms would appear in almost all the bulbs. He regarded the outcome as clear confirmation of his theory that the appearance of the organisms varied concomitantly with the presence of atmospheric dust—that is, that they were carried around on the dusts of the air. If we were to view it as an attempt to establish a precise function between these two variables, Pasteur's experiment would reveal the defect that there was no exact statement of the quantity of dust per cubic foot in the atmosphere in the various places where he opened the bulbs. Had such data been used, and the correspondence between quantity of dust and the percentage of bulbs in which organisms appeared been complete, it would have been possible to state the result with the precision of a mathematical formula—the number of bacteria varies directly with the quantity of atmospheric dust. Pasteur's purpose, however, was satisfied by showing that the microorganisms appeared wherever there was contact with the dusts of the air, and that to avoid them, it was necessary to exclude the dusts. Hence he did not investigate the possibility of a more exact correspondence.

[2] J. E. Creighton, *An Introductory Logic,* p. 475 (copyright 1920). By permission of The Macmillan Company.

Another case—the mechanical equivalent of heat

If the full virtue of establishing laws of physical phenomena in the form of mathematical functions is to be glimpsed, one must turn to the sciences already exact in an unqualified sense —sciences which have succeeded in isolating their field, reducing the properties with which they deal to magnitudes, and expressing each state of any property as a value of the magnitude involved. Thus far, certain branches of physical science only, of which mechanics is the best known example, seem to have reached this point. Let us accordingly take an illustration from the field of mechanics, which will exemplify the method of functional explanation at its best.

J. P. Joule first placed the modern theory of heat on a sound experimental basis. Before him the theory that heat can be treated as velocity of molecular motion within the heated body had been suggested, but no way had been devised of performing the detailed experiments necessary to verify it. In fact, it was necessary to study the motion of molecules of gases carefully before this theory of heat could be stated in quantitative terms so that verifying experiments leading to a functional explanation could be devised. But this point having been attained, Joule

> . . . elaborated the consequences of that theory, and then actually discovered by physical measurements in his laboratory that those consequences did take place. If heat is not a fluid,[3] but is rather the energy of molecular motion within a body, and if those molecular movements take place in accordance with the ordinary mechanical laws of motion, then, when mechanical energy is expended upon a body, say through pressure upon a gas, and is transformed into heat, or the increased motion of the molecules within that body, the energy which disappeared in the act of compression should exactly equal the energy which appears in the increased motion of the molecules, or heat. Conversely, the energy of molecular motion which disappears when a gas expands again (or the heat which it loses on expansion) should reappear as mechanical work done by that expansion. For on the molecular theory we are dealing not with two different kinds of energy—mechanical energy, or the ability to perform work, and an energy of a different sort entirely, heat—but rather with mechanical energy on two different scales, the motion of large bodies, and the motion of very small bodies, or mole-

[3] As had earlier been generally believed.

cules. In other words, the theory could not be regarded as complete until it had been shown that, in the production of work from heat, a certain quantity of heat disappeared and ceased to exist as heat, and that this quantity was the same as that which could be generated by the expenditure of the work produced.

This actual quantitative determination of what is called the "mechanical equivalent" of heat was the task of Joule, and in experimentally verifying that equivalence he conclusively demonstrated the whole kinetic theory of heat. For he succeeded in showing that heat could be regarded as mechanical motion on a small scale, and that it obeyed the laws of all motion. To this end he instituted a number of experiments. He measured the heat produced by a certain amount of electrical energy, and found that it was constant. He measured the heat produced by the expenditure of a certain amount of mechanical energy in compressing a gas, and found that it was exactly equal to the heat produced by the same amount of electrical energy. He found also that the same amount of heat, when converted back into mechanical energy, produced the amount that had been originally expended in creating it. He found that the results so obtained tallied exactly with results when he produced heat by friction. He tried also a number of other experiments, varying greatly his methods each time, and in every case he established the same definite ratio between the amount of heat and the amount of work performed. . . . Since Joule's day many more experiments have been devised, and in every case the same result has been secured.[4]

Summary statement of the virtues of a functional explanation

These illustrations make it very clear that by an explanation of a phenomenon which exhibits the precision of a mathematical function, all the questions answerable by the use of the principles earlier studied are answered, and an extremely important further question besides. We know not only that, given the one phenomenon, the other can be counted on to be present, and that if it is absent the other must be absent likewise,[5] but we also know what exact value of one maintains this dependable relation with any given value of the other. We can then subject our hypotheses or verified laws to

[4] Columbia Associates in Philosophy, *An Introduction to Reflective Thinking*, pp. 133–135. Reprinted by permission of and special arrangement with the publishers, Houghton Mifflin Company.

[5] At least, over a certain range of its values. Over another range, one phenomenon may be present and the other absent. For example, a mechanic is pushing a car across a garage floor. The car's acceleration is functionally dependent on the force of the push. But until a certain quantity of force is exerted, no acceleration takes place at all.

any relevant process of mathematical calculation, thus realizing, as was earlier pointed out, the same degree of exactitude in the conclusions that is realized in the premises from which they are drawn. In this way it sums up in a simple statement far more predictive power for meeting future problems involving the same phenomena than any other kind of explanation that science has been able to devise. It is, accordingly, no wonder that every branch of science strives to establish exact functional laws wherever it can, and tries to organize its field of study, so far as possible, in such a way that functional relations can be discovered.

It is obvious, without specific statement, that in certain fields, such as astronomy, functional laws can be established by observation merely, while in others some kind of experiment is often required. Usually, also, specially devised instruments are necessary as aids in securing the pertinent data. Exact measurement of many variables is impossible without them. Temperature, for example, may be judged roughly by one's feeling of heat or cold, but had the thermometer not been carried to a high degree of perfection no accurate knowledge of its functional relations would have been possible.

When a field of data is analyzed into a system of magnitudes so that the concept of function finds wide employment in the verification of their conjunctions, the concept of cause becomes quite insignificant. It is presupposed in a successful functional explanation, but the latter says so much more about the facts involved that it seems pointless still to use the terminology of causality. However, in two kinds of circumstance this terminology may still find natural employment. When one of the two processes that are functionally related precedes the other in time, or when only one of two simultaneous events can be directly controlled, that one may be appropriately spoken of as the cause and the other as its effect, to bring out the fact that they can be distinguished in this way.

Formulation of the principle involved

Let us now formulate the principle of functional correspondence, again remembering that its role is that of implicit major premise in an inference from hypothesis to conse-

quences permitting detailed verification, when the data have been so analyzed that such correspondence might be established.

III. Functional Correspondence

As the principle of concomitant variations it reads: *Two phenomena are concomitantly conjoined if for every variation of the one in the assumed context the other is found by either observation or experiment to vary according to some constant law.*

As the principle of exact functional relationship it reads: *One phenomenon is a function of another if, both having been reduced to magnitudes, for every quantitative value of the latter a corresponding quantitative value of the former is determined according to some constant law.*

It is instructive to consider somewhat further than has yet been done the process of analyzing a phenomenon so as to reduce it to a variable magnitude with respect to some important property. Obviously, the discovery of such a mode of analysis in the case of data that have previously appeared refractory to it is one of the important ways in which science progresses. Such discovery renders possible the establishment of exact functional laws about matters which hitherto have only been capable of explanation in less precise terms. It is natural, then, that at any given period in the history of factual science an educated man or woman can distinguish, from this standpoint, four classes of scientific phenomena. First, there are phenomena which we have so long been accustomed to think about in terms of a quantitative order that it never occurs to us they could ever have been viewed in any other way. Such are the phenomena of distance (or length) and of time. But somebody must have originally discovered the numerical order now taken for granted in these facts, and even relatively recent history shows progress in replacing less adequate units of measurement in dealing with them by more adequate ones. That a quantitative analysis here was not provided by instinct but had to be laboriously achieved, is indicated by a number

of interesting circumstances, such as that some primitive peoples possess only the vaguest notion of such an order in these matters, and that certain units of exact measurement now used (*e.g.*, the foot) openly betray their origin in exceedingly inexact instruments for determining distance. Second, there are phenomena in which the discovery of a quantitative order, together with an appropriate method of measuring it, has occurred sufficiently recently so that it is possible to compare the situation thus produced with the state of affairs obtaining previously. Heat, just referred to, is a phenomenon of this kind. Prior to the era of recent modern science, heat was generally regarded as a specific substance, which entered a body when the latter became warm and left it when it became cold. Since this substance was never directly observed, no way of accurately measuring it seemed discoverable. Modern physicists came to conceive heat as revealed in the rapidity of motion of the molecules of which a heated body is composed. Now velocity of motion is obviously a quantitative conception, in terms of which a law of the expansion of heated bodies could be definitely proposed and verified, and a simple method of measuring heat (the expansion or contraction of a column of mercury in a tube) could be devised. Thus heat came to be viewed as a property rather than a substance, and one identifiable with a magnitude; every distinguishable degree of heat became a value of this magnitude, and it was then possible to test, in the manner carried out by Joule, functional correspondences between heat and other magnitudes that seem to vary with it. Third, there are phenomena in which attempts at the discovery of such an order, exhibiting some measure of success, are currently under way. Until recently the educated public has regarded human intelligence as an essentially qualitative affair, whose variations are not susceptible of any precise measurement. It is now aware of the fact that systematic efforts to analyze intelligence quantitatively have been going on, and that scales have been constructed in terms of which objective measurements of comparative intelligence are possible. And the concept of "intelli-

gence quotient," by whose aid the results of such measurements are stated, is already a part of the vocabulary of the man in the street. These scales still challenge improvement in many ways, but they are already providing more dependable guidance in dealing with practical problems about the intelligence of people than many of the methods previously employed.[6] Fourth, there are, of course, properties which appear at present quite incapable of quantitative analysis, but in which some pioneer of the future may discover a mathematical order to provide the basis of functional explanations in their field. Charm, or conscientiousness, will exemplify such facts. All know that these qualities vary, but so far as I am aware no numerical scale has as yet been devised in terms of which a systematic reduction of their different states to the values of a magnitude can be successfully achieved. This is, however, no proof that an analysis of the kind described is impossible.[7]

Progress in establishing general functional laws

Moreover, science progresses in its functional explanations in another important way. A moment's consideration will make it evident that scientific laws may be more or less general in scope. Consider, for example, the two following causal laws: (1) Severing the trunk of a tree causes it to fall; (2) Removing the support of any heavy object above the earth's surface causes it to fall. Obviously (2) is much more general and comprehensive than (1); it includes all cases of (1) as well as many other phenomena which are not included in (1). And obviously, for this reason, (2) is a far more important law to conceive and verify than (1), since it provides much fuller theoretical and practical guidance. Cases which exemplify it are met far more frequently. Now precisely the same considerations apply to functional laws, as illustrations already familiar will indicate. Laws which cover the motions of observable bodies, in their varying velocities and accelerations, are themselves very broad; a large number of important facts in

[6] Cf. below, pp. 461 f.

[7] I do not mean to deny that even when such an analysis is performed, the magnitude has a qualitative aspect which for certain purposes is very important. This holds of any magnitude.

the physical world become amenable to functional treatment when such laws are established. But still more general and inclusive laws are at the disposal of science when it is found possible to extend these generalizations so that they cover the motions of the invisible molecules of which these observable bodies are composed. Then phenomena which have previously appeared in the form of sheer qualitative differences, such as the increase or decrease of heat in a body, become susceptible of explanation in the same terms as phenomena which obviously involve varying velocities, such as the revolutions of the heavenly bodies. The functional laws by which changes in temperature can now be accounted for are simply corollaries, under certain conditions, of the general laws which apply to all energy expressed in motion. In more technical parlance, they constitute a *special case* of the latter. Thus a still larger realm of natural fact is brought within the scope of a single inclusive scheme of explanation; in place of a considerable number of separate and unrelated laws science has discovered a systematic functional unity throughout the vast area of nature in which these quite general laws hold good. Why the scientist and the philosopher are so eager to reach comprehensive explanations is not difficult to appreciate when the virtues of such general laws as contrasted with laws of more limited scope are considered.[8]

Other principles guiding explanation may be viewed as special cases of the principle of functional correspondence

The conception that laws of limited scope are sometimes special cases of more general laws may, indeed, be applied to the relation between the principles of scientific verification discussed in the preceding chapter and the principle of functional correspondence. Since the latter accomplishes all that the earlier principles do, and more, the earlier may be viewed as special cases of the latter, and once the idea of systematic concomitance between phenomena has been mastered, a certain economy and coherence in our grasp of scientific procedure are gained by so viewing them. How shall the interrelations of these principles be described from this standpoint? It has been noted that the essential difference between the

[8] Chapter 19 develops this theme further. See below, pp. 409 ff.

other principles and that of functional correspondence is that the latter takes account, in the explanations it provides, of the detailed variations of the phenomena involved, while the former take account merely of the occurrence or nonoccurrence of those phenomena. That is, the former constitute an application of the functional principle in the special case where only two distinguishable values or states of each phenomenon enter into consideration—the state of its taking place, and the state of its failing to take place. Thus the principle of difference becomes nothing but the principle of functional correspondence when applied under the limitations of this special situation. In fact, from this viewpoint all three of the principles thus far introduced may be stated in such a way as will show just how each of the other two really falls under that of the one now discussed. The principle of difference is functional correspondence in the situation where merely the two values of the occurrence and nonoccurrence of each of the phenomena are shown to be regularly conjoined with the corresponding values of the other. The principle of agreement, lastly, is functional correspondence in the still more limited situation where only one value of a phenomenon—its occurrence—is shown under varying conditions to be regularly conjoined with the corresponding value of another. A cause, for example, is found to be constantly conjoined with a certain effect, but its absence may not be regularly conjoined with the absence of that effect.

Further application of this conception of special cases of a more general principle to problems of scientific method will appear when the idea of functional correspondence, as employed in factual science, is itself shown to constitute a special case of a still more general principle, that of correlation, which unites under a comprehensive concept all responsible investigations of the conjunctions of phenomena.[9]

Symbolic representation of how the three principles are used

Let us bring together briefly the central threads of our analysis of the three principles thus far discussed, and consider in their light an important aspect of scientific method

[9] See below, pp. 498–500.

not yet mentioned. In investigating clues pointing toward a universal conjunction between two or more phenomena one may (1) employ observation alone or engage in experiment; (2) be guided by the principle of agreement alone or also by the principle of difference; and (3) consider the occurrence of the phenomena dealt with alone, or likewise, when possible, their quantitative values. The way in which, in each of these cases, the conjunction is disclosed, may be helpfully symbolized as follows:

A = the cause, B = the effect
a = any instance of the cause
b = any instance of the effect

a_1, a_2, a_3, etc. = quantitative values of the cause
b_1, b_2, b_3, etc. = quantitative values of the effect
c, d, e, f, etc. = other phenomena occurring with the cause or the effect, assumed to be irrelevant

Principle of Agreement

$$\begin{array}{l}
e\ g\ a \longrightarrow b\ l\ m \\
f\ h\ a \longrightarrow b\ k\ i \\
l\ i\ a \longrightarrow b\ h\ g \\
m\ e\ a \longrightarrow b\ f\ k
\end{array}$$

Principle of Difference

(by experiment)

$$\begin{array}{l}
cd \longrightarrow fg \\
cda \longrightarrow fgb
\end{array}$$

if the cause is introduced, other conditions remaining constant

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$$\begin{array}{l}
cda \longrightarrow fgb \\
cd \longrightarrow fg
\end{array}$$

if the cause is removed

(by observation—the so-called "Joint Method")

$$\begin{array}{l}
acde \longrightarrow bfgh \\
cfad \longrightarrow ikbe \\
hega \longrightarrow dbkc
\end{array}$$

compared with

$$\begin{array}{l}
cde \longrightarrow fgh \\
fkc \longrightarrow gdh \\
gik \longrightarrow fdc
\end{array}$$

Principle of Functional Correspondence

(direct)

$$\begin{array}{l}
a_1 \longrightarrow b_1 \\
a_2 \longrightarrow b_2 \\
a_3 \longrightarrow b_3 \\
a_4 \longrightarrow b_4
\end{array}$$

(inverse)

$$\begin{array}{l}
a_1 \longrightarrow b_4 \\
a_2 \longrightarrow b_3 \\
a_3 \longrightarrow b_2 \\
a_4 \longrightarrow b_1
\end{array}$$

Just how then does one tell, in examining any step of a scientist's investigation, which of these principles is being exemplified? By noting exactly what he is trying to find out, as shown by his procedure. If he is merely gathering various instances of a phenomenon in order to see what other phenomenon regularly accompanies it, he is employing the principle of agreement. If he is comparing instances of the occurrence of a phenomenon with situations otherwise similar in which it does not occur, in order to see what one-one universal conjunction can be verified, he is employing the principle of difference. If he is taking into account detailed variations in the occurrence of two conjoined phenomena, endeavoring to discover a law according to which a specific value of one is regularly conjoined with any given value of the other, he is employing the principle of concomitant variations. And in the latter case, if he is assuming that the phenomena are reducible to magnitudes, so that the law can be expressed as a mathematical equation, the principle of concomitant variations becomes that of exact functional correspondence.

Self-corrective character of inductive method

A question may here be raised whose answer will clarify in a new way the nature of sound methodology in factual science. In formulating and analyzing each of these three principles we have said that the investigator's sample must include a variety of instances of the hypothesized causal relation, sufficient to justify the assumption that they are representative of all instances in that context. But how great a variety is sufficient? The answer is that no general rule can be laid down on this matter; one must use his best judgment under guidance of whatever relevant knowledge about the subject matter with which he is dealing has already been accumulated. Here is another illustration of the difference between the methodology of factual science and the procedures for testing validity in formal science. But an important point emerges upon consideration of this circumstance; it is that whenever there appears reason to fear that the sample already secured is insufficient, it is necessary simply to enlarge it by further employment of these same principles. Inductive method is, in this

significant sense, a self-corrective method. And this is one of its greatest virtues—to have become so conceived and formulated that whatever defects appear in its use will tend to be corrected by its continued, more extensive use.

This does not mean that we may safely regard these principles as absolutely perfect, in the form in which they are now conceived. That would be a serious error. Just as further experience may at any time compel the revision of a scientific law which has seemed well verified, so may it at any time suggest helpful revisions of what have appeared to be the best techniques for pursuing factual truth. Our conceptions of method must be held tentatively, as our particular explanations must be held tentatively; improvements will appear in both, in the future, just as they have appeared repeatedly in the past. Such changes in method come slowly, of course, and only as definite defects are revealed in our present procedures which some suggested modification promises to correct. But they come. And so far as present methods are held tentatively rather than as finalities, science is self-corrective in this respect also. Wherever scientists become aware of inadequacy in their way of pursuing truth, modifications of method can be suggested which have a chance, when carefully tried out, of providing the needed remedy, and of equipping responsible inquirers with a still better way of realizing the end that all honest inquiry seeks.

Theme of the following chapter

It is time to return now to the two modes of scientific progress above referred to—discovery of new factual exemplifications of quantitative order, and establishment of more general laws including those already verified as their corollaries under limited conditions. So essential is it for the student of the methods of factual science to understand how science advances in these ways that the next chapter will be devoted to the theme thus briefly stated. It will, perforce, be a rather lengthy chapter, for the only way in which such an enterprise can be carried out is to select for illustration some extended course of scientific achievement, beginning with a state of affairs in which the treatment of some important area of fact

has not yet been revolutionized by the persistent search for functional correspondences, and ending with the verification of certain very general functional laws as applicable throughout an imposingly wide realm of nature. And the exposition will also illustrate certain other aspects of scientific inquiry, to which reference in the preceding chapters has been made. One is the manner in which a scientist's problems, like those of everybody else, arise out of the interests of practical life or traditional beliefs which are found to conflict with each other. Another is the extent to which fruitful scientific advance is necessarily a cooperative enterprise, each investigator building upon the accomplishments of his predecessors and stating his own problems in the terms which their researches have helped to define. Parallel with other phases of progress, of course, there appears a growing emancipation from prescientific assumptions, and a fuller application of scientific results to the invention of tools and machines; some to use in further experimental work itself, some to control nature in more practically serviceable ways. No theme in a study of science more fully reveals its guiding ambition than the process by which the scattered novel discoveries of a pioneering age are first separately established, and then gradually come to be incorporated within a more exact and inclusive system conceived and verified by a later genius. For the purpose of such an illustration we shall select the history of astronomy and of mechanics from Copernicus to Newton, concentrating particularly on the discoveries which, in due time, came to be unified in a single systematic explanation under the latter's law of universal gravitation.

EXERCISES

Exercises covering the material of this and the preceding chapter will be found in Appendix B, pp. 748 ff.

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CHAPTER 18

HOW SCIENCE PROGRESSES

The science of motion in the Middle Ages

First consider briefly the main characteristics of the generally accepted way of explaining the motions of bodies in the medieval period prior to the epoch of science on which our attention is to be centered. As might be expected from the comments in the preceding chapter, motion was studied by the dominant schools in a manner reflecting very different assumptions from those of modern mechanics—assumptions which gave no encouragement to the systematic quantitative analysis of motion or to the search for functional laws in which to explain it. The characteristics exhibited by the analysis then pursued were determined by the general view of the world widely believed among medieval thinkers, which may be described in a brief phrase as requiring a qualitative and teleological rather than a quantitative and experimental approach.[1] To say that the analysis was qualitative is to say that thinkers in their search for explanations were willing to stop with the attributes of things immediately disclosed to sense perception, such as red, loud, sour, heavy, hot, etc. Few attempts were made to discover underlying quantitative correlates of these attributes. For this method of approach, steam, water, and ice were quite different substances, with hardly anything in common, because, of course, their qualities as directly perceived are quite different, and almost nobody dreamed that these qualities might depend on hidden factors differing merely in their velocity of motion. To say that the approach was teleological is to say that in their quest for a causal understanding of these qualitatively discriminated

[1] There were other scientific movements, but they were undeveloped and lacked wide influence.

events thinkers assumed that the appropriate clue was to be found in terms of their purpose or use. Now since the glory of God was generally accepted as the supreme end of the drama of creation and the course of history, the divine being became, from this standpoint, a final principle of explanation; all that happens in the inorganic and subhuman realms was likely to be construed in terms of its use for man's earthly pilgrimage, and all that happens in human life was believed to have ultimate significance in its bearing on man's quest for life everlasting in the presence of God.

A few illustrations

The outcome of all this, as may well be supposed, was that scientific explanations were very different from those built up by experimental study of the quantitative structure of events in modern physics. Why does the sun shed its light throughout the universe? To symbolize Christ, who is the sun of righteousness to all who believe. And this teleological point of view pervaded even the simplest statements about the motions of inorganic bodies. A stone falls, when released from one's hand above the surface of the earth, because, being a bit of earth substance, it is out of its proper place when up in the air. Accordingly, when given an opportunity, it seeks its proper place and, having reached it, stays there until forcibly displaced. No incentive was provided here for seeking a quantitative analysis of what happens. It was enough to give a purposive reason for its fall. Galileo remarks that, although it had been known that falling bodies descend with accelerating velocity, "It is requisite to know according to what proportion such acceleration is made; a problem that I believe was never hitherto understood by any philosopher or mathematician, although philosophers, and particularly the peripatetics [followers of Aristotle], have writ great and entire volumes touching motion." [2] The same teleological principle was ultimately responsible for the assumption in astronomy to which reference will soon be made, namely, that the motions of the heavenly bodies must follow a circular path. For the circle, whose circumference is everywhere equidistant

[2] *Dialogues on the Two Great Systems* (Salusbury translation), p. 144.

from its center, is, it naturally seemed, the most perfect curve, and since these bodies were believed to be created and moved by the most perfect being, God, to express his glory and symbolize for man the immutable realm of his perfection, it seemed inconceivable that they should move in any other way.

The bearing of such a viewpoint on the investigation of problems about motion need hardly be treated at length. So far as distinctions were drawn touching the nature of motion [3] itself, they were qualitative rather than quantitative. Motions were distinguished according as they followed a straight line or a curve; they were also divided into natural motions, as the fall of a stone, and violent motions, as that of a hurled projectile. And it was taken for granted that the proper way to account for any motion was to do so in teleological terms: the answer to any problem about it was an attempted answer to the question *why* the motion took place, in terms of its purpose or use in the plan of the universe; not to the question *how* it took place, in terms of the quantitative structure of the motion itself.

The science of astronomy in particular

So far as the science of astronomy in particular is concerned, it reflected these teleological assumptions, but there was no systematic connection in detail between it and the study of terrestrial motions. It was a quantitative science in a sense—the sense, namely, in which geometry is quantitative. As has just been noted, everyone took it for granted that the celestial bodies revolve in circles, and the main problem was to construct a system of circular motions which would fit the detailed observations that had accumulated in the records. A mathematical analysis of such data in any other sense was quite foreign to prevailing habits of thought.

In order to understand these habits and their effects in astronomical inquiry more fully, it will be well to give some attention to the major contrast between this science as conceived by most pre-Copernicans in medieval times and the astronomical viewpoint with which modern minds are fa-

[3] What we now call "motion" was, in the period here referred to, technically called "local motion," to distinguish it from other types of change.

miliar. Prior to the publication of Copernicus' famous work, *On the Revolutions of the Celestial Orbs*, in 1543, it was a practically universal belief in Europe that the earth was the center of the universe—sun, moon, and stars revolving around it. Certain early Pythagorean philosophers and a few later investigators, such as Aristarchus of Samos, had suggested a motion of the earth, or even a solarcentric system, but these suggestions had not been taken very seriously, and Ptolemy's ingenious way of explaining astronomical facts on a geocentric basis had, in its general outline, reigned as authoritative between his time (the second century A.D.) and that of Copernicus.

The cosmology which supported it

What were the main reasons inducing medieval thinkers to believe so readily that the earth is the center of the universe? Some were, of course, religious reasons, connected with the Christian philosophy of history and man's proneness to suppose that the drama of his life is of focal importance in the universe. But there were also certain perceived facts which had become the basis of a rough but comprehensive physical science embodying the same belief. Primitive observation and crude classification had analyzed the physical world into four kinds of substance—earth, water, air, and fire. Some added a fifth substance, the ether, pervading and surrounding the other four. Now to perceptual observation, of course, the earth is by far the most solid, heavy, and stable of these substances. Massive, hard, and firm, it is not readily thrown into motion by external force; it remains an almost unyielding foundation for all processes of change. Water is not quite so heavy, nor is it solid, or hard. It is more pliant and plastic to the play of force, as is seen in the flow of rivers and the waves or tides of the heaving sea; accordingly, it seeks its proper place above and around the earth. Next in order comes air, still lighter and more easily stirred in motion, as every passing breeze and mighty wind tell us; its place is therefore to surround water and earth at their upper limit. Fire, evidently, is the lightest and most mobile of these four substances. In every flickering flame or roaring conflagration we see how inces-

santly its parts are convulsed in motion, and how desperately they struggle to reach their proper place above the air enclosing them. The true home of fire is, accordingly, in the upper reaches of the universe, where the sun, planets, and stars rest in their proper element and pursue regular and placid courses around the firmament. For to the senses these objects appear to be disks or spots of fire,[4] and before the invention of the telescope it is hardly to be expected that more than an occasional hardy thinker would entertain the supposition that they are made of the same substance as the earth, or are more than the minutest fraction of its size.

In this background of thought about the structure of the cosmos, based as it was on rough common-sense observation, how thoroughly natural and consistent is the conception of the earth as center of the universe, and how very radical and unnatural would be the suggestion that the earth moves round the sun! The most patent objection to the hypothesis that the earth moves was expressed by Ptolemy himself when he said, "If there were motion it would be proportional to the great mass of the earth and would leave behind animals and objects thrown into the air." In the absence of even the first foundations of our modern theory of the earth's gravitation, how could such an objection be adequately answered?

Accordingly, the picture of things encouraged by the entire intellectual orientation of the age was that of a solid earth resting immovable at the core of the universe, covered by a partial layer of water and a complete one of air, while at a greater distance these bits of starry flame were gently swept around it, their genial light and ordered courses witnessing the providential love and dependable power exerted upon them by the Divine Creator, who embraces all in His everlasting presence.

The cosmic hierarchy of perfections

This last phrase is to be taken quite literally. The cosmological scheme just portrayed permitted a systematic correlation between the main spatial areas naturally distinguished

[4] That they were chiefly if not solely composed of the fiery element was the prevailing theory, although there were other views.

by one living on the surface of the earth, and the various grades of perfection in the universe which the teleological principle required thinkers to recognize. Surrounding the whole, in the empyrean beyond the sphere of the fixed stars, where he ultimately controls all that happens in the cosmos, is the Deity himself, the embodiment of supreme and changeless perfection. Next below, in the astronomical realm, are entities only slightly less perfect. In their shape and motion they exhibit ideal mathematical forms, and they were believed to be composed of a pure, incorruptible substance, far superior to the impure and manywise defective materials with which our earthly experience is familiar. As one approaches the surface of the earth irregularity and corruption are on the increase, as is everywhere attested by meteorological phenomena. The surface of the earth itself is the scene of the manifold transitions, irrationalities, and pains of which the course of organic history is full, while inside the earth lies demon-peopled hell, its deepest pit reaching to the very center of the entire universe where the most heinous sinners are tormented and Satan sprawls on his foul seat. The hierarchy of levels of perfection was thus also a hierarchy of spatial zones between center and outermost circumference, each zone having its distinctive character and displaying the operation of distinctive laws.

In brief outline, such is the picture of the science of motion in general and of astronomical motions in particular, in their cosmic setting, as conceived by the vast majority of orthodox interpreters at the time of Copernicus. Our story of the historical development which ensued will commence with the latter's replacement, destined gradually to win general acceptance, of this geocentric astronomy by a system which placed the sun at the center and attributed to the earth an annual revolution around it.

The nature of Copernicus' problem

How is this replacement to be understood? How, in the first place, was Copernicus himself induced to contest the hoary and appealing cosmic system just described? Did he break all at once from its presuppositions and invent a quite

revolutionary system? Not at all. The original idea with which he started was that the complexity of the geometrical picture of astronomical facts to which Ptolemy had felt himself forced was inconsistent with the dependence of the motions upon God. The ultimate divine cause is single and is uniform in his modes of operation; the universe ought to reveal, therefore, the presence of simple and uniform laws. But instead it is such a cumbrous and intricate affair (as explained by the accepted

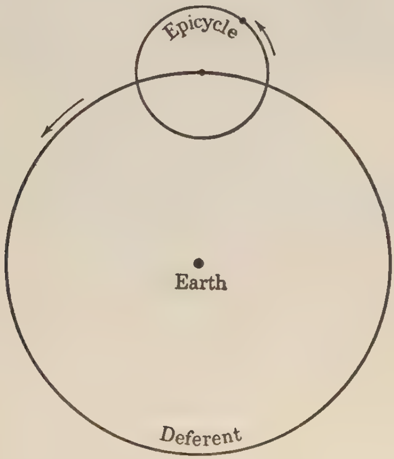

Fig. 15. Ptolemy's Scheme of Deferents and Epicycles.

interpretation) that one is tempted to remark, as the Spanish prince is said to have done to his tutor, "If I had been present when the universe was made I should have seen to it that it was made much more simply." The fact is that, by the time of Ptolemy, such extensive observations had accumulated that no way of accounting for the facts on the assumption that the earth is at rest had presented itself except in pretty complex guise. The best technique that Ptolemy could devise to deal with the situation was a system of deferents and epicycles.[5] A quite uniform motion around the earth as center

[5] He used other technical devices, too, but they hardly need specific mention.

could, of course, be represented by a single circle, but where there are variations from uniformity such a curve would not do. To account for the main variations then known, Ptolemy imagined that the earth is the center of an invisible circle called a "deferent," and that the planet to be accounted for moves on the circumference of another circle, an "epicycle," whose center is borne at uniform velocity on the circumference of the deferent. By selecting suitable velocities and radii for these deferents and epicycles, Ptolemy was able to make his scheme fit the most obvious irregularities of motion with considerable success. The total number of epicycles required to represent the facts by this method was very large—seventy-nine in several standard interpretations of his system—and that there should be any irregularities at all seemed to Copernicus inconsistent with the unitary and perfect nature of their divine author. Any want of uniformity in motion, he says, "must arise either from irregularity in the moving power . . . or from some inequality of the body in revolution. . . . Both of which things the intellect shrinks from in horror, it being unworthy to hold such a view about bodies which are constituted in the most perfect order."[6] His mind became open, accordingly, to some hypothesis that might explain the facts without falling into this apparent inconsistency.

Formulation of the new theory

The next point which seems to have become clear to him, perhaps after beginning some of the reading to which reference will soon be made, was that motion is intrinsically a relative affair. The apparent motion of the sun from east to west, for example, proves nothing more than that a certain change is taking place in the relative positions of the sun and the earth. We tend naturally to think of our own position as at rest, and attribute the motion to the sun, but the change observed does not need to be interpreted thus; it might equally well be explained by supposing the sun at rest and attributing a rotation to the earth on its axis from west to east.

This suggestion was pretty radical, however. Copernicus needed encouragement to pursue it further; moreover, he,

[6] Quoted in A. Berry, *A Short History of Astronomy*, p. 101.

like other thinkers, could not generate a solving hypothesis out of nothing.

> Wherefore, [he says], I took upon myself the task of rereading the books of all the philosophers which I could obtain, to seek out whether anyone had ever conjectured that the motions of the spheres of the universe were other than they supposed who taught mathematics in the schools. And I found first that, according to Cicero, Hicetas had thought the earth was moved. Then later I discovered, according to Plutarch, that certain others had held the same opinion. . . .
>
> When from this, therefore, I had conceived its possibility, I myself also began to meditate upon the mobility of the earth. And although the opinion seemed absurd, yet because I knew the liberty had been accorded to others before me of imagining whatsoever circles they pleased to explain the phenomena of the stars, I thought I also might readily be allowed to experiment whether, by supposing the earth to have some motion, stronger demonstrations than those of the others could be found as to the revolution of the celestial sphere.[7]

Its verification

Making, then, the hypothesis that the sun and the fixed stars should be taken as at rest and that to the earth should be attributed, first a daily rotation on its axis directed toward the pole star, and second an annual revolution around the sun in an orbit between the orbits of Venus and Mars, Copernicus examined the known astronomical facts to see whether from this point of view the motions would exhibit a simpler structure. The outcome was, of course, that they did gain greatly in simplicity when viewed in terms of this hypothesis.

> Thus, supposing these motions which I attribute to the earth later on in this book, I found at length by much and long observation, that if the motions of the other planets were added to the rotation of the earth and calculated as for the revolution of that planet, not only the phenomena of the others followed from this, but also it so bound together both the order and magnitude of all the planets and the spheres and the heaven itself, that in no single part could one thing be altered without confusion among the other parts and in all the universe.[8]

The total number of epicycles needed to chart the astronomical facts was reduced from seventy-nine to thirty-four.

[7] Copernicus, *De Revolutionibus,* Letter to Pope Paul III.
[8] *Ibid.*

That Copernicus was not able to eliminate more was due mainly to the fact that he never questioned the assumption dominant since Greek times that all astronomical motions must be circular. But it was a great gain to simplify to this extent Ptolemy's complex geometry of the heavens, and on the ground of the greater simplicity assured by the solar-centric explanation, Copernicus offered it as the true picture of the astronomical facts. We may sum up his thinking thus: Since all motion is relative to the chosen point of reference, and since a unitary and perfect being is the creator of the cosmic order, that point of reference should be chosen in terms of which the facts are reducible to the greatest uniformity and simplicity possible. This means that the sun rather than the earth is to be placed at the center.

How did Copernicus meet the serious difficulties which stood in the way of this hypothesis from the standpoint of the general intellectual convictions of his age? For there were a host of such difficulties. There was the difficulty Ptolemy had noted. People asked, "If the earth is in rapid motion from west to east, how is it that an object projected vertically from its surface does not fall far to the west of the point of projection?" Copernicus could only answer that forces emanating from the earth attract adjacent objects to it, and sweep along birds and clouds in the same direction and with the same velocity as that of the earth itself; the true answer could only be given later when the foundations of dynamics had been laid by Galileo. There was the difficulty that the fixed stars appear always in the same relative positions, whereas if the earth traverses vast distances through space in its annual orbit the stars ought to appear in somewhat different positions today from those which they occupied six months ago. Copernicus could only answer this objection by saying that the stars are at such enormous distances that while they do reveal such a shift in position it is too small for us to observe. In course of time this was proved to be the true answer, for in 1838 the astronomer Bessel verified it in the case of a few of the stars nearest the earth, the change of position ap-

proximately equalling the diameter of a penny when viewed at a distance of three miles. But in Copernicus' day this meant attributing such a vastness to the sphere of the fixed stars as seemed ridiculously fantastic. The greatest difficulty of all, however, lay in the general background of accepted cosmological ideas. It upset the whole traditional hierarchy of substances—earth, water, air, and fire—in their relative differences of weight, mobility, and position, to suppose seriously that the solid and heavy earth is actually revolving like a planet around a disk of fire like the sun, and the accompanying conception of various cosmic levels of perfection was likewise jeopardized.

Adoption of the Pythagorean cosmology

Copernicus met this last difficulty by turning to an alternative cosmological view, which fortunately had recently undergone a considerable increase of influence. This was the ancient Pythagorean conception of the universe as ultimately composed of numbers, by which the Pythagoreans meant, following a way of mathematical thinking for which geometry and arithmetic were essentially interrelated, minute figured portions of space. In accordance with this conception, all the four kinds of substance, even the heavy earth, were supposed to be reducible to spatial atoms, the shape of the latter and their way of forming larger combinations determining the appearance and behavior of whatever substance they compose. The Pythagoreans, too, had been aesthetically interested in the mathematical harmony which this notion made it possible to find in the universe. In short, this Pythagorean cosmology offered a set of beliefs furnishing positive support for the guiding principles of geometrical simplicity and uniformity which Copernicus was convinced on theological grounds must be revealed in the astronomical motions.

The observed facts were explicable on either view. But the solarcentric scheme explained them in terms of fewer and less complicated figures. Granted the shift just noted in the cosmological ideas involved, it seemed more in accordance with the observed facts while contradicting none. If, for example, the earth and the sun are both, in essence, geomet-

rical patterns, the inertia of the one and the mobility of the other, to the senses, does not necessarily imply that the former must really be at rest while the latter moves. And of course the solarcentric view harmonized better with the religious doctrines which, to Copernicus, seemed fundamental. So it could win his firm belief.

The contribution of Tycho Brahe

The next thinker to whom we turn is Kepler. Born in 1571, nearly thirty years after Copernicus' epoch-making book was published, he became an adherent of the new astronomy while studying under Mästlin at Tübingen in the late 'eighties, and after some years of teaching high-school mathematics at Gratz, he joined Tycho Brahe, the great Danish astronomer, a few months before the latter's death in 1600. Tycho's greatness lay in the number and exactitude of his astronomical observations, of which Kepler secured the use after his death. These observations set an entirely new standard of precision in recording the positions of the heavenly bodies. Gaining from the King of Denmark large financial grants and possession of the little island of Hveen, Tycho had, in 1576, built a splendid observatory equipped with the best instruments that could then be devised or obtained. Realizing the importance of exact observations, he had himself invented some new mechanical devices far superior to anything previously available. Among these were large fixed instruments (capable of restricted adjustments only and therefore much more steady than instruments that could be pointed to any part of the sky), specially constructed sights to aid the eye, and a method of graduating his instruments by transversals, which increased the accuracy of the data. Moreover, he formed the habit of taking a number of observations of the same star under different mechanical conditions, so as to allow for the imperfections of his instruments, and of observing from different positions in order to permit correction of displacements due to the refraction of the atmosphere. With these aids Tycho built up a star catalogue and bequeathed a vast collection of observations, so much more exact than anything previously secured that it was now possible for the first time to test

various mathematical theories in astronomy without constant fear that the margin of error in the recorded observations might be too large to permit confident decision between them.

Kepler's main problem

Sharing quite definitely and strongly the theological and aesthetic interests of Copernicus, and being, like him, a convinced Pythagorean, Kepler found his main problem take the following form: If the Copernican system is true, many other mathematical harmonies in the celestial order must be discoverable by deducing its implications, which would not be consistent with the Ptolemaic cosmology. Accordingly, with Tycho's exact data for testing his hypotheses, it became the passion of Kepler's life to disclose, for the "fuller knowledge of God through nature and the glorification of his profession," these further harmonies. And the harmonies that he had in mind, as would be expected from the context of thought just described, were not the functional laws needed to facilitate exact prediction and to unify astronomy with the science of terrestrial motion—the conscious search for these only came later—but simply whatever geometrical correspondences might help to show that the solarcentric astronomy was more congenial than Ptolemy's system to the Pythagoreanized religious cosmology which he devoutly accepted.

Approaching his work with this purpose, it is natural that Kepler should discover a great many interesting patterns in the astronomical facts which were of no value whatever for meeting the needs of later science. For any sort of geometrical harmony deducible from the Copernican system, while irreconcilable with the geocentric view, was of equal value to him. But, happily, he managed to devote some of the energy spent on this task to problems concerning functional relations between the velocities of the planets and their distances from the sun, and the harmonies he discovered here were indispensable to the later work of Newton.

Taking the especially troublesome irregularities in the orbit of Mars as an object of special study, he clung for a long time to hypotheses of the traditionally accepted type—that is, combinations of deferents with epicycles, or other eccentrics.

He laboriously deduced by calculation the implications of each and then carefully compared them with Tycho's charts. At one stage he arrived at a geometrical pattern squaring with the observations if errors up to eight minutes be regarded as possible. Here was a temptation to Kepler's accuracy and loyalty to fact which he was fortunately able to resist.

> Since the divine goodness [he said] has given to us in Tycho Brahe a most careful observer, from whose observations the error of eight minutes is shown in this calculation . . . it is right that we should with gratitude recognize and make use of this gift of God. . . . For if I could have treated eight minutes of longitude as negligible I should already have corrected sufficiently the hypothesis.[9]

In other words, an error as great as eight minutes in a planetary observation by Tycho was quite out of the question, hence in faithfulness to fact the scheme had to be thrown aside.

Discovery of the laws of planetary motion

Accordingly, Kepler started hypothesizing anew, and after trying various other combinations of circles, he plucked up courage to take a radical step. As previously noted, it had been a universally accepted assumption that the paths of all the heavenly bodies are circles or could be portrayed by some combination of circles, because the circle is the perfect form of curve and alone worthy of the perfect creator of the stars and planets. And before Tycho had appeared with his exact records, the known facts were not irreconcilable with this conception, the possible margin of error being always large enough to include every deviation from a circular path. But Kepler's continued failure with all imaginable combinations of circles led him now to try the hypothesis that the orbit of Mars was an oval of some kind, discarding the hoary traditional conviction. At first he tried an egg-shaped oval, but was soon forced to abandon this guess. Then he tried the ellipse, and found to his delight that deductions from it satisfied entirely the available data, if the sun were placed at one focus of the ellipse. The eccentricity of the ellipse he deter-

[9] Quoted in Berry, *op. cit.*, p. 184.

mined as about one tenth. Accordingly, he was ready to formulate the first of his famous three laws of planetary motion: *The planet describes an ellipse, the sun being in one focus.* And assuming, as he did provisionally, that the same law applies to the orbits of the other planets, he found it possible to eliminate most of the epicycles which Copernicus had been forced to retain. Thus a great new gain in geometrical simplicity was achieved.

Kepler then attacked the problem of formulating the law describing the varying velocity of the planet in different parts of its orbit. Observation showed that Mars moves faster when nearest the sun, and more slowly when distant from it. He tried several hypotheses for expressing the law of this variation, and after many difficulties with their mathematical implications finally hit upon a simple law which proved to agree with recorded facts. This he stated as the second law: *The straight line joining the planet to the sun sweeps out equal areas in any two equal intervals of time.*

The reader will notice that this is an exact functional law, as the latter has been defined in the preceding chapter. That Kepler hit upon it did not indicate, however, that he had become specifically interested in such laws; the discovery is to be explained by the fact that the second term of this functional relation—the area swept out by the planet vector—was a geometrical pattern of the kind which he had long been accustomed to study.

His third law is also a functional law of the same general type. It is that: *The squares of the times of revolution of any two planets about the sun are proportional to the cubes of the mean distances of their orbits from the sun.* And again its verification was in large part due to factors which, from the point of view of the later developments which it helped to render possible, are quite irrelevant. Since ancient times the belief had been common, especially among thinkers influenced by the Pythagorean cosmology, that musical harmonies are produced by the celestial bodies in their revolutions—the so-called "music of the spheres." One of the problems to which Kepler

devoted much labor was the problem of formulating in conventional notation the music generated by each of the planets in our solar system. Now this third law, destined to be of the greatest importance to Newton's quite differently motivated researches, was discovered almost accidentally in the course of his attempt to solve this problem.

Speculations regarding its cause

One other point deserves mention before we leave Kepler. Before his time, as has been noted, astronomy had enjoyed no connection with problems of terrestrial mechanics, save a very general metaphysical one which varied considerably among the traditional cosmologies. How, then, was the fact that the planets and the other celestial bodies are in motion rather than at rest accounted for? What had started these revolutions, and what now continues them? It had been believed by most thinkers that the source of planetary motion was the ever-present divine power, exerted directly from the empyrean on the primum mobile,[10] and communicated by the latter first to the sphere of the fixed stars and then as a result of their motion to the spheres of the various planets. But Copernicanism obviously put this set of ideas out of date. The fixed stars were now supposed to be immobile, and the sun also appeared fixed in spatial position, although observations soon showed that it, like the earth, rotates on its axis. And since, from the Copernican standpoint, the other planets were conceived as composed of the same material as the earth, the question became pressing: Whence the force required to sweep these bodies around in their elliptical orbits, and how is it applied to them? Kepler offered some speculations on this head. Proceeding by analogy from the phenomenon of magnetism, on which he had learned much from Gilbert's epoch-making treatise on the subject published in 1600, he suggested that the sun's magnetic attraction pervades the space within which the planets move, and carries as far as it reaches a moving power arising from the sun's axial rotation. The planets thus share, proportionately to their distances, the

[10] The outermost revolving sphere.

sun's own rotating motion, being swept around in the vortex of the sun's magnetic field.

Galileo as a scientist

Galileo was born in 1564 and died in 1642, his life thus overlapping that of Kepler, who died in 1630. With him, so far as concerns the particular sequence of events on which our attention is focused, we meet for the first time a man consumed by the typical interests of a modern exact scientist, searching systematically for the kind of explanation that permits detailed predictions and is capable of formulation in general functional laws. This quality of mind is best exhibited in the radical revision of the science of terrestrial motions which culminated in his work and which will be examined with some care. First, however, let us notice briefly certain of his astronomical achievements, whose main significance lies in the fact that they provided further confirmation of the Copernican theory.

Early in 1609, Galileo learned of the invention by a Dutch spectacle maker, a few months earlier, of a telescope. Realizing at once the great value of such an instrument for astronomical observations, Galileo, without any detailed information about the spectacle maker's construction, set to work on one for his own purposes. After a few trials he succeeded in arranging a convex and a concave lens in a tube in such a way as to enlarge threefold the apparent size of an object. With further experimentation he learned how to make telescopes which, in the same way, magnified thirty times. The earliest results of his use of the telescope were published in 1610 in a little book entitled the *Sidereal Messenger*.

Some of his astronomical discoveries

His first systematic observations were on the moon, which hitherto had been commonly believed to be perfectly smooth and spherical. Galileo at once recognized that many of the dark spots were shadows of lunar mountains cast by the sun, and that the bright spots near the boundary of the illuminated and dark portions were mountaintops catching the light of the rising or setting sun. From these observations he calculated the height of some of the more prominent mountains, esti-

mating the largest to be about four miles high. This result is not very different from present estimates of the greatest height on the moon. These discoveries first offered factual confirmation of the conclusion deduced from the Copernican theory that the celestial bodies, within our solar system, at least, are essentially like the earth. This deduction could now contend with better success against the hoary assumption that they are more perfect than the earth and different in substance.

The most striking discovery announced in the *Sidereal Messenger* concerned four of the satellites of Jupiter. On January 7, 1610, Galileo happened to turn his telescope on Jupiter, and his attention was caught by three faint spots of light close to the planet and in nearly a straight line with it. He thought they must be fixed stars, but on inspecting them again the next night saw that they had changed their positions relatively to Jupiter, and that the change was not such as could be due to Jupiter's own motion. Continuing his investigations two nights more, he was forced to the conviction that the new bodies were not fixed stars, but were revolving around Jupiter. A fourth body was noticed on January 13. Multiplying his observations again, night after night, by the time the book was published he had determined with fair accuracy their periods of revolution, which varied from forty-two hours for the nearest to about seventeen days for the most distant.

Here again was a heavy blow to traditional ideas. In the first place it supplied definite confirmation of the new doctrine that the earth is not the only center of motion in the universe. Jupiter is such a center too. In the second place, the moon ceased to be an anomaly in the Copernican system. The notion of the revolution of a body around a planet which is itself in motion had thus far been a real stumbling block to the acceptance of the Copernican theory. How could the moon, people asked, revolve around the moving earth without falling behind it in space? The discovery of the satellites of Jupiter did not, of course, answer this question, but at least it showed that the case of our moon is not unique.

We turn now to Galileo's work in laying the foundations of

the science of terrestrial dynamics, described mainly in his *Dialogues and Mathematical Demonstrations Concerning Two New Sciences*. Here he wrote a still more decisive chapter in the history of exact science.

The context of Galileo's problem in dynamics

His central problem here, like that of Copernicus in astronomy, arose from a serious conflict that by this time had emerged between two strands of scientific tradition. On the one hand there was the Aristotelian theory of motion, dominated by the principles described at the beginning of the present chapter. On the other hand, attempts at a mathematical formulation of mechanical laws, beginning with a few Greek physicists and renewed in the later Middle Ages by followers of Ockham and an influential Italian school, had, by the late sixteenth century, achieved considerable success. All serious workers in the field, in Southern Europe at least, were now challenged to square their assumptions and methods with the claims of this movement. In fact, the dominant school in Northern Italy at this time (the school of Padua) exhibited in its way of thinking a quite self-critical Aristotelianism—that is, an Aristotelianism gradually transforming itself by absorbing and applying in detail the method of mathematical analysis. And the mathematical analysis pursued by these men reflected more and more a different temper of mind than that of the Pythagorean cosmologists, the transformation being encouraged by insistent practical and technical problems of the day. The new explorations to the Indies and America had led men to demand a knowledge of the relative positions of astronomical bodies as they vary with time and with an observer's position on the earth's surface, such that a wandering mariner could determine his location and chart his course more accurately than before. The increasing use of gunpowder in warfare had rendered necessary such an understanding of the motion of projectiles as would facilitate aiming them more accurately. Under pressure of practical demands like these, investigators found themselves less and less satisfied with mathematical harmonies so formulated as merely to enhance the scientist's sense of the underlying simplicity of God's in-

tricate universe, which had been a dominant motive with Copernicus and Kepler. They began to seek functional laws of the kind that would permit precise prediction, at the initial or any given stage of a body's motion, of its position, velocity, and direction at any later stage. Further, the time was now ripe for systematic study of motions under the guidance of the supposition that the universe is the scene throughout of uniform quantitative law. The earlier notion of the cosmos as a hierarchy of perfections, exhibiting progressively superior purity and order as one advances from the pit of hell in the center of the earth toward the seat of the Deity in the outer empyrean, had now been drastically weakened by the new solarcentric astronomy.

How to prove the universe a unitary mathematical order?

Galileo inherited this self-critical Aristotelian tradition of Padua, but the faith that the universe is a uniform structure of mathematical law took especially firm root in his mind. His main chosen task was to demonstrate the validity of this faith, and since most of his contemporaries—the more orthodox followers of Aristotle—rejected it, he had to devise a general procedure for convincing them. This procedure, in brief, consisted in analyzing this or that kind of motion—such as the rolling of a ball down an inclined plane—to detect the functional law exhibited in it, deducing the implications of any hypothesized law by mathematical calculation (specifically, implications inconsistent with those deducible from Aristotle's principles), and then verifying them by careful experiments. As a result of his work the modern theory of dynamics was placed on a solid foundation and given an adequate method for its subsequent growth. This achievement rested largely on his thorough mastery of the methods of the Alexandrian mathematicians, particularly Archimedes, whose works had become available in Italy half a century earlier, together with his remarkable ingenuity in devising experimental procedures capable of reasonably exact and hence convincing verification of the conclusions drawn by mathematical calculation from his hypotheses.

For more than two centuries prior to Galileo's time, at-

tempts had been made in Western Europe to discover a mathematical law of falling bodies. What sort of law was sought varied, of course, with different inquirers. One of the main problems which Galileo set for himself was to express and verify the precise functional law in accordance with which a freely falling body is continuously accelerated.

His studies on the law of falling bodies

To solve it by attempting direct measurements on such bodies proved impossible; the fall was soon too fast, and it was before the day of anything remotely resembling a stop watch. Accordingly, Galileo assumed that if balls were rolled down grooves on an inclined plane, the motion would be retarded sufficiently for exact measurement, while the form of the law obeyed would be unmodified. But how to measure the time accurately was still a problem, for the only timepieces available were the ancient water clocks and sandglasses, sadly inexact and only of value for measuring long stretches of time. To meet this difficulty Galileo constructed a water clock. It consisted of a large vessel of water, having at the bottom a minute opening to be closed with the finger. When the ball began to roll down the plane Galileo removed his finger and allowed the water to flow into a cup adjusted on the arm of a delicate balance. When it reached the end of the path he wished to measure, he closed the orifice. On account of the large diameter of the vessel, the pressure-height of the fluid did not perceptibly change, and therefore the weight of the water discharged from the opening was proportional to the time of its flow.

After considering and rejecting one hypothesis, that the velocity of the ball increases directly with the space traversed, Galileo turned from space to time, hypothesizing that the gain in velocity is proportional to the time of the descent. This means that if one body falls from rest through twice as long an interval of time as another, it will attain double the velocity of the other. He deduced the implications of this hypothesis in the form necessary to guide experiment. Since velocity means so many units of space traversed per unit of time, if we suppose that a body's velocity increases according

to the time, we must infer that the space traversed increases according to the square of the time. Hence Galileo notched off on the groove of his plane distances equal to 1, 4, 9, 16 units, and noted that if his theory were correct these distances ought to be traversed in 1, 2, 3, 4 units of time. By the aid of the water clock above described, he measured the time required for the ball to cover each of these distances, and found the hypothesis confirmed as fully as could be expected under the conditions. A functional law of uniformly accelerated motion—that is, motion in which equal increments of velocity accrue in equal intervals of time—was verified as correctly describing the behavior of a body rolling down an inclined plane.

The concept of inertia clarified

By subsequent experiments, Galileo was able to justify his assumption that the law of descent is the same in form for a freely falling body as it is for a ball rolling down an inclined plane. But a very important further value of his work on inclined planes lay in its clarification of the concept of *inertia*, which became fundamental in modern mechanical science and whose nature was later described by Newton in his famous first axiom of motion. By all those schooled in the Platonic and Aristotelian traditions, motion had been considered less natural than rest; when external forces set a body in motion, it tended to find its proper place in which it could rest. The observable facts on which this conception was founded are sufficiently obvious: owing to various resistances which bodies interpose to one another's motions, no actual case of motion, on the earth's surface, at any rate, is perpetual. Accordingly, from the traditional standpoint, even to maintain a body in motion with constant velocity requires the exercise of force, which failing, the body must soon come to rest. Circular motion being the natural motion for the stars and planets, incessant play of divine energy was therefore supposedly needed to keep these bodies going. But Galileo's experiments implied that it is just as easy, so to speak, for a body to move with constant velocity as to remain at rest. The application of force to a body produces a positive or negative

acceleration, not a uniform velocity. In the absence of external force a resting body remains at rest; a moving body continues to move at the same speed and in the same direction. And inertia is just that property of a body in virtue of which it acts in this way. It is revealed in the resistance of a body at rest to being set in motion, and likewise in the resistance of a body in motion to being brought to rest.

Consider, for example, a body rolling down an inclined plane *AB*, from the level *AF* to the level *BH*. With the velocity which it has attained at *B* let it be projected up another inclined plane, say *BC*, *BD*, *BE*, or *BF*. What will be its motion

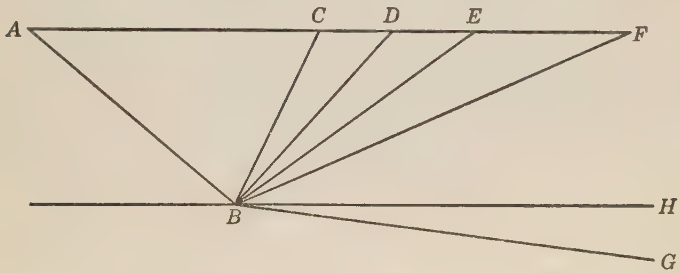

Fig. 16. Inertia and Uniform Velocity.

now? Obviously, it will be gradually slowing down, *i.e.*, it will show a negative acceleration. What, again, will happen if, instead of ascending one of these planes, the body continues downward at *B*, though on a less precipitous slope than *AB*, such as *BG*? Clearly its positive acceleration will continue, though the rate of acceleration will be smaller than before. Where lies the limit between retardation and acceleration? Obviously, the level *BH*, parallel to *AF*. And what will be the behavior of the body if it continue its motion on this level? It will be neither retarded nor accelerated; it will continue to move in the direction *BH* with the velocity reached at *B*, as long as some other force does not interfere. Apart from the activity of other forces, its motion could not cease, nor could the velocity change. Conditions make it impossible directly to verify this, but many experiments verify it indirectly. It can be shown, for example, that the more completely friction and

the resistance of the air are eliminated, the more nearly does motion on a horizontal plane approximate the uniform velocity affirmed by this principle.

Thus the traditional conception of motion as requiring the exercise of force for its maintenance is replaced by the conception that uniform velocity is just as natural an expression of the inertia of bodies as rest.

Galileo points out matters of common-sense observation illustrating the basic principle of inertia, which had not been adequately explained by the traditional theories about motion. There is the case of the rider thrown over the head of his horse by the latter's sudden stop. There are the bodies in the cabin of a moving ship, which have no tendency to lose the motion imparted to them by the ship, but when they fall, do so exactly as if the cabin were at rest. We see here, he pointed out, that while falling, these bodies retain the forward motion of the ship and combine it as an independent factor with the downward motion imparted by gravity.

Application of this concept to planetary theory

What was the bearing of this discovery on astronomical problems? Well, some of the implications of the principle of inertia helped greatly, even in Galileo's first formulation of it, in answering certain objections to the Copernican astronomy; some, on the other hand, led to the posing of new problems about the celestial motions which were not answered till Newton united terrestrial dynamics with astronomy in a single comprehensive science of matter in motion.

Under the first head, the behavior of objects near the surface of the earth was no longer a source of difficulty to the view that the earth is in rapid motion. Why does not a stone dropped from the top of a tower, for example, fall considerably to the west in its descent, if the earth's surface is moving rapidly toward the east? And why are not the clouds, the atmosphere, flying birds, and loose objects on the earth's surface left rapidly behind in a westerly direction as the earth moves? In terms of Galileo's new principle these facts were easily explained. Such objects share the rotational velocity of the earth's surface, and retain it by their inertia; it would

require additional force to push them behind, not to make them behave as they do. Phenomena familiar to everyone were thus shown to be as consistent with the theory of the earth's motion as with the theory that it remains at rest.

Under the second head, the problem now obviously took an entirely new form as to the actual forces responsible for the motion of the planets. The old idea that their natural motion is circular or, as Kepler had taught, elliptical, no longer satisfied. If the general principles of dynamics apply to them, their motion, however originally started, ought to continue at uniform velocity in a straight line, not in an ellipse. How they started to move is almost beyond investigation, but what force holds them in an elliptical orbit? It was not long, of course, before thinkers began to speculate in this connection with Kepler's notion of an attraction emanating from the sun, not now to sweep the planets around because of the sun's rotation, as he supposed, but to restrain them in an orbit instead of permitting them to fly off at a tangent. Here they had found a most promising clue. Further investigation along several lines was required, however, before this problem could be put in a form susceptible of precise solution.

Huygens' theory of uniform circular motion

The significance of Huygens, whose most important results were published in 1673, for the main sequence of thought we are now pursuing, lies in his deductive analysis of Galileo's laws of acceleration in such a way as to show their application to cases of uniform circular motion. He saw that a body moving uniformly in a circle must be deflected from its naturally straight path by some constant force pulling it toward the center of the circle. Otherwise it would slide off at a tangent. Accordingly, such a body must be moving with constant acceleration toward the center, as well as with a constant speed along a tangent to the circle; its observed motion is the net result of combining these two motions. What law expresses the acceleration revealed in such uniform circular motion, *i.e.*, in what functional relation does it stand to the velocity and to the radius of the circle? Such were Huygens' questions; their significance for the astronomical problem just

mentioned is evident. We shall not describe the course of his investigations in detail, but simply state their outcome. By deduction from Galileo's laws and established geometrical knowledge, Huygens proved that *a body moving with uniform velocity in a circle has an acceleration toward the center, directly proportional to the square of the velocity and inversely proportional to the circle's radius.* In symbols, $a = \frac{v^2}{r}$. This law may be helpfully illustrated by its application to a particular case. Imagine a body moving in a circle whose radius is five feet, at a velocity of fifteen feet per second. Then the acceleration of such a body toward the center of the circle is equal to $\frac{v^2}{r}$, or $\frac{15 \times 15}{5} = 45$ feet per second per second. The general truth put in exact numerical form in this law is familiar to anyone who has whirled a sling at varying speeds and with varying lengths of the cord. The faster the speed, the greater the pull of the sling on the hand; the longer the cord, the weaker the pull if the sling is whirling at about the same linear speed.

Another necessary line of investigation concerned the exact size of the earth. If we are to suppose, as Newton came to do, that the earth's gravitative attraction reaches as far as the moon and is the force holding that satellite in its orbit, we must know the radius of the earth and the distance of the moon, in order to verify these hypotheses by Huygens' formula—that is, to be able to compare the acceleration due to gravity at the earth's surface with the acceleration due to the same cause at the distance of the moon. Fortunately, the vastly more accurate instruments of observation available in Newton's day, allowing minute differences in the positions of the celestial bodies to be noted when observations at the same time were taken at different points on the earth's surface, made possible a much more exact measurement than formerly of the length of a degree of the earth's circumference, $\frac{1}{360}$ of the whole. By 1671, a result only a few yards in error

according to present figures had been reached. And from the circumference the radius could readily be determined.

The challenge to Newton's genius

To prepare the way for a full appreciation of Newton's achievement, let us state with due care the main problem that he inherited. It was now widely believed that all motions in physical nature, including astronomical and terrestrial motions alike, constitute a single unified system of mathematical law. On this assumption, once the implications of the new achievements in dynamics had penetrated the minds of thinkers, it had been more and more realized that the real puzzle about the orbits of the planets and of their satellites was not to explain how they are pushed along in their paths—once their motion had started that was explained by the property of inertia—but how to account for the acceleration confining them in their orbits, without which they would fly off at a tangent. Now the force producing this acceleration acts in the direction of the center of the orbit, which is very near the position of the sun in the case of the planets, and the center of the earth in the case of the moon. Some constant influence draws the body toward that center, sufficient to counteract its tendency, in virtue of its inertia, to move off at a tangent to its orbit. In other words, to explain such a motion, and explain it completely in terms of the new dynamics, it is necessary to show that the body has, in addition to its original velocity, however secured, a definite acceleration toward the center of the orbit. Viewed as a purely general question in the theory of dynamics, apart from any astronomical application, the problem is that of Huygens discussed above: Under what conditions can a body revolve with uniform velocity in a circle? Can his results be applied successfully to the revolutions of the moon and the planets?

Newton's genius was equal to this challenge. In him were found all the scientific virtues that have been noted in his great predecessors, and he possessed a boldness of speculative imagination and a clarity of mathematical analysis in which he probably surpassed them all. Born in 1642, he published

the famous *Mathematical Principles of Natural Philosophy,* in which the laws of motion and the theory of universal gravitation are formulated, in 1687.

As with many great geniuses, the clues destined to prove fruitful in all his important lines of achievement came to Newton fairly early in life. A picture of his precocious activity in the field of mathematical discovery, written long afterward, is summed up in the following paragraph by himself:

> In the beginning of the year 1665 I found the method of approximating Series and the Rule for reducing the dignity of any Binomial into such a series. The same year in May I found the method of tangents of Gregory and Slusius, and in November had the direct method of Fluxions [Newton's form of the infinitesimal calculus], and the next year in January had the Theory of Colours, and in May following I had entrance into the inverse method of Fluxions. And the same year I began to think of gravity extending to the orb of the Moon, and having found out how to estimate the force with which [a] globe revolving within a sphere presses the surface of the sphere, from Kepler's Rule of the periodical times of the Planets being in a sesquialterate proportion of their distances from the centers of their orbs, I deduced that the forces which keep the Planets in their orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve; and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the earth, and found them answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded Mathematicks and Philosophy more than any time since.[11]

His bold hypothesis

Newton saw clearly the major problem challenging solution, and the specific point at which to attack it. Here are the planets revolving in orbits not very different from circles, and not far from the center of these orbits is the sun; in a similar manner the moon circles around the earth. If we suppose that the acceleration toward the center evident in these motions is caused by some influence emanating from the central body, it becomes natural to guess that this influence may be identical with the gravitative force which is evident on its surface. This force we are familiar with—it extends upward to the boughs of the apple trees, to the tops of mountains, and as high as

[11] Quoted in Berry, p. 212 f.

projectiles can be hurled from a mountaintop—does it extend as far as the moon? And is it, perhaps, just the force providing the centripetal acceleration needed to maintain the moon in its orbit? Such was the daring hypothesis.

To test this idea it was first necessary to find an answer to the question: Supposing that this is the case, in what functional relation would the centripetal acceleration of uniformly revolving bodies revealing such velocities as the planets do, stand to their distances from the sun? To reach an answer to this question, Newton found that Kepler's third law, together with the principles of dynamics involved in Huygens' deductions, supplied all the help needed. By mathematical calculation he proved that the acceleration of the different planets toward the sun, given the actual velocity of each, must vary inversely with the square of their distances from the sun. Hence this acceleration can be explained as due to the gravitative attraction of the sun if that attraction itself can be shown to vary according to the same law.

Consider the attraction of the sun on the earth and on Jupiter as an illustration. In round numbers the distance of Jupiter from the sun is five times the distance of the earth. Hence if the sun's gravitative force varies inversely according to the square of the distance, its attraction on Jupiter, and consequently the latter's acceleration toward it, will be $\frac{1}{5 \times 5}$ or $\frac{1}{25}$ the corresponding attraction and acceleration in the case of the earth.

Now how was Newton to test this grand hypothesis by which he hoped to unite terrestrial dynamics and planetary astronomy under a single law of gravitative attraction? In the case of the sun and the planets, not all the needed data were available. But here, fortunately, is the moon, whose revolution around the earth presumably exemplifies the same laws as are obeyed in the planetary motions. The acceleration of gravity at the surface of the earth was known, likewise the radius of the earth, and the distance of the moon had been determined pretty accurately as sixty times the radius

of the earth. These were all the data necessary to solve the problem, assuming that the force holding the moon in her orbit is the earth's gravity, and assuming also that the center of the earth is the center of her gravitative attraction both on the moon and on objects near the earth's surface.

Approximate verification of the hypothesis

The essence of Newton's solution is as follows: The moon's distance is sixty times the radius of the earth, and since the circumference of a circle is proportional to the radius the moon must travel in each of her revolutions sixty times as far as a point on the equator of the earth travels in the course of the latter's daily rotation. Now the distance around the earth at the equator is roughly 24,000 miles, and, as everyone knows, the moon completes a revolution in about twenty-seven days. The moon therefore moves about $60 \times 24{,}000$ miles in twenty-seven days, which indicates a velocity in her orbit of about 3,300 feet per second.[12] According to Huygens' formula, therefore, the moon's acceleration toward the earth is proportional to the square of this velocity, divided by the distance between the two bodies. Reducing the earth's radius from miles to feet, where the figure becomes about 20,000,000, and remembering that the moon's distance is sixty times this figure, the moon's acceleration toward the earth is then proportional to the fraction $\frac{3300 \times 3300}{20{,}000{,}000 \times 60}$. This works out to a fraction of approximately $\frac{1}{110}$. Now how ought this fraction to compare with the acceleration of falling bodies on the earth's surface? By Newton's law of the inverse square, the acceleration of a falling body at the surface of the earth, sixty times nearer than the moon, ought to be equal to the square of sixty times this fraction. But $(60)^2 \times \frac{1}{110} = 32.7$. Since the known acceleration on the surface of the earth due to gravity is slightly over thirty-two feet per second squared, the hypothesis is verified as nearly as could be expected in the absence of more exact figures. The moon is thus shown to be essentially a falling

[12] It is desirable here to express distance in terms of feet and time in terms of seconds, to permit comparison of the moon's acceleration with acceleration due to gravity at the earth's surface, which is ordinarily expressed in terms of so many feet per second squared.

body, and its motion around the earth is an instance of the same law that governs the terrestrial motions which Galileo had studied.

Popular illustration of its truth

Newton illustrates this great discovery by drawing an analogy between the motion of the moon and that of a projectile hurled horizontally from some point on the surface of the earth, such as the top of a high mountain. Let a bullet be fired from *A* (see Fig. 17); then, under the joint influence of

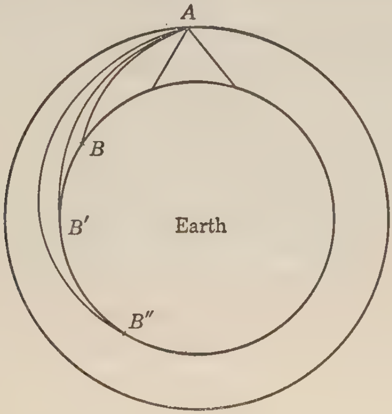

FIG. 17. The Moon as a Falling Body.

its original velocity and of the earth's attraction, it will describe a parabolic curve, reaching the ground at some point such as *B*. If it were shot with greater velocity the curve of its motion would reach farther, to *B′*, before meeting the ground, for the time required to drop to the ground is the same whatever its horizontal velocity. If the velocity were greater still, it would not reach the ground till it had passed around to *B″*. Now with a little effort of imagination it can be seen that, if its original velocity were just great enough, and if the resistance of the atmosphere were not present, the bullet would miss the earth altogether and describe a circle around it, returning to the same place from which it was shot. Its velocity, in other words, is just great enough so that the gravitative attraction of the earth supplies the centripetal force needed for circular motion to take place. This is what

happens with the moon, only this body is sixty times as far away from the center of the earth as the hypothetical bullet, and its original motion was not due to a gun; but once in motion with the appropriate velocity it behaves exactly as a falling body would. Every revolution is a fall toward the earth, but the velocity is so great that it falls entirely around and returns to the same point.

Generalization of the result

How, now, did Newton succeed in generalizing the result thus verified in the case of the moon's revolution around the earth, so as to apply it throughout the solar system, and ultimately to establish a universal functional law of gravitative attraction between any two bodies? For he could not rest without reaching such an all-inclusive law.

Late in the following decade he discovered that his formula for the acceleration of a body revolving in a circular orbit applied also to an elliptical orbit, provided that the acceleration were directed toward one of the foci of the ellipse, and that the motion of the body varied in velocity, being greater when nearer that focus and smaller when farther away. Calculating how much greater the velocity would have to be at the perihelion and how much smaller at the aphelion [13] for the body to remain without displacement in its elliptical orbit, he found that this condition would be met if the velocity were such that the planet vector traversed equal areas in equal times. But this is precisely what Kepler's second law affirmed of the motion of the planet Mars. In other words, he succeeded in uniting completely Kepler's laws of planetary motion with Galileo's foundations of dynamics as developed by Huygens and himself, and as already successfully applied to explain the revolution of the moon. Given Kepler's laws, it follows that each of the planets moves as it would if it had an acceleration directed toward the sun varying inversely according to the square of the distance. Conversely, given the foundations of terrestrial dynamics and the law of the inverse square, and supposing that these principles apply to the

[13] The perihelion is the part of the orbit nearest the attracting focus; the aphelion is the part at greatest distance from it.

planetary system, Kepler's laws follow, for only revolutions in accordance with them are consistent with such dynamical principles. The planets thus also seem to be falling bodies, retained in their elliptical orbits by a gravitative acceleration toward the sun, just as the moon is retained in its orbit by the gravitative attraction of the earth.

The conception of center of gravity and that of mass

Thus far we have been assuming that the center of gravitative attraction in the case of the moon's revolution is the center of the earth, and that in the case of the revolutions of the planets it is the center of the sun. But Newton's further work showed that this is not exactly true, and his discovery of a generalized law of gravitative attraction between any two bodies rests upon its correction by a more exact statement.

The moon is attracted toward the earth, and continually falls toward it; yes, but also the earth is attracted toward the moon, and falls toward it. For the moon, too, is not impotent as an attractive force. The most obvious and interesting fact showing that the moon does attract the earth in the same way that the earth attracts the moon, is the phenomenon of the tides, where the fluid masses of the sea, cohering less strongly than the solid earth, are pulled toward the moon when her course brings her opposite them. Accordingly, Newton saw that it is not quite correct to speak simply of the moon revolving around the earth; the earth revolves around the moon, too. Or, rather, the two bodies constitute a system revolving around some point between them which is their common *center of mass*. Similarly, as applied to the solar system as a whole, this means that the sun is attracted toward each of the planets just as the latter are attracted toward it, and that, therefore, the planetary system does not revolve around the center of the sun, but around the center of mass of the system as a whole.

How can we locate this center of mass in any such system of bodies moving under the influence of each others' gravitative attraction? To answer this question, Newton needed to give a clear and exact definition to the basic concept of "mass," which we have already found it necessary to use.

If we think simply of the moon being attracted toward the earth's center in the same way that objects on the latter's surface are, we could get along merely with the conception that each of these bodies has *weight,* which is the force with which any object tends to fall toward the center of the earth. But the fact that the earth also falls toward the moon, and the sun toward each of the planets, shows that we need a more general concept than this. Every body evidently has a property in virtue of which accelerations are produced in other bodies under specified conditions, including gravitative accelerations which vary inversely with the square of their distance. We call this property the body's *mass.*[14] It is a constant for any given body, and its quantity is determined by the acceleration produced under the influence of a given force. It became a basic concept in the theory of modern mechanics along with the concepts of distance and time. Let us see just how it is defined, and how the concept is then employed to formulate the law of gravitative attraction between any two bodies in quite general terms. At this point we shall turn from Newton's own way of dealing with these matters, which exhibits in some respects the groping state of physical science in his day, to a way of stating them that is more in line with the procedures of contemporary physics.

The general law of gravitative attraction

Two objects interact in space—Jupiter and the sun, let us say. Their accelerations toward one another are found by many experiments to be always in the ratio $\frac{A_1}{A_2} = \frac{M_2}{M_1}$, *i.e.,* the acceleration of each proves to be directly proportional to some property in the other which we are symbolizing by M. This property, by definition, is the other's mass. Now let us multiply each side of this equation by the quantity A_2M_1. The result will be the equation $M_1A_1 = M_2A_2$. That is, in such a system of two interacting bodies the product of the mass and acceleration of one equals the product of the mass and acceleration of the other. The gravitative force which is re-

[14] In the case of objects on the surface of the earth, this property can be measured by their weight.

vealed in the acceleration of any mass under such conditions is defined as equal to this product; in other words, $F = M_1A_1 = M_2A_2$, again, by definition. Let us consider now what is involved in these definitions. Think of one of the masses by itself for a moment, M_1, for example. The force revealed in its motion toward the other body $= M_1A_1$. But the acceleration A_1 is, as we saw, directly proportional to the mass of the other body, M_2, and inversely proportional to its distance—that is, $A_1 \propto \frac{M_2}{d^2}$, or $A_1 = \frac{M_2}{d^2}$, multiplied by some proportionality constant K. We may, then, substitute the quantity $\frac{KM_2}{d^2}$ for A_1 in the equation $F = M_1A_1$, and we then have

$$F = \frac{KM_1M_2}{d^2}$$

That is, on the basis of these definitions, and the manifold experiments which underlie them, *the force expressed in the acceleration of any body interacting with another body in virtue of their gravitative attraction, is directly proportional to the product of the masses of the two bodies, and inversely proportional to the square of the distance between them.*

Such a functional law frees the conception of gravitative attraction from all limitations, and expresses it in universal form, as a law of behavior revealed throughout the entire material world and explaining systematically certain major facts of the sciences of astronomy and mechanics. Its discovery was the supreme achievement of Newton's genius. Thus a splendid chapter in the history of scientific thinking reached its culmination. The most familiar motions of the objects with which men deal on the surface of the globe, together with the revolutions of the celestial bodies, were now shown all to follow from a simple, comprehensive, functional law. No wonder that Locke, England's greatest philosopher before the days of Hume, designated himself, beside the "incomparable Mr. Newton, as an under-laborer, employed in clearing the ground and removing some of the rub-

bish that lies in the way to knowledge,"[15] nor that Pope wrote as a couplet for Newton's tomb in Westminster Abbey:

> Nature and Nature's laws lay hid in night;
> God said, "Let Newton be," and all was light.[16]

Transformation of Copernicus' original hypothesis

To think of the sun moving around the earth, as the pre-Copernicans did, now becomes inconsistent with the very foundations of established mechanical knowledge. But it is instructive also to observe that in this progress of scientific discovery the original hypothesis of Copernicus becomes itself transformed. For it appears that the sun is not, after all, the center of the solar system—to say nothing of the universe as a whole—although it is very much nearer that point than the earth. An entirely new conception has appeared—that of the center of mass of a system of bodies moving under the influence of their gravitative attraction—and it is around that center that the solar system revolves. Such a transformation of an original leading hypothesis by the facts which it helps to explain and by the new conceptions to which it gives rise, is exemplified in every important chapter in the history of science. This is of especial interest at present in view of the fact that some of Newton's major beliefs have now been proved antiquated, and are in course of replacement by more adequate notions.

EXERCISES

1. List the distinctive features of scientific method thus far discussed. Exemplify each of them in the work of Galileo.
2. In what sense did Newton prove that his explanation of the moon's motion is the *only* valid explanation?
3. Read the chapter on Descartes in Oliver Lodge, *Pioneers of Science*. Why did science reject his theory of vortices?
4. In what respects did the Pythagorean cosmology aid in the development of modern astronomy and mechanics? In what respects did it hinder that development?

[15] *Essay Concerning Human Understanding*, Epistle to the Reader.
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CHAPTER 19

CRITERIA OF EVIDENCE—THE ORGANIZATION OF SCIENTIFIC KNOWLEDGE

With this varied material from the history of science at hand, a question may be raised that would earlier have been premature.

What constitutes scientific evidence?

Our attention has already been centered on the principles under whose guidance the scientist proceeds to verify a hypothesized law for the explanation of some puzzling occurrence. Now it was noted earlier that verification is always an appeal to evidence; it consists in the examination of what the investigator takes to be evidence capable of deciding whether the hypothesis is properly substantiated or fails of adequate proof. In considering those principles our main question was: When is the evidence sufficient to warrant the assertion of a causal, or a functional, law connecting the occurrence to be explained with some other phenomenon?

But a question at least equally important, especially when we remember the halting historical process by which science has gradually displaced superstition, is: What *constitutes* evidence when one is dealing with matters of fact? When a thinker is occupied with problems of valid inference, as the preceding Part has shown, appropriate substantiation of the correctness of an inference always takes the form of exhibiting a pattern of strict implication which, examination shows, the inference ought to respect and with which, therefore, it is to be compared. In factual matters, however, evidence is obviously something different. Just what sort of thing is it?

Let us consider a situation which will show the significance of this question. When Galileo, having turned his newly in-

vented telescope upon the moon, announced that its surface was highly irregular, containing mountains several miles high, believers in the traditional doctrine that the celestial realm was geometrically perfect were greatly disturbed. One of his contemporaries, as was noted in Chapter 6, wished to save the imperiled view by maintaining that these irregularities are filled with a transparent glassy substance, so that the surface is really smooth after all. Galileo retorted that so brilliant an idea ought to be extended; he would accordingly assume that the moon also boasts mountains of this same transparent material, ten times as high as any he had observed. To what would one appeal in trying to find a basis for deciding between such competing assertions as these?

Clearly, such questions can only be answered in terms of some conception of what factual evidence essentially is. And certain ideas on this fundamental matter have, in the course of man's intellectual history, become generally accepted among reputable thinkers. They are now so firmly embedded in the structure of scientific method that many investigators are unconscious of their presence and importance. To distinguish them from the Postulates of Factual Science and the Principles of Verification we shall refer to them by an equally brief descriptive phrase. Factual science employs three major Criteria [1] of Evidence, two of which provide the court of appeal when one is establishing specific laws, and the third when one is organizing such laws in a more inclusive system. What are these criteria? Well, let us begin with the former pair.

The criterion of empiricism

The first is the criterion of *empiricism*. By this is meant that proper evidence for the truth of any hypothesis about factual matters must consist in something directly perceived, either by one or more of the senses—sight, touch, hearing, taste, or smell—or in some other mode of direct perception. Whether a suggested explanation can be taken seriously or not as a candidate for verification depends primarily upon

[1] A criterion being a standard by which one decides between competing alternatives.

whether or not it appeals to entities that can be observed in this way.

That some persuasive theory implies it constitutes no proof. That my previous prejudice has been in favor of its truth, or the authority of some tradition vouches for it, or present convention around me takes it for granted, may give it plausibility and even considerable likelihood, but none of these can supply a decisive test. They may differ among themselves, and all of them may be mistaken when tested by the perceived facts. This situation is precisely what is contemplated by the popular adjectives applied to facts thus disclosed—they are cold, hard, implacable, by which we respectfully indicate their power to disappoint our expectations and wishes, our traditions and authorities, and to compel acquiescence in them despite their harshness. A true explanation must square with the relevant events which observation discovers. Only in the guise of such events does the needed evidence appear.

If one is in doubt as to the essential appropriateness of this criterion, let him consider the illustration just cited. Galileo turns his telescope upon the moon, and sees something he did not expect to see; it also betrays no mercy for metaphysical and religious prejudices built up through long ages, nor for the authoritative astronomical tradition. He sees that the surface of the moon, instead of revealing a smooth and regular curve as accepted theory about the purity of the celestial bodies required, is covered with the most monstrous irregularities, far larger in proportion to its size than those on the surface of the earth. Shall one evade the natural conclusion by suggesting that these irregularities are filled with a transparent glassy substance? Galileo's retort to this supposition enforces the obvious necessity of empiricism as a criterion of evidence. If such a hypothesis is permissible, it is equally permissible to maintain that there are huge mountains of this same substance on the moon. The transparent material, by its very nature, cannot be seen; its presence cannot therefore be verified, and it may not be properly invoked, either in defense of the ancient belief against the new observations or in exten-

sion of the latter at the observer's whim. Galileo was right in believing what he saw, and because the observed fact was there, in the minds of those who came after him the ancient theory gradually faded, and men's beliefs about the heavens were accommodated to the perceived reality. As long as our knowledge purports to apply to the factual world and to give us dependable guidance in dealing with its phenomena, there seems to be no tenable alternative to the criterion of empiricism. In the last analysis a fact is something perceived or is securely grounded in something perceived.

Apparent exceptions to its employment

Scientific explanations do on occasion, however, appeal to things that cannot be directly observed. The astronomer talks about another side of the moon than the side inhabitants of the earth are able to see, and the physicist refers to the force of gravity and to minute entities called electrons, neither of which, strictly speaking, is capable of being observed. What do such circumstances prove? Well, in each of these cases there is an answer to the charge of inconsistency, which shows that when the scientist is confident of the existence of these things he has not abandoned empiricism.

First, there are entities which cannot be perceived under conditions at present realizable, but which could be perceived under conditions capable of definite description that might sometime be realized. The criterion of empiricism does not exclude these, but only entities conceived as intrinsically imperceptible, like the glassy substance just mentioned. Since the moon, except for its irregularities, is a spherical object, its supposed other side is continuous with the side turned toward the earth, and one could specify unambiguously the conditions under which that further side would be observed. The fact that at present no one is able to realize those conditions does not seem to be vital. By the same right I am sure that a steel bar which I hold in my hand is not just a shell but has a solid inside, even though I possess no tool capable of breaking it open and rendering that inside perceptible.

Second, such entities as gravity, electricity, evolution, etc., are intrinsically imperceptible, but for another reason than

that which applies in the case of the glassy material on the moon. They cannot be observed for the same reason that furniture cannot be observed—that is, furniture as something distinguished from the particular chairs, tables, stands, and bookcases with which we are all familiar. One perceives "pieces" of furniture; so he perceives "manifestations" of the force of gravity, namely, all those phenomena whose behavior Newton's gravitative law describes. In other words, gravity is not a particular object but a class concept under which we gather together all the varied interactions in physical nature which exemplify a certain law.

Third, the electron, for most scientists who appeal to it, is in a somewhat different case, although some are endeavoring to find a way in which it can be treated successfully as a purely mathematical concept. Others usually regard it as a particle carrying a unit charge which, since it occupies some position and possesses a velocity, could be observed if appropriate instruments that are delicate enough were at our disposal. In this case the instruments would not only have to possess high microscopic power but would also have to illuminate the electron to be observed without displacing it. At present, physicists cannot conceive any way in which this might be done. But they can assign a number of definite properties to this unperceivable entity, these properties being required to account for quite a variety of unquestioned facts of observation, in accordance with laws amply confirmed elsewhere. The word "electron" then means for them the inferred physical reality whose position and velocity, together with these other properties, provide a coherent explanation of an important group of observed facts. From this point of view its status becomes that of a scientific theory justified by the criterion of simplicity, which is soon to be discussed. If the objection is made: But perhaps some other explanation of these facts might be equally or more tenable, the physicist will reply, "Produce it. And if you can show that it offers a better systematic account of all the relevant observations than the electron theory does, we will accept it and help you develop and test it further."

The invisible substance of Galileo's contemporary cannot be justified in any of these ways. It is obviously proposed merely to buttress a traditional belief which newly observed facts prove to be mistaken. Nothing further can be said in its support.

The second criterion is that of *objectivity*. By this is meant that the perceived facts appealed to in verification of a suggested law must be such as can be experienced in the same way by any observer with normal sense organs and a normal nervous system. If evidence taken as confirming an explanation does not present itself as such under appropriate conditions to other competent investigators, the evidence must be held dubious and the verification incomplete.

The criterion of objectivity

Should emphasis on such a criterion be thought superfluous, it will be well to glance at a few common situations which enforce its importance. It has been historically a long and difficult lesson for people to learn how to distinguish clearly between objective and subjective experiences, and in the last analysis appeal must be made to cooperative verification to tell the one kind from the other. The most prominent group of observations which lack objectivity is that indicated by the terms "dream," "illusion," and "hallucination." These we generally discover to be subjective without appeal to other persons because they lack constancy and dependability even for our own experience. Another group is suggested by the words "dragon," "sea serpent," "fairy," "ghost," "devil." In sophisticated communities these creatures are no longer likely to be observed, but it was not easy for men to discover their subjective character. Given appropriate prevalent beliefs and accompanying emotional attitudes, many people had repeated and persistent experiences which they interpreted as indicating the presence of such beings; the latter seemed as real as any familiar object in nature. Many communities, indeed, still appeal to such entities in explaining certain puzzling events.

As a third illustration, a group of experiences may be mentioned whose subjective character civilized people have readily learned. This is composed of our emotions, and those

spontaneous likes and dislikes that vary greatly from individual to individual and in the same individual at different times. Consideration of these cases enforces an important point which must be made if confusion about objectivity is to be avoided.

Obviously, emotions can be studied objectively—and this is one of the tasks assumed by the scientific psychologist—provided that their status as internal occurrences, likely to vary as such from one person to another, be recognized and universally agreed upon by those who study them. Confusion and misunderstanding would only arise if they were supposed to be external objects comparable to rocks and houses, or properties of such objects. In that case, the more they were invoked as evidence the more would disagreement between different investigators appear, since in many situations no two men would be likely to feel exactly the same emotion in relation to the same object. But if they are admitted to be relative to the individual who feels them, and are analyzed and classified on this basis, certain truths about them can be verified just as can be done in the case of an external event which changes in accordance with varying conditions. This circumstance shows that a similar conclusion should be drawn about dreams and illusions; likewise about fairies, dragons, and the rest. If such things are regarded as relative to some individual or group—that is, are supposed to constitute, as we say, "imaginary" entities—then they can be studied without confusion, and certain objective truths established about them. For fairly dependable psychological conjunctions between them and other experiences are capable of being discovered, *e.g.*, between certain kinds of dreams and certain family situations of early childhood. Only if this imaginary status is not recognized, and they are assumed to exist externally like physical things, do they lack the objectivity requisite in phenomena which are to be used as evidence for scientific conclusions. Successful study of any event must place it properly in its context of relations with other events. This principle must always be kept in mind.

Is the method of introspection objective?

These considerations suggest a more general question of some importance. Does acceptable evidence include the facts revealed by introspection,[2] or are such facts intrinsically so lacking in objectivity that they should be flatly denied evidential status? Well, the latter would of course be the case if "objective" had been defined to mean "external," *i.e.*, capable of being perceived by one or more of the bodily senses through which we become acquainted with things around us. And it is true that agreement can be most readily secured in the case of such facts; we can unambiguously point them out to each other, we can study their conjunctions together, we can measure them by instruments familiar to all investigators, etc. By contrast, inquiry into the things discovered by introspecting what goes on in our own minds lacks these virtues, and it is much more difficult to reach cooperatively demonstrable propositions about them. It is largely for these reasons that some influential schools of psychologists have felt happier when they believed it possible to restrict their field to the physiological accompaniments of mental happenings, or to other kinds of externally observable behavior. It will be noted that several of the phenomena just mentioned as illustrations of subjective occurrences are phenomena that must be discovered largely, if not exclusively, through introspection.

In view of the above analysis, however, there seems no justification for going to this extreme. For as long as introspectible facts are recognized as in various ways relative to the individual in whom they occur, many of their properties and conjunctions can be established in a form on which different investigators can agree; otherwise introspective psychology would never have satisfied any serious scientific thinker.

The more general situation which it exemplifies with unusual vividness is that there are cases in which a relevant fact can be directly observed by only one person. But this circumstance does not, in itself, mean that his testimony must be

[2] Introspection is the process of inward observation, by which one discovers what is going on in his own mind, *e.g.*, that he has a certain emotion, that it is succeeded by a certain desire, that the latter is accompanied by certain beliefs, etc.

rejected by others. Historical events have sometimes been witnessed by only one observer, *e.g.*, the passage of a certain flock of birds over a given point on the earth's surface at a given time, but if the testimony of that observer conforms to the canons that experience has shown to be applicable for its evaluation, it may justifiably be accepted by others as true. Such canons are discoverable in introspective psychology. All persons can observe what goes on in their minds in the varied interconnections which it exhibits; common patterns can be found which each can verify in his own experience, illustrated by the laws of association discussed in Chapter 4. By the aid of these patterns, principles for evaluating testimony based on introspection can be formulated and systematically employed. So far, then, as that is the case, introspective facts can provide acceptable evidence; introspection may be admitted as an empirical and objective procedure even though it reveals significant differences from perception through one of the sense organs. To be sure, its handicaps are very real and must not be blinked. They constitute one important reason why the sciences which try to explain human activities have thus far appeared much less adequately grounded than the physical sciences, so far at least as they have attempted to deal with the most challenging problems in their field. But in whatever fashion and to whatever extent results can be established that competent investigators agree upon, the criterion of objectivity is satisfied.

The essence of objectivity, then, as demanded in the evidence acceptable to factual science, is the capacity to exhibit the same status and conjunctive relationships under critical observation by any normal person who understands the problem under investigation. No proposition can be considered completely and satisfactorily verified as long as the facts to which it appeals cannot be so presented that they reveal this character. To assume otherwise is to assume that one's own perception and judgment, or those of some limited group sharing the same peculiarities, are more trustworthy than the consensus of perception and judgment in all competent think-

ers. The mere statement of this alternative is sufficient to reveal the essential place in verification of the criterion of objectivity.

The manner in which the criteria of empiricism and objectivity interact in the history of science may be briefly suggested by referring again to Galileo's observations on the moon. By the criterion of empiricism Galileo believed what he saw, even against the universal conviction of his time. But by the criterion of objectivity, unless what he saw proved just as capable of being seen in the same way by all competent seekers for truth, those who came after him would not have held belief in it to be verified.

A great modern scientist, Sir William Herschel, has summarized in a single phrase these two criteria: "The grand and indeed the only character of truth is its capability of enduring the test of *universal experience*, and coming unchanged out of every possible form of fair discussion." [3]

The problem of organizing scientific knowledge

In discussing these two criteria for the determination of what is relevant evidence for an explanatory hypothesis, we have been thinking of specific laws proposed for the explanation of this or that particular phenomenon. The third major criterion appears when we note also that one of the human needs expressed in the search for true explanations is the need for a systematic understanding of the world, that is, a need not only to know what cause universally produces a given effect, but also to know how that causal relation stands to other regular conjunctions—what broad, underlying laws are exemplified in all of them. We want to know not only that A is conjoined with B, and E with F, but also, if possible, a more inclusive generalization from which both of these specific laws can be deduced. Such unification of knowledge about the world has just been exemplified on an imposing scale in our study of the work of Sir Isaac Newton. Laws comparable to his general law of gravitative attraction not only satisfy curiosity in a way that nothing less quite does, but they are of practical and emotional value as well.

[3] *Discourse on Natural Philosophy*, p. 10. Italics ours.

The main reason for their having this further value is that the inclusive law, when discovered, expands the range of our intellectual grasp, and gives an increased sense of security to our trust in any particular law which becomes one of its corollaries. Then the latter does not, so to speak, stand merely on its own feet; behind it is the warrant, not only of the data on which it directly rests, but also of the facts explained by all the other special laws which, together with it, are embraced by the general law. Much more, in other words, in which we have scientific confidence must also be dubious if it fails to hold. This point will become clear on considering, for example, the difference, in our understanding of the physical world, between knowing a number of specific conjunctions, such as that unsupported objects fall to the ground, that a projectile follows a parabolic curve, that the planets move in elliptical orbits, etc., and knowing an inclusive generalization such as the law of gravitation, from which these and many others can be deduced when the special properties of the bodies to which they refer are given.

The criterion of simplicity

The further criterion employed by the scientist comes to our attention when we ask how such inclusive laws as these, or "theories," as they are sometimes called because of their difference from specific laws, are established. They are not verified directly, as is the case with the latter, but only indirectly through the verification of the latter; their ability to imply all the specific laws which they embrace, and thus to give those laws unity, is their ultimate justification. They are a step more remote, so to speak, than the specific laws, from the particular facts they purport to explain, and this circumstance gives rise to difficulties, in determining what verification requires in their case, that do not trouble us when dealing with laws explaining this or that particular phenomenon. For example, the general theory of evolution, or that of relativity, cannot be confirmed by direct appeal to observed fact. It is possible only to verify directly the specific hypotheses which, as these theories by implication indicate, ought to hold under certain conditions.

The most important difficulty of this sort arises from the circumstance that sometimes more than one inclusive theory seems capable of covering in this fashion all the specific laws that are empirically and objectively verified within a certain range of observed fact. How is the scientist to proceed in such a case? By what guide is he to decide which of these theories to accept? Evidently, the criteria of empiricism and objectivity must be supplemented by some other guide when this situation appears.

The criterion which functions here is *simplicity*.[4] But this concept, in the role now considered, has two senses which, in the interest of clear understanding, need to be distinguished. In one sense it means economy in the number of propositions needed for a complete statement of the theory; in the other it means economy in the number of exceptions to the main principle of the theory which have to be recognized. The way in which these two aspects of simplicity are related in any given case is a matter of art; no automatic rules can give guidance at this point. These two senses of the criterion will now be explained, in each case with an illustration from scientific history.

In its first sense, the idea of simplicity was enunciated many centuries ago by William of Occam in the famous rule: "Entities [of explanation] are not to be multiplied beyond need." In other words, it tells us, when we face the situation just described: Choose the theory which renders possible the deduction of all the relevant specific laws from the smallest number of independent propositions. Any theory, when clearly stated, will consist of one proposition, or a set of propositions which are logically independent of each other. Now, other things being equal, we demand a theory which is least complex in this respect. And to justify this demand it is necessary merely to point to two considerations. The most important one is that a theory possessing simplicity in this sense reveals unifying connections between the facts explained which a more complex theory does not. The less important one is that

[4] Sometimes also called "parsimony," or "economy."

it is much more convenient to work with. Why should anyone insist on an intricate explanatory scheme when a less intricate one will do all that an explanation is asked to do just as well?

The Copernican theory in astronomy possessed no superiority to the Ptolemaic in terms of the requirements of empiricism and objectivity. But it explained the facts in terms of a much more economical geometrical pattern. By conceiving the sun as the center of the planetary system instead of the earth, a number of facts hitherto accounted for only by independent eccentrics and epicycles—for example, the annual motions of the planets farther from the sun than the earth—were shown to be deducible from the single assumption that the earth revolves in an annual orbit around the sun. Thus a very intricate scheme of explanation was reduced to a quite simple one in this sense of simplicity.

But choice between theories cannot always be decided by the principle of simplicity in this meaning alone. In the case of any theory which requires two or more independent propositions for its adequate statement, there is usually a key proposition or central principle which may vary in its comprehensiveness, *i.e.*, in the range of data which it covers. The other propositions then constitute "supplementary" hypotheses, requisite to take account of relevant facts which appear to be exceptions to that central principle. Now two competing theories may be equally simple in the sense already explained, and yet the key idea of one may be more comprehensive than the key idea of the other. In such a case the former is simpler in the second sense of simplicity, and is to be preferred. For example, there was a stage in the history of biology when many thinkers were choosing between two main theories of the way in which living creatures are generated. One theory held that most species are generated from parents of the same species, but that certain kinds of species such as worms, maggots, etc., arise spontaneously from inorganic matter. The other held that all species are generated from living parents, including those believed by the other theory to be sponta-

neously generated, but that microorganisms constitute an exception to the law.[5] Now these theories are equally simple in terms of the number of independent propositions required. Each consists of one main proposition, and a second proposition to take account of exceptional cases. But the latter is simpler in the second sense of simplicity. It allows fewer kinds of exceptions to its main principle of explanation; hence, so far as this consideration is concerned, it is to be preferred. Even if the microorganisms had to be accounted for by the aid of several different hypotheses instead of one, so that this theory is less simple in the first sense than the other, many scientists would have regarded it as preferable nonetheless. It points more encouragingly toward the completely unified explanation which is desired. This illustrates the statement that the way in which these two aspects of simplicity are combined is an affair of art; no definitive rule can be laid down as to which should be weighted more heavily than the other when they lead in different directions.

In the subsequent history of biology the simpler of these two theories was soon confirmed by experiments, and in course of time the whole problem was placed in a new light by the discovery of modes of generation which had not been known to earlier investigators. A striking analogy with the Copernican theory in both these respects thus appears; as we have seen, the latter, too, was soon confirmed as against its Ptolemaic rival by further data, and later the conception of the center of mass in a system of motions put the whole problem of explanations in astronomy in a new light.

The conception of supplementary hypothesis

The conception of supplementary hypotheses, however, needs somewhat fuller examination. Such hypotheses are required whenever there are facts which seem to be relevant but do not conform to the main idea which lies at the heart of an otherwise satisfactory theory. They are strictly a part of the theory because of the relevance of those exceptional facts. But the scientist thinks of them as "supplementary" because they are required simply for completeness' sake, and he will

[5] This was the situation in Europe in the late seventeenth century.

hope that the need to appeal to them will prove to be merely temporary. Further investigation, he trusts, will succeed in showing either that they do not constitute exceptions after all, or that they occur under conditions which the main theory should not be expected to cover. The logical problem met by such hypotheses may be illustrated in a familiar situation of daily life. John, Paul, and Henry fell in the brook yesterday, and had a long walk home in their wet clothes. Henry and Paul caught cold. We explain the cold by yesterday's accident. But why, then, did John escape a cold? Obviously, some answer to this question must be provided if we are to hold (and no alternative seems available) to our explanation of what happened to Paul and Henry. The answer would, of course, depend on what relevant facts were discovered. It might be that John has a more vigorous constitution than the others; appeal, again, might be made to the circumstance that he had been swimming in the pool frequently and was used to the water, or that he took better care of his condition on reaching home. Such an answer, proffered to take account of John's case, would be a supplementary hypothesis.[6]

None of the most outstanding achievements in the history of science can be understood without recognizing the essential place which the demand for system occupies in scientific thinking. The world, as science conceives it, is always the best integrated order that can be established consistently with the known facts. To explain a particular phenomenon is to show a dependable conjunction between it and another; to explain a group of related conjunctions is to derive them from a theory realizing economy and unity in the highest degree possible.

When simplicity may override empiricism

As the above discussion has implied, in general the principle of simplicity is to be regarded as subordinate to the criteria of empiricism and objectivity whenever they conflict. Science would run into many vagaries if it were permitted to override perceived facts, cooperatively confirmed, in the in-

[6] Supplementary hypotheses which cannot command the justification here illustrated, and whose only excuse lies in the fact that they offer plausible protection to a theory in the presence of embarrassing exceptions, are sometimes called "*ad hoc* hypotheses."

terest of upholding a theory merely because the theory is a simple one. In that case, Kepler, for instance, would have been foolish to allow himself any concern because of the facts which were irreconcilable with the supposition that the planets move in the simplest curve, a circle. But under certain special conditions the principle of simplicity may properly outweigh what an objective empiricism alone would lead us to accept. Consider the most important of such conditions.

When an astronomer records his observations of a planet's revolution, they never conform exactly to an elliptical curve, even when perturbations arising from the attraction of other planets, etc., are taken into account. They do, however, cluster more or less closely around such a curve. Shall he maintain, as empiricism alone would require, that the planet actually moves in this highly irregular orbit? No, because if he repeats the observations the two sets never quite coincide. Shall he appeal to the criterion of objectivity? Well, if someone else makes his own observations, even at the same time, the two sets never coincide either. What shall he do? The plausible answer, itself supported by objective empirical considerations, is: Ascribe such irregularities to defects in the instruments used, to varying conditions in the intervening medium, or to a never fully eliminated personal equation in the observer. If that is done it seems clearly permissible to attribute to the planet the simplest curve toward which the various sets of observations converge, even though perception, whether employed by one investigator or by many, never confirms the law thus asserted. At any rate, no alternative to this procedure appears rationally justifiable.

Further problems concerning the unification of knowledge

Consideration of scientific theories, however, whose role is to bind together separate laws in a unified system, raises further important questions, which may best be considered at this point. One is: How far, at any given time, may such explanatory unification extend? Newton's law of gravitation illustrates the circumstance that a wide range of facts in a single branch, or even more than one branch, of science may

be coherently organized under a happily conceived theory. Are there any traces of a still broader unification in the scientist's treatment of his facts? And if there are, another question arises: What is the logical status of the concepts in terms of which such unification is achieved?

The answer to the first of these questions, amply illustrated in the preceding chapter, is that very definite traces of this sort are always at hand. For there is nothing in the nature of human curiosity, expressing itself in the quest for inclusive explanations, that makes it satisfied with any limited degree of systematization; the most complete integration that proves possible, the better. Hence the scientific mind, in those individuals especially preoccupied with questions of theory, continually reaches out for more comprehensive systems than any already established, and even hopes for a theory capable of binding together harmoniously all the laws verified and facts accumulated by the various branches of science, leaving no refractory data of any sort outside. By way of preparing ourselves for an answer to the second question, let us see how this search for a wider organization of factual knowledge actually proceeds. And here science and philosophy join hands.

What happens is somewhat as follows. At any given time in the history of science, some theories have been established in certain of its branches, realizing in those fields a limited degree of systematization. They exhibit some measure of success in achieving the scientist's goal of unified understanding. Such success suggests that a more inclusive unity of the same sort may be possible—perhaps that one of the key concepts, which have thus demonstrated their organizing power in a limited way, provides a clue to the structure of nature as a whole. Certain philosophic minds, enticed by the possibilities thus suggested, take over this selected idea, either with convinced faith in its success or as a guiding general hypothesis, and proceed to sketch the picture of the cosmos which would issue from its systematic elaboration and detailed application. They show what the whole universe would look like were this the

key by which to unlock its secrets. As a result of their work, investigators in other sciences are moved to see what they can do with the same idea in meeting their own puzzles. If considerable success is achieved at their hands too, a rather challenging conception has emerged as to how nature is ordered—a conception of some fundamental pattern in which all or a large part of the totality of facts harmoniously find their place. The doctrine thus arising may attain widespread influence over the inquiries of several generations, or even centuries, before it has shown how far it is capable of effectively organizing the masses of accumulated data into a unified whole and is superseded (or at least relegated to a severely limited field) by some other and now more promising notion.

How one cosmic theory succeeds another

When a change from one such cosmic theory to another comes about, it is almost always supported by two factors. One is the steady acquisition of facts which prove persistently recalcitrant to the previously dominant view. The other is the emergence as a social power of a different human interest, which leads those who share it to seek systematic understanding in another direction than that which the ruling theory has pursued. At the time of Kepler and Galileo, both these factors can be easily detected. Careful investigation had, on the one hand, disclosed many facts which did not harmonize with the conception of nature as a purposive structure, as that conception was then interpreted. On the other hand, scientific thinkers were becoming increasingly insistent on establishing such explanations as would permit confident prediction and control of Nature's ways. They wanted the kind of system into which these explanations would readily fit. No longer were they satisfied with theories whose virtue lay primarily in their encouraging contemplative appreciation of how the whole cosmos fits into an eternal divine plan. The mathematical conception of natural law, expressed in the quest for exact functional correspondences, proved to be a successful and satisfactory answer to both these needs alike, so far at least as large areas of inorganic nature are concerned.

In the history of Western science there have already appeared at least three major clues to the organization of scientific knowledge in this fashion, which have attained very wide currency and exercised influence over a long period of time. A brief examination of them will contribute to our understanding of the process just described and will clarify the logical status of such key ideas.

Hierarchical classification the main organizing clue of Greek thought

When Greek thinkers faced the problem how best to build upon the loose-knit organization of knowledge attained by common sense, they followed a basic clue suggested by the principles of logical division and definition. This presupposed that the fundamental pattern in nature is the inclusion or exclusion of one class by another, and that the proper way to build what we know into a comprehensive system is to establish a hierarchy ranging from the most general class at the top to the most specific classes with their individual members at the bottom.[7] All the classes at any given level of the hierarchy below the summit will, on any given scheme of division, exclude each other, and all will be included in a smaller number of classes at the next higher level. Each class is distinguished from others by its possessing, or failing to possess, certain attributes, namely, those which inductive examination proves to be essential to it. On this presupposition scientific knowledge is expressed in a system of accurate definitions, each of which locates the class to be defined in the next higher or more inclusive class and distinguishes it from other entities in that class by the attributes which are its differentiating characteristics. Thus "man" can be defined as "rational animal," animal being the next higher class including other species as well as man, and rationality being the essential attribute which marks man off from those other species.

The pattern of thought determined by this key to the systemization of knowledge was taken for granted by the majority of thinkers for many hundred years, but it finally gave way to another. Its major weaknesses were lack of quantitative

[7] Greek mathematics suggested another method, but it did not assume a widely challenging form until much later.

exactitude in the relations between the classes entering into the system, and the circumstance that it did not permit prediction and control of the changes going on in nature. Changes could be known, but only in the same way in which other facts were known, *i.e.*, by locating them as species within some broader class. Such knowledge could not guide anticipation of the future course of any change that happens to be under study; it could only tell what kind of change it is.[8] Hence this sort of knowledge came to appear quite unsatisfactory when a fundamental shift of scientific interest took place, demanding a quantitative analysis of phenomena on a scale and in a form hitherto beyond the imagination of all but a few of Europe's intellectual leaders, a form permitting detailed prediction of what is going to happen in the future as a result of the motions now taking place.

Its use in present-day science

To be sure, in the less developed sciences, systematic division into classes, freed from the rigid framework of an Aristotelian cosmology, has persisted as a fruitful preliminary way of placing facts in some sort of order, and as in many cases offering the best organization of available material possible at the time. Science cannot proceed to further conquests till the facts which it studies have been identified as members of certain definite classes. In the exact sciences, such classification has been already achieved and can be taken for granted, but wherever it has not been achieved it becomes an initial problem for investigators to solve. In the less advanced sciences this is often evident.[9] As recently as the middle of the eighteenth century, the Swedish botanist, Linnaeus, marked an epoch in his field by a much more thorough classification of plants than had been made before. His method revealed no significant logical novelties as compared with that of the Greek biologists; he analyzed minutely the structure of plants, discriminating the more general properties

[8] This limitation was also encouraged by the teleological assumptions characteristic of Greek thought. The natural urge of everything is to realize the form of the species to which it belongs, *i.e.*, simply to be what accurate definition says that it essentially is.

[9] See below, chap. 21, pp. 455 ff.

from the more local and specific ones, and erected a table in which the position of each distinguishable species in its genus, order, and class was determined by its possession or nonpossession of one or more of these properties. The existence of many intermediate forms difficult to fit into the table soon betrayed to discerning eyes the weakness of the classificatory method when conceived as more than a preliminary and tentative aid in dealing with these matters. And in the social sciences, often the most that can be done with confidence in dealing with a puzzling group of facts is a careful classification according to significant differences of structure and function. For example, the scientific study of governmental agencies still consists largely in a classification according to such differences; there are unicameral and bicameral legislatures, appointive and elective judges, executives with veto power and executives without, etc.

The ideal of a universal mechanism

A different and still more daring ideal for the organization of knowledge than that of division according to essential attributes came to be actively pursued during the period of rapid development of the physical sciences in early modern times. And it has been actually realized over a wide enough area for many scientists and philosophers to picture it today as the legitimate ideal for the systematization of all knowledge. It was the dream inspiring each of the great thinkers whose contributions were described in the preceding chapter. It presupposes that its subject matter has already been classified and defined sufficiently for purposes of accurate identification, and for the discovery of the more obvious causal conjunctions which classes thus defined may reveal. But it goes beyond this achievement in seeking a quantitative analysis of the phenomena thus classified and causally related. The underlying assumption is that the superficially perceptible differences of quality and behavior in objects and events are really but the effects of quantitative differences of structure and motion that can be assigned to the entities of which they are composed. For modern physical science, different objects appear and act as they do because of differences in position,

mass, velocity, etc. (all mathematically expressible differences), of the atoms or other ultimate units of which the objects are made. In the field of mechanics, and to a less extent in other branches of exact science, this ideal of knowledge has already been largely attained; we can analyze the phenomena studied into masses moving in space and time according to some quantitative law, and facts which would seem very different when defined in terms of perceptible attributes become merely different mathematical values in the equation by which that law is formulated.

What does this sort of analysis imply as the proper way of bringing together in a single system items of knowledge drawn from different fields? The answer is fairly clear. It implies that the objects or events with which thinkers are anywhere occupied in these fields should be conceived as simply different combinations of the same ultimate elements which, considered in themselves, move according to precisely the same quantitative laws. So far as this can be successfully done, the behavior of all phenomena thus reduced can be expressed in the same terms; all can be brought together in one system of matter in motion, and their performances viewed as corollaries implied, under varying specific conditions, by the equations which describe the motion of the ultimate elements.

As an illustration of what the mechanical theory of nature renders possible in unifying knowledge, consider again the work of Newton. The essential condition of such a scientific achievement is the analysis of all the phenomena dealt with into physical units now termed "molecules," conceived as moving according to universal laws statable in the form of functional equations, in which mass, distance, and time are the basic magnitudes. These molecules in motion, then, compose the field of mechanics, and every event analyzable in terms of the motions of such units becomes, by that fact, capable of a functional relation to all other events that are so analyzable.

Functional explanations presuppose it

The ideal of scientific explanation, of course, from this point of view, would be to merge into one the separate branches of

science by discovering some still more ultimate unit whose laws of behavior could be shown to imply the causal and functional relations previously appearing ultimate in each of these branches. In the inorganic realm, much has already been accomplished in this direction. Progress has been made toward uniting chemical and mechanical phenomena within the still broader field of electromagnetism, and a formulation has been sought of the laws of the electron (at present apparently the final unit in this field) such that the verified truths of physics and of chemical reaction will follow from them. This quest has run into unexpected difficulties, arising in part from the fact that the electron acts in certain ways that appear irreconcilable with the assumptions of molecular mechanics. These difficulties suggest that there may be definite limits, even in the inorganic sciences, to the fruitful application of this cosmic theory in its traditional form. Moreover, little progress has as yet been made in finding a common unit or law in terms of which the inorganic and organic realms may be united in a single system. Now that Aristotelian conceptions of how inorganic behavior is to be explained have been abandoned, the former realm seems definitely irreducible to the latter, and likewise the most interesting performances of organic creatures seem incapable of satisfactory explanation in terms of the laws of inorganic change. Quantitative analysis has found significant uses in the organic and social sciences as well as in others, but not in ways which make their material clearly reducible to electromagnetic phenomena.

Clue derived from the theory of evolution

It may be, then, that there are phenomena revealing properties which cannot be analyzed in terms of the kind of units that such an interpretation requires. So far as this is the case, they will not fall into a system of the type just described. Gradually increasing concern about this possibility has made scientific minds in recent times hospitable to the promise of a third major ideal of the unification of natural knowledge. During the last century and a half, and especially since Darwin successfully applied the notion of organic evolution to certain biological problems, thinkers have been increasingly

impressed with the idea that evolution is perhaps a more pervasive principle in nature than mechanism. Every living thing, at least, reveals both a genesis and a history; its adventures seem more than the successive values of a quantitative series; and even in the inorganic world every individual object, like the earth or the solar system, has come to be what it is by a temporal process not exactly predictable from the antecedents, and perhaps irreversible. Since we need to systematize as far as we can our understanding of these historical phenomena, a way of doing so by discovering universal aspects or stages of their course of development promises too many virtues to be ignored. The various concepts bound up with the scientific idea of evolution afford the key to this mode of organizing knowledge into a comprehensive system. This idea, too, in its most constructive forms, aims to include the mechanical interpretation of all phases of nature that can be so interpreted, but to set them in a larger historical context. Viewed in this context, nature falls into a series of levels which are empirically related in a certain temporal order, life having evolved out of physical matter, and mind out of life. Within each level, functional and causal conjunctions are discoverable, but in terms appropriate to the distinctive character of the entities belonging to that level. Nature, as a whole, is thus an evolutionary process in which new and more complex forms emerge out of older and simpler ones, some proving able to maintain themselves in their distinctive modes of behavior under the environing conditions which they face. In these terms, the ultimate explanation of any phenomenon consists not in analyzing it into its basic constituents with their causal or functional relations, but in locating it, as thus analyzed, in its appropriate place in this all-inclusive evolutionary process.[10]

At the present time, science is undergoing radical transformations which suggest, among other things, that a still more novel conception of how verified knowledge is to be compre-

[10] I am thinking here of the common features in such theories as those of Alexander, Lloyd Morgan, Whitehead, Dewey.

hensively integrated may be emerging. One who dared to forecast what form this conception will take would be very bold. It would be reasonable to anticipate, however, that it will give greater weight than previous notions have done to the statistical interpretation of natural laws [11] and that it will generalize some idea now proving fruitful in the rapidly advancing social sciences—perhaps the idea of "sociality" itself. These branches probably have yet to make their distinctive contribution to scientific theorizing in general.

Theories intermediate between cosmic theories and specific laws

With this analysis of such all-inclusive theories in mind, we are in a position to describe more definitely the status and function of theories less ambitious in scope, *e.g.*, the theory of gravitation so impressively verified by Newton. They fall logically into an intermediate position between specific laws established by direct observation or experiment and some cosmic theory of the kind just considered; they are hit upon as a result of using the latter to guide imagination when it attempts to formulate hypotheses by which the unification of a larger or smaller group of specific laws may be established. In choosing between any pair of theories, at any intermediate level, the criterion of simplicity plays the role that has been described above.

To indicate the logical status of the all-inclusive ideas dominating science at any given epoch, and to distinguish them from less inclusive theories, we have called them *cosmic theories*. How far the distinctive demand of the third criterion can be fulfilled in their case—which means the realization of an all-embracing system with a maximum of simplicity—is determined simply by historical success, *i.e.*, by whatever power they gradually disclose to organize a large variety of phenomena in a manner capable of satisfying the major interests shared by most scientific thinkers at the time.

EXERCISES

1. Define: empiricism, objectivity, simplicity, system, cosmic theory.
2. How ought a scientist reply to an objector who turned aside

[11] See below, pp. 498 f.

scientific proofs on the ground that he felt certain something different was the case? Explain.

3. The principle of simplicity has been criticized on the ground that nature's ways are apt to be complicated rather than simple. Develop and answer this criticism.
4. On reading Campbell, *What Is Science?*, especially chapter 2, write an essay on the objectivity of scientific thinking.
5. Read the fifth chapter of the same book. Discuss the distinction and relation between theory and law.
6. In what respects do you think that the three cosmic theories described in the chapter are mutually exclusive? Justify your answer.

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CHAPTER 20

PROBABILITY

With the concept of probability a new phase of the methodology of factual science demands attention. Not that the idea expressed by this word is now introduced for the first time. On the contrary, its importance has been recognized more than once. But the themes which it brings before us have not yet been explored.

Are formal inferences merely probable?

It has been noticed, for example, that the essential difference between formal and factual science lies in the circumstance that the conclusions of the latter are subject to the approval of a court which can be entirely neglected by the former, the court of perceived fact. Reasoning in formal science has but one responsibility—to reach results whose denial would contradict the premises with which the reasoning began. As Part II amply showed, mistakes are sometimes made in the attempt to reach them, so that an invalid conclusion is taken to be a valid one; even a well-trained logician or mathematician will occasionally blunder, and there is always a slight possibility that the error may pass unnoticed in his or others' subsequent traversing of the same steps. Hence, strictly speaking, no conclusions formally established are absolutely certain; they are affairs of high probability. But while recognizing this truth, reflection on the methodology of science has thus far found it inappropriate to apply the concept of probability in this field, for two reasons. In the first place, the history of the formal sciences seems to justify the presumption that blunders by such thinkers, when they occur, are quickly noticed and corrected. They are quickly noticed, because any new formal demonstration is of interest to the author's col-

leagues who work in the same field; it naturally receives their critical examination at once. And while they, too, may fall into some logical inadvertence, there seems no reason to expect that all who check a given proof will commit the same error at exactly the same point. As soon as any mistake is noticed it is, of course, corrected by concentrated attention, and there are very few cases, if any, where such a correction, agreed upon by experts at the time, has been later found to be itself invalid. Hence, in the case of these tested formal proofs, there appears ample justification for subsequently disregarding the possibility of error (unless some special ground for suspecting it turns up), and for treating the results established as quite assured. In the second place, even if we should attempt to cover these inferences by the concept of probability there seems to be no way of measuring in any given case the degree of probability involved, or of comparing the probability of one demonstration with another. To do this, it would be necessary in each inference to form a definite estimate as to how likely it is that the conclusion will later be corrected, and on what basis could such an estimate be made? Probability is by its very nature a comparative idea; one statement is more or less probable than another; adjudication in terms of some scale or standard is therefore always implied. Accordingly, where no method of rational comparison seems possible it would appear inappropriate to apply the idea at all. Established conclusions in formal science are, then, not absolutely certain, but it is best to treat them as though they were so except when doubt in some special case happens to arise.

Probability in factual reasoning

Conclusions in factual science lack certainty in another way; not only must they be formally valid but they also lie always at the mercy of what subsequent perception may disclose. Does a scientist offer a careful description of some event? As a record of his own (now past) experience of that occurrence the description is doubtless final, except for some possible modification which his memory may come to suggest. But the description is of no scientific or practical use unless it is found to hold good of events of the same class perceived

later, by the same individual or others, or unless it is consistent with subsequent perceptions that throw further light on the original event. Does he offer a generalization purporting to explain some phenomenon? Again, if this is to be more than a summary statement of a conjunction regularly repeated in his past experience—if it claims predictive power with reference to further instances of the phenomenon—it must be verified by perception of such instances. Now the preceding chapters have reminded us of an important lesson which life everywhere teaches—namely, that there is no guarantee that future experiences of a phenomenon will find it embedded in the same network of connections that past experiences of it have led us to expect. All that we can do is to guide our anticipations by conclusions which seem thus far to have been more fully confirmed than any alternatives. Here an element of uncertainty is confronted that is distinctively different from the possibility of a merely logical blunder.

Hence it is very desirable to find some method of systematically taking into account the circumstance that conclusions claiming to apply to future perceptions are in varying degree probable rather than certain. And this means not merely discovering, wherever possible, techniques whereby less assured results can be replaced by more certain ones—an aspect of scientific progress amply illustrated in Chapters 16 to 18—it involves two further tasks that must now be described.

First, it involves perfecting methods whereby the degree of confirmation realized when any particular conclusion is reached can, so far as is feasible, be clearly indicated. Inquirers need to know how the various results, proffered by science at any given time, stand with respect to the degree of confidence that may properly be placed in them. Are they but "first approximations" to the true result? Or are they more precisely and securely buttressed by wide ranges of relevant evidence? The same motives that make men eager to discover truth make them anxious to tell how far short of assured truth any particular conclusion has fallen, or at least how it compares in dependability with other conclusions, so that they

may avoid placing more trust in it than is warranted. To use a technical term that will later be defined more exactly, science needs a way of indicating the "probable error" when an assertion is made about factual matters.[1] In a broad sense this enterprise has already occupied us. It was noted in Chapter 16 that causal laws verified by applying the principle of difference are more likely to be dependable than those established merely by the principle of agreement, and that inductions confirmed by controlled experiment are more certain than those which rest upon observation alone. This is to say that, other things being equal, the probable error in the former cases is smaller than it is in the latter. Now science has succeeded in meeting this need in a fairly promising fashion; in many types of situation, indeed, investigators now use devices which give a quantitative measurement of the probable error.

The significance of probable conjunctions

Second, it involves a problem about the nature of the conclusion itself. Hitherto, we have been thinking about conclusions in the form of universal laws. But when the subject of factual science was first introduced, a group of phenomena were noticed which science has no intention of neglecting, but to which it has not been possible hitherto to give detailed attention. The reader will recall that besides "universal" conjunctions between phenomena there are "accidental" and "probable" conjunctions. Now science can do nothing for purposes of prediction or explanation with accidental conjunctions; when a phenomenon is conjoined indifferently with various others, so that there is no basis when it occurs for expecting this or that particular happening to accompany it, we have no clue to any orderly pattern as a part of which the event may be understood.[2]

But with probable conjunctions the situation is different. When two phenomena accompany each other more often than

[1] See below, pp. 483 f.

[2] As will be shown below, however, statistical procedures can be applied even to conjunctions that appear merely accidental, and sometimes they succeed in uncovering clues that otherwise would have caught no one's attention. See chap. 21, pp. 489–493.

not, even though the conjunction is not universal, science is far from helpless. For one thing, accumulation of further instances, guided by careful analysis, may lead to the discovery of a universal conjunction and the establishment of a causal law. For another, even though this does not happen, prediction of future occurrences of one or the other phenomenon can be made, guiding our expectations as rationally as is possible. According to the postulate of predictive uniformity, the same degree of interconnection between two phenomena that has been observed in previous experience may legitimately be expected to continue in subsequent experience, barring the influence of unknown factors. And it is obviously far better to be able to predict that phenomenon A, in its proper context, will be conjoined with phenomenon B four times out of five than to be unable to make any justified prediction about them at all.

Illustrated by the fall of rain

Consider a further illustration of both of these points. Anyone who has taken an interest in the weather knows that when the barometer falls, in most areas of the United States, rain within a few hours is more likely than not. That is, a spatio-temporal conjunction between instances of these two events, while not universal, is much more frequent than would be the case if the connection were merely accidental. In many parts of the northern hemisphere a person who keeps records of this conjunction over a considerable period will find that on the average, out of every ten cases of a falling barometer, rain comes within, say, twenty-four hours in about seven of them. Now, even though such information had led to no further knowledge, it would be far better as a guide to our expectations for the future than nothing at all. The city dweller can plan his picnics and outings, the farmer his sowing, hay cutting, and the like, with reasonable assurance that disappointment will overtake him in not more than a minority of instances. But, as everyone knows, further knowledge in meteorology has been attained—knowledge which has made possible much more dependable predictions than these and which encourages the hope that invariable causal laws

about such matters will sometime be established. This knowledge is secured by supplementing the records just described by observation of other conjunctions. For example, by bringing together barometric readings at various locations on the earth's surface at the same time, charting the results, and comparing them with similar charts recording readings at other times, it becomes evident that the cause of rain is the approach of a low pressure area, ordinarily from the west, bringing condensation of moisture in the atmosphere. This discovery indicates where further study needs to be applied in the interest of obtaining more exact and complete causal knowledge; if it could be found out just what accounts for the formation of these areas, what determines their depth at every point in their path, and why each follows the particular course that it does, far more adequate explanations could be given than are now possible. But even in advance of such an achievement, forecasts of rain can be rendered considerably more accurate and better buttressed than with only the records first mentioned. The weather bureau's predictions are correct eight or nine times out of ten instead of seven.

The major problems concerning probable conjunctions

Such considerations make it evident that science cannot afford to pass these conjunctions by in haughty disdain, as lacking the universal dependability that she would prefer. Their value is very great. And it is an important part of her task to teach men how to render their judgments about the probability of this or that event as exact as possible—as consistent as may be with what the available evidence justifies. To return to the illustration just employed, a man who has no meteorological knowledge and has kept no records of relevant observations can make only the vaguest predictions, unbuttressed by definite evidence; he can say merely that when the barometer falls rain is "likely" to come soon, and when asked for evidence he can only add that this is the way it has "usually" happened. Such statements often seem sufficient to common sense, but they lack the quality that science always insists upon when it begins to improve on common sense. The trained meteorologist can make probability assertions of a

very different kind. They will be more apt to prove correct; they can more often be stated with quantitative precision; and definite evidence can be invoked to warrant whatever assertions are made.

Practically all branches of science now find it necessary to express some discoveries in terms of probability statements, and in many sciences the most important results thus far reached are of this sort. In the social sciences, especially, very few causal or functional laws have proved possible; the bulk of our valuable and promising knowledge there consists of carefully recorded conjunctions and the correlations of greater or less probability established by their aid. It is imperative that assertions in this field, as elsewhere, be more than vague guesses, and the way to render them so lies in intelligent quantitative application of the concept of probability.

We begin in the present chapter with the second of these problems, passing to the first in the next chapter.

What are the main questions of logical concern about assertions regarding the probability of events? They are these: Just what is it that a probability statement says? What sort of evidence is required to warrant this or that particular form of probability statement? Can probability always be expressed in some quantitative ratio? If not, why not, and what can be done in such cases to approximate exactitude as closely as possible? What are the foundation theorems used in the calculus of probability? And, finally, what is the ultimate criterion to which we must appeal in determining probability?

Answers to these questions will now be sought by considering in some detail certain types of situation to which the concept of probability is admittedly relevant. Let us begin with situations that are familiar to everyone, are relatively simple, and in which probability assertions are evidently capable of quantitative formulation.

Quantitative formulation of probability assertions

When one tosses a coin, what is the probability that it will fall heads? Well, we know by the law of gravitation that it will not remain poised in air; and it is so constructed that it

will not come to rest standing on its edge. It must therefore fall on one or the other of its two sides, heads or tails. And it cannot lie on both of these sides at once. There are thus but two alternative outcomes, which between them *exhaust* the possibilities, and which are *mutually exclusive*—that is, the occurrence of one precludes the occurrence of the other. Furthermore, the evidence available in advance of the toss indicates that these alternatives may be regarded as *equally likely*. Each of the faces, so far as we know, is equally distant from the center of gravity of the coin, and the variations in the toss which determine which of the two sides comes uppermost are too slight to be controlled; the thrower is just as likely to impart the motion that will result in heads as to impart the motion that will result in tails.[3] Under these circumstances, how is the probability of heads to be indicated? In ordinary language the answer is: There is one chance out of two that the coin will fall heads, and likewise one out of two that it will fall tails. Mathematically, the probability of either event can then be indicated by a simple fraction. The denominator will represent the total number of equally likely and mutually exclusive alternatives, in this case, two. The numerator will represent the particular one of these alternatives whose probability is being sought, in this case, heads. (Technically, this is called the *favorable* chance—that is, it is the chance favoring the particular alternative that we have in mind.) The probability of heads, then, may be quantitatively measured as ½. And for the same reasons, the probability of tails is also ½.

The toss of a die is another simple illustration. Here we know that the die will not come to rest on any of its points or edges; it will fall on one of the six faces. Again, these are exhaustive and mutually exclusive; it must rest on one or another of those six sides, and if it rests on one it cannot rest at the same time on any of the others. And for the same

[3] Later on we shall consider more fully just how the phrase "equally likely" is to be interpreted. See below, pp. 439–444.

reasons that apply in the case of the coin, we may, on the evidence available in advance, regard each of these alternatives as equally likely. This assumes, of course, that the die is approximately cubical in shape and is made of homogeneous material. What, then, is the probability of a deuce falling uppermost? The fraction here is evidently $\frac{1}{6}$. For there are six equally likely and mutually exclusive alternatives, of which the deuce is one. What is the probability of a number less than five falling uppermost? The answer this time is $\frac{4}{6}$ or $\frac{2}{3}$. For there are now four favorable chances out of the total of six chances; the prescribed conditions are met if either the one, the two, the three, or the four falls uppermost. Only two of the six possibilities are in this case unfavorable.

Relation of probability to necessity and impossibility

With the die still before us, a question may now be raised whose answer will bring out a further important point about the nature of probability. What is the probability that a number less than seven will come uppermost when a die is thrown? Following the principles thus far explained, the answer obviously is: $\frac{6}{6}$ or 1. For in this case any of the six possible alternatives is favorable, which means that the result asked for could not fail to happen—it is, in the assumed context, *necessary*. Necessity is thus the upper limit of probability; it denotes the situation in which there is no chance whatever that the prescribed conditions will not be fulfilled. Again, what is the probability that the toss of a die will bring uppermost a number greater than six? The answer now is $\frac{0}{6}$ or 0. For there is no chance of this eventuating at all; none of the faces of a die has any such number stamped on it. This outcome then is *impossible*, and impossibility, measured by 0, is the lower limit of probability. These considerations make it evident that any probability capable of quantitative statement, under conditions which exclude necessity and impossibility, may be measured by a proper fraction lying between 0 and 1. The nearer it approximates the former of these numbers, the smaller is the degree of likelihood; the nearer it approaches the latter, the greater is the degree of probability.

By a *calculus of probability* is meant a series of theorems about probability assertions, demonstrated by the usual operations of formal reasoning. By its aid one can translate a probability statement in one form, or about certain entities, into a statement of another form, or about other entities, without the necessity of any appeal to factual matters. It is thus like any branch of mathematics, such as arithmetic, where through some permissible operation, say addition, a statement about four and three may be translated into a statement about seven. Now there are two theorems in the calculus of probability that are foundational to all its applications in other than the simplest cases, and they can be readily explained on the basis of the considerations thus far discussed.

The calculus of probability

The first is the "theorem of the addition of probabilities." The word "addition" in this phrase means what it did in our study of disjunction in formal science, except that here the alternative possibilities are assumed to be mutually exclusive. We are adding "either A or B" in situations where "both A and B" is ruled out as impossible. Suppose that one has a full deck of playing cards, properly shuffled. By the procedures just outlined, the probability of drawing a ten is $1/13$. For there are fifty-two cards, each of which is equally likely, and four of these are favorable to the prescribed condition, since the pack contains the ten each of spades, hearts, diamonds, and clubs. The probability fraction then is $4/52$ or $1/13$. Similarly, the probability of drawing an eight is also $1/13$. What, now, is the probability of drawing either a ten or an eight? Well, the denominator, representing the total number of alternatives, obviously remains the same, 52. But the numerator is now $4 + 4$ or 8, since any one of eight cases is favorable: spade ten, spade eight, heart ten, heart eight, diamond ten, diamond eight, club ten, and club eight. The fraction is thus $8/52$ or $2/13$. The probability of the ten and the probability of the eight are simply added, and one immediately apprehends that this procedure validly applies to all similar cases. Let A stand for one of the events involved—in this case, the drawing of a ten.

Fundamental theorems—addition of probabilities

Let B stand for the other—in this case, the drawing of an eight. Then it is clear that

$$\text{prob}\ (A + B) = \text{prob}\ A + \text{prob}\ B. \quad (1)$$

As one application of this theorem, it appears that when the events whose probabilities are added together are not only mutually exclusive but also exhaustive of all the possibilities, their sum must be equal to 1. For in that case it is necessary that one or another of them take place. What, for example, is the probability (the assumed context remaining unchanged) that on the toss of a die either the one, the two, the three, the four, the five, or the six will fall uppermost? From the theorem just formulated,

$$\text{prob}\ (A + B + C + D + E + F) = \frac{1 + 1 + 1 + 1 + 1 + 1}{6}$$
$$= \frac{6}{6} = 1. \quad (1.1)$$

This result is obviously in harmony with what would be expected from the relation between probability and necessity, pointed out above. Another simple illustration of the same theorem appears when two events are such that the assertion of one stands to the assertion of the other in the relation of contradictory. What is the probability that it will either rain today or not rain? Clearly, whatever probability may be assigned to one of these alternatives, the sum of the two probabilities must be equal to 1. For it is necessary that rain will either fall or fail to fall.

Multiplication of probabilities

The second is the "theorem of the multiplication of probabilities." Consider again a pack of cards, only this time let us leave out the diamonds and the clubs. We shuffle the hearts separately on our right and the spades separately on our left. What, now, is the probability of drawing the king of hearts with the right hand and the queen of spades with the left? Well, the probability of drawing the king of hearts is evidently $1/13$; there are thirteen equally likely alternatives, of which only one is favorable. Similarly, the probability of

drawing the queen of spades is $1/13$, for the same reasons. And it is clear that neither draw affects the other; the probability of the king of hearts or the queen of spades appearing is just the same as it would be if the other draw were not being made at all. In other words, the probability in each case is entirely *independent* of the probability in the other. Now any of the thirteen alternatives in the first draw might be combined with any of the thirteen alternatives in the second; there are thus 13×13 alternatives possible in the two draws taken together, and only one of these—king of hearts combined with queen of spades—is favorable. The probability of meeting the conditions specified is thus $1/13 \times 1/13$, which $= 1/169$. There is thus only one chance of success in a rather large number. The principle here exhibited evidently applies to any case of computing the probability of the occurrence of both one event and another event from the probability of each considered separately, when the two probabilities are independent of each other.

$$\text{prob (AB)} = \text{prob A} \times \text{prob B}. \quad (2)$$

For another illustration of the same kind of situation, suppose that we have a bag containing three white and six black balls of the same size. Let us draw a ball at random and then, after replacing it and mixing the balls in the bag, draw again. What is the probability that in two such successive draws white balls will be taken both times? Here also the two probabilities are independent of each other. The first is obviously independent of the second; and the second is likewise independent of the first, for before the second draw is made a condition similar to that under which the first draw took place has been reestablished. The probability of a white ball on the first draw is obviously $3/9$ or $1/3$, for there are nine balls altogether, of which three are white. Similarly, the probability of a white ball on the second draw is $1/3$. And since any one of the nine balls that might be drawn the first time could be combined with any of the nine that might appear on the second draw, the probability of getting a white ball both

times is $3/9 \times 3/9$ or $1/9$. That is, we multiply the two independent probabilities to find the probability of the combination specified.

Now let us consider a situation in which the second of two draws is *dependent* on the first, instead of being unaffected by it. We take a bag of three white and six black balls, as before, only this time we shall not replace the ball appearing on the first draw when the second draw is made. What is the probability now that in two successive draws, at random, white balls will be taken both times? Here the second draw is obviously dependent on what happens as a result of the first draw; it will be a draw from among the balls left after the first trial is over. Well, at the beginning there are nine balls, each of them equally likely to appear, and three are favorable to the appearance of a white ball. Hence the chance of getting a white ball on the first draw, as before, is $3/9$ or $1/3$. But this time the second draw is made from the eight balls that are left. And any one of the nine possible balls in the first draw might be combined with any one of the eight remaining balls in the second draw, so that there are 9×8 or 72 possible combinations in the two draws. How many of these are favorable? To answer this question we need to consider that only if a white ball has appeared on the first draw will there be any chance at all of getting white balls on each of the two draws; if a black ball appears the first time we might as well quit the experiment and start over again. Hence the second draw, if worth making at all, will be made from eight balls, of which only two are white. Therefore, the chance of getting a white ball this time is only $2/8$ or $1/4$. So the answer to our question is given by the product of the numerators of the two fractions reached, just as the total number of chances is given by the product of the denominators. The probability required is thus $3/9 \times 2/8$ or $1/3 \times 1/4 = 1/12$. There is thus one chance in twelve of getting two white balls under these conditions.[4]

[4] The correctness of this reasoning, and of the general formula which it applies, will be seen clearly if this problem is analyzed in detail. In order

The multiplication theorem for cases of this kind, where the probability of one occurrence is dependent on the other, is:

$$\text{prob}\ (AB) = \text{prob}\ A \times \text{prob}\ B, A. \quad (2.1)$$

The last symbol in this formula is to be understood, of course, in terms of the conception of dependence which has just been explained. It means the probability of B on the supposition of the prior occurrence, in the context described, of A. In this particular instance B means the appearance of a white ball on the second draw; A the appearance of a white ball on the first draw.

What is the ultimate criterion of probability

It is now time to broach another fundamental question. What is the final court of appeal in determining the probability of a single specified event under prescribed conditions? What we have just been noting is how, given such and such probabilities for single events A and B, it is strictly implied

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to identify the individual balls, let us number them: W1, W2, W3, B1, B2, B3, B4, B5, and B6. The possible combinations then in the two draws may be tabulated as follows:

First Draw	Second Draw
W1	Either W2*, W3*, B1, B2, B3, B4, B5, or B6
W2	" W1*, W3*, B1, B2, B3, B4, B5, or B6
W3	" W1*, W2*, B1, B2, B3, B4, B5, or B6
B1	" W1, W2, W3, B2, B3, B4, B5, or B6
B2	" W1, W2, W3, B1, B3, B4, B5, or B6
B3	" W1, W2, W3, B1, B2, B4, B5, or B6
B4	" W1, W2, W3, B1, B2, B3, B5, or B6
B5	" W1, W2, W3, B1, B2, B3, B4, or B6
B6	" W1, W2, W3, B1, B2, B3, B4, or B5

That is, if the white ball numbered 1 is drawn on the first try, the second ball might be any one of the other eight listed in the top row of the column at the right; if the black ball numbered 2 is drawn, the second might be any one of the remaining eight in the row to the right of B2. There are 72 possible combinations, the product of the number of alternatives on the first draw and the number of the remaining alternatives on the second in each case. The favorable combinations are marked by an asterisk, and are clearly six in number. As the table shows, they represent the product of the favorable chances on the first draw (the three white balls) and the favorable chances that are possible on the second draw in each case when the first draw has been successful (the other two white balls remaining). The validity of the general theorem for cases of this kind may thus be intuitively seen.

that the probabilities for the compound events denoted by "either A or B" or "both A and B" are what the above theorems affirm. The present question is: How do we tell, in the last analysis, just what probability should be assigned to any given event under prescribed conditions, such as the events A or B in the above cases? If anybody questions the probability fraction that you or I propose in any given case, to what evidence do we ultimately turn in order to test its correctness?

In the illustrations thus far used it has been tacitly assumed that this can be done merely by inference from the conditions laid down in the problem to be attacked. Thus, in the case of tossing a die it was assumed that if each die is approximately cubical in shape and is made of homogeneous material, it can be taken for granted that any of the six faces is equally likely to turn up. Similarly with a coin; we have assumed from its structure and the way in which it is tossed that heads and tails are equally likely. But suppose actual experience with any die or coin should seem to belie these assumptions? Suppose that when a die is thrown a considerable number of times, one face turns up far more often than would be expected from the probability assigned on the assumption mentioned. What should be concluded from this?

A crucial instance

The suspicion would certainly be encouraged that, as a matter of fact, the six faces are *not* equally likely. Perhaps the die is loaded, we would say, or possibly its shape is not as near that of a cube as had been supposed. At any rate, it would hardly be appropriate to preserve, in the light of this factual evidence, undoubting assurance that the faces are equally likely. And how would the question whether they are or are not be put to a more decisive test? The shape could be checked by some tool of precise measurement, and the die could be opened up to see whether there are any indications that the material is not homogeneous. If such investigations confirmed our suspicions the question would be settled; the original assumption would be abandoned in favor of one justified by the evidence now discovered. If it did not, we

would surely proceed by tossing the die a much larger number of times and carefully noting the results. The reasoning behind doing this is as follows: It is consistent with the laws of chance that the same face of a die might turn up several times running, or might appear for some time more frequently than ⅙ of the throws even if the faces are equally likely. But if the latter is really the case, no face having any genuine advantage over the others, a larger number of throws will confirm that circumstance and the relative frequency of the various faces actually shown will more closely approximate the ratio expected. It can be proved that where the probability of an event under certain conditions is some specified ratio, say ⅙, and where there is no factor determining the instances to happen in any particular order, its relative frequency, as actually shown in the first few trials, may vary quite considerably from the true probability fraction; but that as the number of trials increases, the difference between the two fractions will become less and less. The last two clauses of this statement constitute an important theorem in the theory of probability, known as Bernoulli's theorem. More precisely formulated, it is: *We may expect that the relative frequency of an event A in a series of independent trials with constant probability P will differ from that probability by less than any given fraction greater than 0, provided the number of trials is taken sufficiently large.*[5]

Two outcomes are possible from this larger series of tosses. The relative frequency with which each of the different faces turns up may converge toward the ⅙ expected. In this case we would conclude that the unexpected result in the earlier tosses was due merely to chance; the faces are equally likely, after all, and ⅙ is the true probability. But the relative frequency in the more numerous trials might converge toward another fraction, perhaps a fraction not very different from that shown in the earlier tosses. What would be the proper

[5] The validity of this theorem depends on assuming that the "constant probability P" is measured by the relative frequency of the event in the long run, *i.e.*, that the ultimate criterion insisted upon in the present discussion is the criterion actually applied.

conclusion now? Should we still insist that the true probability is ⅙, or should we revise our estimate of the true probability and make it conform to the ratio that actual lengthy experience with the die has approved? Which of the two ratios would it be more reasonable to expect in the future, if the number of trials were yet further increased?

The criterion is relative frequency in actual experience

The answer would seem to be clear, if one holds consistently to the objective empiricism that is everywhere characteristic of factual science so far as concerns its ultimate criteria of evidence. In the axioms about equal likelihood stated early in the present chapter, which underlie the deductions of the probability calculus, we were really talking not about actual coins or dice but about *ideal* coins and dice—that is, solids of perfect geometrical shape, with their centers of gravity exactly where they should be, and tossed in such a way as to exemplify complete randomness. Of such solids the axioms are doubtless true, and to such solids the deduced theorems involving the notion of equal probability doubtless apply. But no *actual* coin or die conforms fully to this ideal description. It is inevitably affected by whatever variations from the ideal are in fact present, and these variations may not always be such that they cancel each other out as the number of tosses is increased. The center of gravity, for example, may be nearer one side than the others, and if so that side will be definitely favored as against the others; a larger number of trials will simply indicate more clearly how much it is favored. In the last analysis, the probability of any factual event, providing guidance to reasonable expectations for the future, is determined by the relative frequency with which that event actually occurs under the specified conditions, as the number of trials is indefinitely increased. And we shall therefore define "probability" (which has not yet been formally done) in the light of this criterion—we shall mean hereafter by the probability of any event under given conditions that property of it which is determined in this way by its relative frequency.

This problem of the ultimate criterion is rather challeng-

ingly posed by what has actually happened in tests made by tossing a coin. A considerable number of investigators have tested the assumption of equal likelihood for heads and tails by keeping records over a large number of trials, five thousand to twenty thousand or more. In most of these tests the outcome has definitely favored heads, although not by very much—the ratio actually verified, in a selected sample of such tests, being about 51½ per cent heads to 48½ per cent tails.[6] If this had been the outcome only in about half of the tests, tails being similarly favored in the others, the natural conclusion would be that any difference from the 50 per cent deduced on the assumption of equal likelihood is due to chance; the difference is not greater than might be expected to appear on that hypothesis even in a rather long series of tosses. But if the sample just described is a representative one, the outcome reached from its examination points toward a different conclusion—that in coins of the kind here used there is some constant factor slightly favoring heads—and the natural anticipation would then be that similar coins in the future would be more likely to show for heads a relative frequency of something like $^{51.5}/_{100}$ than to show a precisely equal probability as compared with tails.

This does not mean that it is illegitimate ever to apply to actual events the assumption of equal probability, except when lengthy tests have shown that the relative frequency of the alternatives is approximately equal. Often we have not time enough to make such tests; sometimes the tests that have been made on similar objects support the assumption of equal probability closely enough so that the difference may be justifiably neglected; and it is more convenient to calculate anticipations of future occurrences where equal probability may be assumed than where it may not. For these reasons it is permissible in many cases, as a matter of practice, to assume the equal probability of the alternatives involved. But it should always be remembered that this is an assumption made

[6] I do not recall what kinds of coin were used in these tests. Presumably, there are coins that are more heavily weighted on the tails side.

for practical purposes, that at best it is only approximately correct, and that a systematic appeal to the relevant objective evidence, *i.e.*, through a long series of trials, checked by various investigators, might establish a somewhat different probability in each case.

This conclusion confirmed in cases of another type

It is still clearer that relative frequency, as exhibited in actual instances when the number of such instances indefinitely increases, is the ultimate criterion of the probability of any factual occurrence, when we turn to certain other situations to which the notion of probability is applied. In the cases now to be considered, probability seems capable of quantitative measurement, as it is in those already examined, but the concept of equal probability has no relevant application. Suppose the question is raised: What is the probability that a resident of the United States who is now sixty years old will live to be sixty-one? The alternatives here are, of course, two —his surviving a year longer and his failure to do so. These obviously are mutually exclusive and exhaust the possibilities; hence it is necessary (*i.e.*, the probability is 1) that he will do one or the other. But these alternatives are not equally probable. There is an age for which they are approximately so, but it is a much more advanced age than sixty. Indeed, there are rather few situations where, with alternatives exhausting the possibilities, their probability is equal or nearly equal; the cases thus far considered have been exceptional in this respect. However, the probability of such an event as this is capable of quantitative determination; were it not, life insurance and annuity companies could not possibly proceed with anything like the confidence and success that have actually characterized their operations. Inexact judgments play no part in their work; the premiums charged are based upon tables which provide a definite probability fraction for the life expectancy of every group whose insurance they are prepared to write.

Consulting these tables for the proper answer to the question just raised, we find that the probability asked for is .734, that is, that the chances are 734 out of 1000 that the man who

has passed his sixtieth birthday will live to be sixty-one.[7] On what evidence does such a fraction rest? Well, certainly not on analysis of man's physical structure, as seemed to be possible in the case of coins and dice, or any other analysis that could be made in advance of experience as to what actually happens to people sixty years old. But census figures show that in the recent past in the United States, out of every 1000 persons who attained the age of sixty, 734 attained the age of sixty-one. They show, furthermore, that for some years this ratio has been fairly constant. Now such evidence seems to be the only relevant support that can be produced, and it seems to be adequate. Barring any specific knowledge regarding any person of sixty years that would throw light on his chances for survival, beyond the fact that he belongs to the class of United States residents of that age, it seems reasonable to test the probability of his surviving another year by the relative frequency with which other members of the same class have survived another year. At least, no alternative way of testing such a probability appears able to claim superior justification, and companies that have systematically used this method have found that in the long run it works out successfully. The basis of their reasoning is, of course, the postulate of predictive uniformity explained in Chapter 15—the postulate that subsequent conjunctions of two phenomena in the assumed context are likely to appear with the same degree of regularity as previous conjunctions of the same pair.

The ultimate test everywhere, then, it would appear, when one is dealing with a quantitatively measurable probability of factual occurrences, is the relative frequency with which they take place under the conditions specified, as the instances examined increase in number.

Hence, what is meant by the probability of some particular event?

But if this is the case, just what is meant by talking about the probability of such and such a particular event, *e.g.*, the probability of my living to be ten years older than I now am? In common usage the concept of probability is applied in this

[7] This figure applies to life expectancy in the United States during the 1930's.

way, and in the interest of avoiding confusion it is wise to clarify what may properly be meant. Suppose that, on the basis of well-verified life expectancy tables, we affirm that my chances of surviving ten years more are four out of five, and then find that, as a matter of fact, I die within a year, as may very well happen—does that disprove the assertion and show that the probability fraction was mistaken? Not at all. A highly improbable event may at any time happen; that is why we call it improbable rather than impossible. What the assignment of this fraction meant was this: On the basis of the relevant available evidence, it is reasonable to expect that in the long run, of a large group of people belonging to the same class, approximately four-fifths will be found to survive ten years. The predictions implied by a probability statement are predictions about this larger class and not about any individual belonging to it. Despite the apparent reference to a particular individual, no attempt was made to forecast what was going to happen to him or to any other member of that class; there was no prediction as to which of them would conform to the class average and which would not. Hence, what happens to any individual cannot disprove the assigned probability fraction. All it can do is add further relevant evidence which should be taken into account in formulating probability fractions for the class concerned in the future.

Probability statements are relative and responsible to the evidence

An equally important lesson enforced by the considerations thus far discussed is that, in the case of factual matters, probability statements are made on the basis of the objective empirical evidence which is relevant to our knowledge about the class involved. They are thus explicitly relative to that evidence, and they are likewise responsible to it. By the former of these two clauses, it is meant that they are always hypotheses arising from the evidence available at any given time, and dependent for their continued verification on the continued appearance of similar evidence about other members of the class in question. It also means that when the evidence changes, the probability that may properly be assigned any member of the class changes accordingly; probability esti-

mates are continually revised so that they will conform to the most recent and complete data at hand. Thus, if more sixty-year-olds are now reaching the age of sixty-one than was formerly the case, this circumstance will be reflected in the available evidence, and the probability fraction that will be assigned a new member of the class will be modified in the manner indicated by this change.

By the latter of the two clauses—that probability statements are responsible to the evidence—it is meant that the probability assigned to members of a specified class must take into account whatever definite differences there are between the evidence regarding that class and the evidence relevant to all other classes. This is especially important when two classes are so connected that one includes the other. For example, the probability that persons aged sixty who live in an area where standards of living are low and competent doctors few will survive a year, would naturally be lower than the probability for residents of that age in the country at large. Similarly, the probability that persons of the age in question who belong to certain professions will live another year will presumably be different from that of sixty-year-olds in general. For a probability fraction can be intelligently assigned to individuals in each of these groups only on the basis of the relative frequency of deaths at that age in the specific groups contemplated. If, lacking such evidence, one makes probability assertions about members of any such group, he can properly do so only by considering them as members of the more inclusive class about whose relative frequencies information is available. That circumstance must then be indicated in the form of the assertion made; if the probability is really about the whole class of sexagenarians, one must say so, and not blithely assume that the probability for sexagenarian slum dwellers or sexagenarian professors will coincide with the probability established for sexagenarians in general.

Bayes' theorem and its scientific significance

A situation of special interest arises when a phenomenon A is conjoined by a certain relative frequency with another phenomenon B, and by a different relative frequency with

other phenomena such as C or D. Consider three men shooting arrows at a target. B hits the target in one-sixth of his shots, C in one-fourth of his, and D in one-half of his. All three men now fire simultaneously, and one of the arrows hits the target. The arrows are alike; there is no way of telling by direct inspection whose bow it came from. What is the probability that it was shot by B, C, or D? The answer is given by using a theorem in the calculus of probability known as Bayes' theorem. This theorem summarizes in a formula considerations which, as applied to the special case at hand, may be described as follows: Imagine that each of these men shoots twelve arrows at the target with average success. Thus, in all, thirty-six arrows will be shot, and since each man shot one-third of them the probability that any particular arrow was shot by B, by C, or by D, will be determined in this way. Of B's arrows only two hit the target, since his chance of hitting it is $\frac{1}{6}$; similarly, of C's arrows three hit it, and of D's six do so. Out of the total number of thirty-six arrows, then, only eleven hit the target, of which two came from B's bow, three from C's, and six from D's. If now we know about any arrow simply that it hit the target under these conditions, the chance that it was shot by B is $\frac{2}{11}$, by C $\frac{3}{11}$, and by D $\frac{6}{11}$. The general theorem which enables anyone to reach correct results in such problems is: If an event A only occurs when brought about by one or another of a set of events B, C, D, etc., the individual probabilities of which are P(B), P(C), P(D), etc., and if A is known to have happened, the chance that it was preceded by any particular one of the events B, C, D, etc., is a fraction, the numerator of which is the product of the individual probability of this event by the chance that it will be followed by A, while the denominator is the sum of exactly similar terms, one for each of the events B, C, D, etc.[8] In symbols,

$$P(B, A) = \frac{P(B) \times P(A, B)}{P(B) \times P(A, B) + P(C) \times P(A, C) + P(D) \times P(A, D) \text{ etc.}} \qquad (3)$$

[8] For the sake of symmetry, the effect here is symbolized by A rather than by B, as has previously been our practice.

The substitutions to be made in the particular illustration used, if we are inquiring what is the chance that a successful shot was fired by B, are:

$$P(B, A) = \frac{\frac{1}{3} \times \frac{1}{6}}{\frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{2}} = \frac{2}{11}$$

This theorem is often referred to as the theorem for determining the probabilities of causal hypotheses. Such a description is only appropriate, however, when certain particular situations where causal explanation is sought are intended. Suppose it has been discovered by the principle of agreement that an effect A may be brought about by any one of several causes B, C, or D, and no one has as yet succeeded in analyzing these causes so as to discover the common factor which is always present when A happens and always absent when it fails to happen.[9] Suppose also that enough occurrences of A have been examined so that we know what proportion of them, on the average, are caused by B, what proportion by C, and what proportion by D. Now an instance of A happens, but its antecedents were not observed. What is the probability that its cause was B? Was C? Was D? Clearly here are all the conditions requisite for applying Bayes' theorem. The student will find it good practice to work out for himself an illustrative case. The chief value of applying it in such situations is often merely that the result will indicate the most reasonable order in which to investigate the possible causes of a given effect. If B is the cause more often than C or D, obviously it should be systematically studied and brought under control first, other things being equal.

Nonquantitative probability

Thus far, the discussion of probability has been restricted to cases in which quantitative formulation of the probability of an event in terms of some proper fraction seems to be possible. But the concept is also employed in situations where

[9] Or this analysis may have been made; but in the particular problem at hand it is important to take account of the differences between the possible causes and not merely of their common features. A might, for example, be an epidemic of some disease, and B, C, D, etc., the various ways in which it could have been started.

no such formulation is feasible, at least at present. It is very important, of course, in these situations as well as in those already examined, to secure as much relevant evidence as is available to supply the basis for a probability judgment, and to make the latter conform exactly to the evidence. How this is to be done in these more difficult cases is a question that probability theory has not yet completely solved. In facing this problem we stand at the front line of scientific progress; the discovery of adequate principles of guidance here remains for the most part a task for the future. At present, science passes into art at this point; alert skill, based on varied experience, in discriminating the comparative likelihood that appropriately fits each special case, is the primary need. Two kinds of illustrative problems will be briefly examined.

Where relative frequency is still the court of appeal

In at least some of these further uses of the concept, the principle of relative frequency still appears to provide the court of appeal for testing the validity of whatever assertions are made, although it is very difficult to apply it. For example, suppose that one morning the headquarters of a labor union, in some city marked by frequent and violent labor disputes, have been invaded and ransacked. What is the probability, on the evidence, that this act was perpetrated by a gang of vigilantes, by spies from the employers' association, or by the police? Put in such a form, the question can hardly be answered—certainly no one could assign to each of these alternatives a definite fraction, as could be done in the case of solutions to the problems discussed above. But it may be possible to decide rather confidently which is the *most* probable alternative, and in reaching the decision the principle of relative frequency will play an essential role. An investigator will analyze the scene presented on that morning into its main observable phenomena—broken windows and a smashed door, desks overturned, files ripped open and several of their drawers missing, papers scattered about. In the case of each of these events thus discriminated, the question will be: What sort of persons would have been most

likely to do what has been done in just this way, leaving this precise evidence now before us? And how can any such question be answered, except by appealing to the relative frequency with which each phenomenon has been observed, in the past, to have been brought about by persons belonging to this or that kind of group, using this or that set of tools, and urged by this or that complex of motives? Here is the state of the files, for instance. Similar situations have been observed in the past. What is the relative frequency with which they have been found due to persons seeking incriminating evidence in general, to persons seeking evidence against particular individuals, to persons aiming mainly at wholesale destruction, etc.? When, in this way, the most probable description of the perpetrators has been decided upon, in the case of each of the pieces of evidence at hand, the several descriptions will be united into a composite picture of the authors of the entire act of vandalism. Then the final question becomes: Which of the various groups that might have been responsible fits most closely this composite description? The impossibility of assigning a definite probability fraction to the solution here reached will be due to three circumstances: (1) the relative frequencies by which the various partial solutions have been justified will ordinarily not be the same; (2) the uniting of these partial solutions into a composite whole is a qualitative integration, not a mere quantitative addition; and (3) the comparison of the total picture thus established with each of the possible groups of perpetrators is again a matter of qualitative discrimination rather than mathematical computation.

Where appeal to relative frequency seems impossible

There are, however, many situations in which the concept of probability is naturally used but where appeal to the principle of relative frequency is apparently inappropriate.[10] In general, these are situations incapable of analysis into factors of which each is a member of some class of recurring events,

[10] In dealing with these, therefore, in view of our definition of probability, we shall use the less precise term "likelihood." Probability is likelihood appearing under circumstances permitting appeal to relative frequency as a basis for its measurement.

whose frequencies can therefore be noted and recorded. Suppose that it is asked: What is the likelihood that, twenty-five years hence, all nations of the world will be living under some form of state socialism? The question is a meaningful one, at least from the point of view of our constant practical need to anticipate the future as wisely as may be, and to prepare ourselves for any eventualities. But since modern socialism is something new, showing, at best, merely partial analogies with political structures of the past, one cannot analyze it confidently into factors which have been historically repeated. Hence he has no adequate basis in terms of relative frequency for asserting what chains of preceding events are most likely to lead to its appearance. A still clearer example, perhaps, is found in a comprehensive scientific theory, which has already been supported by some data but which has not by any means been adequately established. What is the likelihood that it will be fully verified? Again, it seems quite impossible to analyze any such theory into elements repeating themselves sufficiently in the history of scientific explanations so that one can appeal to the relative frequency with which they have proved true to buttress his judgment. Each theory is new—a distinct, qualitative whole. The theory of relativity, for instance, involves many subsidiary ideas—about space, time, light, gravitation, inertia, etc. But none of these can be identified as a member of any class of scientific ideas whose chances of verification can be indicated by the frequency with which they have been confirmed in the past. In all these matters, judgments of likelihood seem to be an affair of art rather than of precise determination grounded on the application of definite rules.

This does not at all mean that in making them one guess is as good as another, and that scrupulous respect for whatever evidence is relevant can be dispensed with. Far from it. Here, too, the expert shows the same superiority over the amateur that he holds in any field of knowledge. Here, too, the competence of anyone's judgment varies primarily with the scope of his available knowledge, and with that disciplined

mastery of the data which issues in a sound feeling of what is important in any complex subject matter and how it bears upon other factors whose status is in question. In all matters of fact "probability (or likelihood) is the guide of life," and in our effort to render it an intelligent guide we must expect no magical shortcut.

EXERCISES

1. Define: probability fraction, equally likely alternatives, favorable chance, calculus of probability, dependent probability, relative frequency, nonquantitative probability.
2. Solve the following problems by use of the addition or multiplication theorem:
   a. What is the probability of an even number falling uppermost on the toss of a die?
   b. A die is tossed three times. What is the probability of an odd number falling uppermost each time?
   c. What is the probability of drawing a face card from a well-shuffled pack of cards?
   d. A lunch counter serves the employees of two factories which are equally distant and whose noon whistles blow at the same moment. In one, 14 out of 36 employees are Negroes; in the other, 20 out of 50. Assuming that there is no cause making it easier for members of any particular race to reach the lunch counter first, what is the probability on any given day that the first to reach the lunch counter from each factory will be a Negro?
3. Explain clearly the relation between probability and necessity. Between a probability fraction and a causal law.
4. Study carefully the following problem in circumstantial evidence. Indicate in detail how the principle of relative frequency would properly be used in reaching a solution to it.

   Mr. A is charged with arson, resulting in the destruction of his neighbor's house. There is no direct testimony to his guilt, nor can it be established by direct testimony that he was elsewhere at the time of the fire.

   a. The following items of evidence are introduced: He has been known to nourish a grudge against the injured neighbor. This grudge had been inflamed anew on the day preceding the fire. He was absent from home at the time the fire began. Footprints leading away from the burned building are such as might have been made by his shoes. He had

been hitherto a law-abiding person. He gives a plausible explanation of his activities at the time of the fire.

b. What difference in the probability of his guilt would arise if the following additional items of evidence are presented? A knife found near the burned building belongs to him. The fire began in a tool shed which was the scene of the altercation on the preceding day.

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CHAPTER 21

STATISTICS

In Chapters 16 to 18 our discussion of scientific method contemplated situations in which the available data, supplied either by common sense observation or by previous scientific inquiry, suggest a plausible causal or functional hypothesis. The immediate task of science in such cases is, of course, to test the hypothesis by the procedures described in those chapters and to see whether it can be verified, or, if not, whether some other equally adequate explanation of the data can be established in its place. **Introductory considerations**

The chapter just concluded, however, has not only called attention to the circumstance that even the most fully verified of such explanations are tentative rather than final, subject to correction by subsequent experience and therefore to be regarded as properly justified rather than as absolutely certain; it has also reminded us that there are many conjunctions in nature about which, for purposes of practical guidance, the most exact information that can be secured is needed, but which are not universal and hence give no immediate encouragement to the expectation that a causal or functional explanation might be established. We are tolerably certain, for instance, that every human being of a given age will sometime die—here is a quite regular conjunction—but if the question is raised *when* he will die no answer in terms of any universal law appears possible; we must fall back upon merely probable conjunctions, that is, upon the verifiable fact that while individuals of that age living in a given area die in various subsequent years, more of them die in a particular year than in either any earlier or any later one. Yet, if insurance protec-

tion is to be available on any dependable basis, knowledge about such probable conjunctions must be rendered as thorough and accurate as the relevant facts permit. In the less exact sciences, notably in the biological and social fields, it is such conjunctions as these that scientists ordinarily confront. One interprets their occurrence in harmony with the postulate of causality by noting that any process in nature is affected by a large number of forces, and that in some situations many of them are beyond our experimental control. They cannot be isolated and tested individually. Moreover, sometimes even small forces may affect the outcome of a process very noticeably, *e.g.*, the force which decides whether a coin falls heads or tails. In these situations discovery of a universal law is usually beyond our powers for the time being; the most promising conjunctions that can be detected are merely probable ones. By what main procedures is information about them gathered and thrown into such a form as will permit, wherever possible, an exact measure of their probability? How shall we bring together relevant data, and how formulate the results of their systematic study, so that, where a causal or functional explanation is as yet unrealizable, we may apply the postulate of predictive uniformity as confidently as possible to subsequent instances of the same phenomena?

The problems of statistics and of correlation

These questions lead to two closely related problems. The first may be briefly stated in terms of the distinction drawn in Chapter 15 between explanation and description. As was there noted, the concept of explanation is ordinarily restricted to those investigations of phenomena which succeed in tracing constant spatio-temporal conjunctions with other phenomena, the ideal hoped for being attained when it is found possible to reach an explanation in the form of a functional law. But, where the data already available about a phenomenon do not lend support for any hypothesis of a functional or causal explanation, there is obviously a preliminary problem that must be faced. That problem is to secure the most exact and systematic description of the facts that can be secured—the description which, on the one hand, will express present avail-

able information about them as accurately as possible, and on the other hand, will provide a basis for measuring the probability under specified conditions of whatever conjunctions appear. By such a description the way is also prepared as well as may be for eliciting causal or functional hypotheses should such become feasible. How does one go about studying any phenomenon or group of phenomena in such a way as to attain exactitude of description in this sense?

Here is the problem of *statistics*. By this word is meant just that part of the scientific enterprise which is devoted to the collection and organization of data so that they will best serve these ends of systematic and accurate description. Acquaintance with the methods employed here is important not merely for those who expect to engage in statistical investigations. All who live in the contemporary world need to know how to interpret the results of such inquiries and to criticize the conclusions proffered by others. One who cannot tell under what conditions statistical conclusions are trustworthy, and thus cannot detect fallacies in them, may be easily imposed upon by persons who are more eager to justify biased conclusions than to reach the truth about the matters concerned.

The second problem follows hard upon the first. In the course of achieving a precise and objective description of any fact, various conjunctions will be noted between it and other facts, and even though no universal connection is revealed, clues will be suggested as to which of the less than universal conjunctions are most promising. How are the available facts about these conjunctions systematically investigated; how is the degree of approximation toward regular conjunction measured; and how are the results expressed in as exact a form as possible?

This is the problem of establishing *correlations* between phenomena.[1] All such cases can be viewed as exhibiting some degree of conjunctive relationship which varies from merely

[1] I use the word "correlation" here to cover association of attributes as well as the conjunction of variables. Most statisticians, it should be noted, include correlational investigations within the general field of statistics.

accidental concomitance (called "independence" of attributes or variables in statistical theory) at one end of the scale to the perfect detailed correspondence exhibited in a functional law at the other. This measurable degree of correspondence is what is meant by "correlation."

The present chapter will deal with the main topics of general significance which cluster around the amassing and tabulating of statistical knowledge, while the following chapter will deal similarly with the subject of correlation.

Prescientific vs. scientific description

A study of statistical procedure may well begin by considering the difference between prescientific and scientific descriptions of phenomena, and the corresponding difference between the kind of information which, from each of these viewpoints, is naturally sought. Take the facts of plagues and epidemics as an illuminating example. In most areas of the world, before the development of a scientific approach, these dire events were unquestioningly regarded as punishments inflicted by some divine power on the hapless communities affected. When this is the way in which they are conceived, what data about their occurrence would it seem relevant to secure? Obviously, such considerations as the following will be regarded as important: the particular community that is being scourged, the deity or sorcerer who is responsible for the plague, the motive which animates him in inflicting it, and the methods which may be counted on to appease him and win his favor instead of his curse. In this context of ideas there would be no encouragement to look for any of the information that a modern statistician would regard as significant except the first item. Now suppose that this climate of thought is changed; the notion dawns that disease may be an entirely natural event which might be predicted and prevented by human effort without regard to any supernatural beings. What sort of information will be needed, once this conception has come to be accepted among the intellectual leaders of the community? Attention will, of course, be focussed on the conjunctions between cases of disease and circumstances within man's control about whose occurrence

knowledge can be hopefully gained. And since, at first, nothing may be known as to which conjunctions offer promising clues in this regard, the need that must be initially met is to keep an exact record of the cases as they occur, in all the conjunctions that seem at all relevant. This means noting *how many* of them take place during such and such a time, in each of the localities where they appear, under such and such attendant circumstances which might be found to vary with them and thus to throw light on their possible causes—circumstances like war, flood, famine, insect migrations, climatic changes, etc. It also means noting the *degree* of severity of each of the cases recorded. In brief, it means *counting* instances of the disease under these various conditions which experience suggests as worthy of notice, *measuring* (by whatever method of measurement is available) their severity, and keeping the results in the form of a record designed to bring out as clearly as possible whatever scientific values they may possess. Only by the careful accumulation of such data can worthwhile clues be suggested as to which of these attendant conditions are more frequently conjoined with the phenomenon studied than others, and hence indicate points at which effective remedies might be sought and applied. Statistics is thus an affair of systematic counting and measurement, carried out for the purpose of furthering scientific understanding and intelligent control of the phenomena involved.

The process of counting

Counting seems a simple process, but just what does it involve? To count anything, we must first have a way of telling what is a single or unitary instance of the phenomenon under study, so that each recurrence of such a unit may be counted as one. It is easy to forget the essential character of this requirement because the statistician often confines himself to problems prescribing by their very statement the unit to be used, and in dealing with which no difficulty arises in telling whether a given fact is or is not a unitary instance of the phenomenon studied. Thus when statistics are secured on the number of births, deaths, bank failures, earthquakes, etc., in a given community during a certain time, no question arises

about the unit to be used—we assume from the very statement of the problem that each case of these events is to be counted as one. And whether a given happening is or is not an instance of any of these phenomena is usually not hard to determine. This is the situation with the many investigations of social problems in which each person in the community contemplated is obviously a relevant instance, so that each becomes a unit for the purpose of counting. It is a fortunate circumstance that many other familiar facts, too, are readily identifiable as either members or not members of the class of events which is under investigation. That the situation is not always as simple as this, however, is revealed if we imagine ourselves assigned the task of counting cases of, say, piety in a specified community. As stated, this problem involves two ambiguities. First, what is to be taken as our unit—each instance of a pious individual or each instance of a pious act? Clearly we cannot embark on the counting until we know which of these alternatives is to be chosen. Furthermore, after this has been decided, a more difficult problem must be solved. By what criterion are we to decide whether a given individual, or a given deed, is pious or not? Piety is an indefinite property; different definitions of it are plausible, and hence there are many borderline cases which some would regard as pious and others not. Hence, without a confidently applicable criterion of what constitutes piety, instances of it cannot be counted in any objective and dependable fashion. Here is intimated one of the most challenging tasks that science must face whenever it has not been performed already—the task of rendering the vague terms of common sense description more definite, or of replacing them by more precise concepts, so that the phenomena referred to may be investigated with some hope of reaching definitive results.

The nature of measurement

When objects or events which have been identified as members of a given class, and thus rendered capable of being counted, are observed to vary in degree, the problem of measurement arises. Measuring, like counting, is a form of quan-

titative analysis; as briefly shown above,[2] it presupposes the discovery of some property in the phenomenon measured which can be treated as a magnitude, and hence with which the number series can be correlated. Many such properties are already familiar, and in their case measuring itself becomes a simple problem in counting. Some value of the magnitude is counted as one; it then becomes the unit to be applied in determining the numerical result. Suppose, for example, I am asked to measure a man's girth at the waist. I employ some conventionally accepted unit of distance which is appropriate in view of the size of what is to be measured, say the inch, and count the number of times it is contained. There are many phenomena, however, in whose case no such measurable property has as yet been clearly analyzed, so that no appropriate unit has as yet been fixed. As noted by anticipation in Chapter 17, we might wish to measure mental phenomena which all human beings reveal, but in varying degree, such as intelligence, courtesy, initiative, memory, ambition, and the like. To do this, we must find some way of viewing these characters quantitatively, and it is a significant gain of the present epoch in psychological and social science that much progress has already been made in securing numerical equivalents of some of them. But this can only be done by discovering some magnitude which, for all essential purposes, can be treated as equivalent to this or that property in the differences of degree that it exhibits. Then every instance of the property investigated can be located on a quantitative scale; it becomes a multiple of some clearly defined unit or is numerically related to some chosen standard. In the case of intelligence and memory, this has been done by experimenting with various types of questions to be solved; considerable progress has been made in finding a combination of questions such that, in the opinion of competent students, ability to solve a certain percentage of them in a given time would constitute an adequate quantitative equivalent for intelligence or memory as

[2] See chap. 11, pp. 204 ff., and chap. 17, pp. 345 f., 353 f.

possessed by the normal individual at the age studied. At least, in the so-called intelligence tests, a method of measuring intellectual competence has been created which educational institutions are finding superior for the purpose to some of the traditional ways of judging intelligence. But it is important to remember that somebody was first to discover even those tools for measuring physical facts which we now take for granted in all our exact thinking about nature. It is clear that no exact—that is, mathematically formulated—information about anything can be secured until such quantitative equivalents have been discovered, and satisfactory techniques of measurement have been established.

Ability to deal more and more effectively with the most vital human problems is dependent, in part, on the rapidity with which we learn how to perform this quantitative analysis on the other features which, in the interest of larger human welfare, need to be understood precisely and predicted confidently. When we can measure numerically in terms of an accepted unit or standard such properties as tolerance, artistic ability, generosity, statesmanship, sincerity, and inventiveness, and thus can search hopefully for correlations between them and more controllable events, statistical science will have contributed what it can to the enterprise of education through which man endeavors to eliminate undesirable traits and realize to the full those which lead to creative happiness.

Two kinds of scales

There are two general kinds of measurement which it is important to distinguish. One is the measurement of *additive* properties, exemplified in the common methods of measuring length. If we have three sticks, one two feet long, one four feet long, and one six feet long, it can be said that the third is just as long as the other two added together; also, the second can be said to be just twice as long as the first. This is what is meant by calling length an additive property. But if two individuals have an intelligence quotient of .70, and a third an intelligence quotient of 1.40, we cannot say that the third is just as intelligent as the other two put together, or that he is twice as intelligent as either of them. Here the property

measured is *nonadditive*. This is revealed in the circumstance that the quotient 1.00, by reference to which all other degrees of intelligence are calculated, is not a unit, as the foot is a unit of length, but is rather a standard, representing the average intelligence of individuals belonging to a given class. The quantitative difference, then, between any person's quotient and this normal figure simply measures his deviation from the average of his group.

If the reader is puzzled by this difference, thinking that wherever numbers are used they should have all the characteristics that they exhibit in the series of arithmetical numbers, and be subject to all the operations that can be employed on the latter, he should recall the discussion of mathematical series in Chapter 11. A series may have all these characteristics, in which case the property identified with such a magnitude becomes a property measurable on an additive scale. Weight is a property of this sort; the time occupied by an event is another familiar illustration. When time is measured in terms of any conventional unit, such as the second, hour, or year, it can be dealt with by any of the operations of arithmetic in the same way that distance can. But a series may have certain of these characteristics without having the rest. It may, for example, merely be such that the relation between any pair of terms is asymmetrical and transitive, and yet we might find it convenient to use the series of numbers to refer to the successive terms. Clearly, the numbers in such a case will only be subject to those arithmetical operations that are implied as permissible by the asymmetrical and transitive character of the relation, not to others.

Suppose that a group of a dozen men are ranked in order of height, without the exact height of each being measured. The series thus established is asymmetrical and transitive. For, of any two men of the group, if one is taller than the other, the other cannot be taller than the one. And of any three of the group, if one is taller than a second and the second is taller than the third, the first must be taller than the third. Now it would be convenient to use the series of num-

bers to identify the rank of each man in this group with respect to height. This could be done in various ways; a very simple way would be to designate the shortest by the number 1, the next in height by the number 2, and so on, the tallest being numbered 12. But obviously, in this series, the person assigned number 4 is not twice as tall as the one assigned number 2, nor is number 8 equal to the sum of 5 and 3. Indeed, even the differences in height between any two pairs of contiguous individuals will not, except by chance, be the same. Number 4 might exceed number 3 by an eighth of an inch, while number 10 might exceed number 9 by three-quarters of an inch. Hence whatever operations are employed must be consistent with these circumstances. Another way, following the clue employed in reaching intelligence quotients, would be to assign number 1 to the individual who comes nearest to the average of the group, designating those who exceed him in height 1.1, 1.2, 1.3, etc., and those who are shorter .9, .8, .7, etc.

Now in this situation we are treating height as though it were a nonadditive magnitude, although we know perfectly well that it can be treated as an additive one, by measuring the exact height of each member of the group and assigning to him the number reached by such measurement. There are many properties, however, that we do not as yet know how to treat in this way, but in whose case an asymmetrical and transitive series can be established. Intelligence is such a property, hardness is another, temperature is a third, importance is a fourth. And it is convenient to rank the successive values by numbers, but when this is done it is essential to remember that the numbers are only subject to certain operations that are determined by the relation present, and not to others. Consider the last of the properties just mentioned. When a man of affairs comes to his desk in the morning he will compare in importance the tasks which he hopes to get done that day, ranking them in order. Thus, in case it proves impossible to do all of them, he will be sure to perform the ones of greatest importance. And very likely he will assign

numbers to them on his memorandum pad, the most important being numbered 1, the next 2, etc. But it is clear that this is a nonadditive scale, not an additive one.

Purposes guiding statistical inquiry—1. to throw accurate light on a single phenomenon

We now turn from this brief examination of counting and measuring to a general classification of statistical investigations. This classification will be in terms of the purposes which may guide a statistical inquiry, for the selection of data which are going to be subjected to counting and measurement, and the way in which these processes will be performed in detail, depend, in the last analysis, on the purpose behind the inquiry.

Most investigations that can be called statistical fall, in this regard, under one or another of three heads. And a brief discussion of these will naturally lead to the two themes of general logical interest with which the balance of the chapter will deal—the ways in which a statistician selects a representative sample, and the forms in which he condenses his data so that their significance will be clearly revealed.

First, one purpose which may guide a statistical study is that of throwing accurate light on a single phenomenon, which for one reason or another has challenged investigation. Situations in which this is attempted vary greatly in detail; we shall give two illustrations which bring out this variety.

Among the more ambitious projects which fall under this head is that of securing an exhaustive description of some complex phenomenon, in all its possibly significant properties and relations. In the social sciences the procedure employed to this end is called the "case study" method. Two well known examples of this method are found in *Middletown* and *Middletown in Transition,* by R. S. and H. M. Lynd. These books attempt an exhaustive description of a middle western American city of moderate size, bringing out all the social problems which urban life under such conditions presents.

Less ambitious in scope, though often complex enough in the details that are involved, is an analysis of some single phenomenon to the end of solving some specific problem about it. Where there are many possible solutions, the method

pursued will reflect this circumstance; where the solutions are narrowly limited by the nature of the problem, a method is appropriate which may be called that of "converging evidence." Such and such are the possible solutions; toward which of them do the relevant data, as accumulated and when considered together, point, so that it may be regarded as the most probable one? The data will, of course, be counted and measured whenever the evidence will be revealed more clearly by doing so.

On June 30, 1930, baby boys were born in a Chicago hospital to Mrs. William Watkins and Mrs. Charles Bamberger. They appeared on the same day, Mrs. Watkins' baby at half past four in the afternoon and Mrs. Bamberger's shortly before seven in the evening. Nine days later the babies were taken home with their mothers. Soon after they were brought home, Mr. Watkins noticed that the name on the bit of adhesive tape affixed to the back of his baby was Bamberger. He called up the Bamberger family. In the wastebasket at the Bamberger home a bit of tape was found bearing the name Watkins. The problem faced by the parents was to determine whether the babies were in the right homes or not. The evidence already at hand strongly suggested that they were not, but by itself was hardly conclusive. There are only two possible solutions: Either the babies are in the right homes or they should be exchanged. On which of these conclusions does the relevant evidence, when gathered and carefully examined, converge?

The testimony of the nurses who had cared for the babies, which might have been quite conclusive, was meager and ambiguous. The general appearance of the two babies was sufficiently alike to allow no decision on that ground. Accordingly, appeal to indirect and circumstantial evidence was the only resort available.

Dr. A. H. Kegel, Health Commissioner of Chicago, who was asked to assist in the matter, appointed a committee of eight experts to test the parents and babies and make a report. The

tests consisted of all the comparisons which present physiological and anthropological knowledge indicates might throw light on hereditary relations. The dermatologist who examined the birthmarks on the Bamberger baby and Mr. Bamberger found them without evidential value. Two fingerprint experts disagreed, one holding that the babies were in the right homes on the strength of the prints, the other that they furnished no definite indication. The neurologist who examined the reflexes of the parents and babies found that the baby in the Watkins home and Mrs. Watkins both had exaggerated reflexes, but held that this was not a very significant factor. The anatomist and anthropologist, who studied the head measurements of parents and babies, was confident on the basis of his comparisons that the babies were in the wrong homes. The expert in blood analysis, however, was most positive in his report. He said that the blood of the baby in the Bamberger home was identical with that of Mr. and Mrs. Watkins, which is listed as number four, while that of the baby in the Watkins home was number two. As the blood of Mr. and Mrs. Bamberger was at wide variance, he believed that the baby with number two blood belonged to them. On the basis of these findings five of the committee voted it as their opinion that the babies were in the wrong homes, one that they were in the right homes, while two felt sufficient uncertainty to offer no judgment. After some weeks of dispute the babies were traded in accordance with the majority vote of the committee.

In this case, as in many of the specific problems faced by statisticians, no one of the lines of evidence examined was alone sufficient to answer the question which baby belonged to which parents. But, as all were relevant, it was necessary to bring them all together, to express them in exact form wherever possible, to evaluate the comparative importance of each, and then to decide which of the alternative solutions was on the whole more strongly supported. A situation analogous in every significant respect except that the number of possible solutions may be larger, is the attempt to determine, on cir-

cumstantial evidence, the authorship of a crime, or of such an act of vandalism as was described in the preceding chapter.[3] The principles of probability there illustrated are obviously also pertinent to the case just described.

2. To secure data about all members of a given class

Second, the aim of a statistical study may be to secure data about a class of phenomena, by examining each member of that class. Such a study will engage in such processes of counting and of measurement as seem likely to yield results that will be of value in solving problems that might subsequently be raised about the members of the class. Where the class is small, *e.g.*, the fruit trees in a certain orchard, such an investigation may be quite simple and brief. Where it is large, and the data sought rather numerous, the inquiry may be long, laborious, and expensive. A good example of the latter sort is the United States census. It not only records the number of inhabitants in the nation and each of its political subdivisions, but also specifies how many belong to each sex, each race or nationality, each age, each occupation, and it records many other matters on which exact knowledge might later prove useful. Anyone who projects an inquiry about American residents which involves such information will often find his counting already done for him by the Census Bureau.

3. To investigate a class through a representative sample

Third, the aim may be to reach justified conclusions about an entire class, but to do so without examining all the members of that class. Time and cost often make it impossible to carry on such a study in any other way; also, many classes include members that do not yet exist but will do so only in the future. In such cases it is evidently necessary to investigate the class through a sample, which must be so chosen as adequately to represent the whole. A very large number of statistical investigations are of this kind.

In these situations, the most challenging feature of the problem is that of selecting the representative sample. Suppose that a statistician is interested in studying the subject of suicide. He can examine in detail a limited number of suicides only, but he wishes his conclusion to apply to a far wider

[3] See above, pp. 450 f.

field—a field including suicides in other communities, and in his own community at other times. The suicides that he studies must then be so chosen that the result may legitimately apply by inductive generalization to this wider field of phenomena not yet examined.

The problem of statistical sampling

How, in gathering the limited data that constitute his sample, shall he safeguard in advance this leap of induction? His aim is to establish a result which will permit as dependable predictions as are possible regarding instances of a class that have not yet been observed, on the basis of a sample of instances that have been observed. What principles of selection should guide him, so that his sample will be genuinely representative of the whole class of facts about which he wishes to make predictions? Let us consider this problem with some care.

When one is verifying a hypothesis of a causal or functional law, this question is answered by the principles of agreement, difference, and concomitant variations. There, those three principles give him what general guidance can be given in deciding under what conditions his sample is adequate, *i.e.*, is such that unambiguous results from its examination can properly be expected to apply to any event of the same class. But the statistician is engaged in a process of preliminary description and quantitative analysis of his data; he does not yet know whether a causal (not to say functional) explanation of the material he is trying to clarify will ever be possible. The promising conjunctions of his facts often vary so widely at different times and places that it would be quite absurd even to attempt any predictions about too wide a class. His field, then, will in most cases be severely restricted temporally and spatially—it will be the present population of a particular city, the members now belonging to a given trade union, the crimes committed in a certain country during a specified period, etc. How does he realize, for his more limited purposes, conditions analogous to those formulated in these three principles?

Well, so far as concerns the principle of concomitant variations, analogous conditions are realized by the procedures of

accurate counting and measurement, together with those employed in establishing correlations which will be discussed in the following chapter.

With regard to the principle of agreement, it will be recalled that the essential requisite is to secure a sample exhibiting the supposed cause in all the variety of circumstances in which it may occur in its appropriate context. What constitutes the analogous problem confronted by the statistician, and how does he meet it?

An illustration will help to bring out the precise nature of the difficulty as the statistician faces it. I am asked, let us say, to find out how large a percentage of their waking time people in my county give to political interests, and how this compares with the time given to other interests, such as business, family, recreation, and religion. Suppose that I carry out this task merely by visiting a number of my neighbors and persuading them to note for a few weeks the way in which they spend their time, as itemized under such heads. Having gathered these data, I proceed to compute an average and discover, as a result, that about 5½ per cent of waking time was apparently devoted to political interests, and that four other interests consumed a larger portion of their hours out of bed. What is the significance of a result thus reached? Am I justified in announcing as a general conclusion that persons in my county count politics important only to the extent of one eighteenth of their time, and that they make it subordinate to four other interests?

Evidently, this is not justified at all. I must recognize frankly the limitations of an inquiry carried out in this way. What I have done is comparable, in the field of statistics, to the procedure which, in causal investigations, was called that of simple enumeration. Let us note the more important of these limitations. My neighbors who were kind enough to help me in this investigation are all inhabitants of a particular community within a particular part of the city in which I live. I have no right to assume that they are a typical sample of people in another corner of my ward, still less that they are typical of

the entire city, not to speak at all of the larger area originally assigned. Moreover, when I let this geographical limitation suggest others, I see plenty of reasons why they might not be typical. To live where they do, they must all be people whose incomes are within narrow limits, certain races are practically excluded, and since the majority of them belong to a very few professions, their standard of education and intelligence is not the average standard. Surely such facts are very likely to affect the behavior of these people with reference to their political activities. And very likely the time factor should be taken into account. It may be that the time of year makes a difference, or at least the nearness or remoteness of an election. Obviously, some principle of selection that will guide us in taking proper account of these matters is essential. We want to select in such a way that no group of members of the class investigated has a greater chance of being picked than any other group; otherwise those selected will not be representative.

The two most important principles applicable in meeting this need are those of *random selection* and *stratified sampling*. Each is appropriate under certain conditions.

The method of random selection

Random selection is the simpler of the two. It is appropriate when the data are such that no list of factors like those mentioned in the illustration just discussed need to be specifically considered. This is the case when the material dealt with is sufficiently homogeneous so that, as far as our preliminary knowledge about it can determine, there are either no varying circumstances which might significantly bias the result, or such circumstances are pretty evenly distributed. In such a situation, it is sufficient and correct to pick our sample at random.[4] The assumption is that whatever varying factors are present will be cancelled out by a large enough number of random samplings, so that the entire sample will fairly represent the whole. Consider a familiar example. When an in-

[4] Random selection has often been defined in terms of the general purpose governing all sampling selection, namely, the purpose of securing an equal chance of selection for each individual. But this purpose is shared by stratified sampling, too, hence such a definition is illogical.

spector examines a carload of wheat, he samples the grain from different parts of the car, and there seems to be no reason why a sufficient number of such selections of a supposedly homogeneous article should not adequately represent the whole carload. Of course, if in cases of this sort, actual experience should belie the assumption, one ought to be ready to modify it and to respect whatever varying circumstances become relevant. In the above illustration this would be suggested if handfuls from different parts of the car should show marked differences in quality. But if the random samplings agree with each other in the main as they increase in number, the assumption that no attention need be given to varying circumstances is justified and the random method approved.

It might be thought that random selection is equivalent to chance selection. Such is not the case, however. Selection here and there by chance does, to be sure, secure the elimination of conscious biases, but it does not eliminate unconscious biases, and statisticians have found that these play a considerable role in human choices. Hence it has been necessary to develop artificial techniques to make sure that unconscious preferences are cancelled out, and that the selection is genuinely a random one.[5] The problem thus met is of considerable theoretical interest, but we must not take the space to discuss it here.

The method of stratified sampling

But in many important statistical studies the method of random selection, by itself, would be quite inappropriate. This is so with the illustrative case above discussed. Political interest is not a property homogeneously distributed among the inhabitants of a country. We already have enough information about such a field to know that political interest is likely to vary from one group to another: it will be different in different age groups; it will be different in the two sexes; it will be different among teachers, lawyers, and ministers from what it is among those who pursue other callings; it will be different among persons of different racial strains, etc. A sample chosen at random might not be typical of the whole field at all. A group of city blocks picked at random might contain a dispro-

[5] Such as the use of Tippett's numbers.

portionate number of persons in a certain few professions, or persons whose parents were immigrants from central Europe, or persons of meagre education. Clearly, this possibility must be guarded against. How should a sample be picked in cases of this sort? The answer is: by stratifying the sample in accordance with whatever relevant factors random selection cannot be trusted to take care of. In most cases (including the present one) this amounts to employing a method of proportional weighting. That is, all the important factors which might conceivably bias the study must be distributed in the sample in the same general proportion as they are distributed throughout the whole field. Within any one of these strata the selection is random. Only thus may we have confidence that the sample is representative, so that predictions about the field based on what is discovered in the sample are rationally justified.

Illustrated by a study of nonvoting

It is a well known fact that not all American citizens regularly exercise their right to vote. Professors Merriam and Gosnell, of the University of Chicago faculty, undertook a study of nonvoting in the city of Chicago, to secure more exact understanding than had previously been gained about its conjunction with other facts. In pursuing this study they found it necessary to select a sample of the nonvoters of Chicago. A group was to be picked small enough to make personal interview possible, but representative in every relevant respect of the entire body of nonvoters, so that conclusions reached from its examination should hold within a narrow margin of error for the whole class of nonvoters. How was this to be done? Well, the random method, by itself, obviously would not do. It would not exclude the possibility of unintentional bias which might favor certain groups not typical of the nonvoters as a whole. Hence this possibility must be specifically guarded against. Now in certain respects this could be done quite simply. Records of a recent election showed, for example, that twice as large a percentage of women as of men failed to vote; so the authors made sure that their sample consisted of two thirds women and one third men. But it seemed important

also to guard against the possibility that selection at random, in various parts of the city, might unintentionally favor certain racial groups, such as, say, native whites of native parentage, Negroes, foreign-born whites, and in the case of the latter, particular racial stocks. There were no available figures about the percentage of nonvoters in each of these groups, but census records showed how they were distributed within the city of Chicago as a whole. Hence if the sample were distributed in the same way, it would be possible to render it representative of the entire group of citizens. This would be safer than to run the risk of merely random selection among the nonvoters. By selecting, then, a sample revealing roughly the same percentages of distribution in these respects as obtained in the entire city, it was possible to insure its representative character with reference to such matters. The number of nonvoters originally interviewed was around 6,000, but for the purpose of deriving the significant results nearly 700 were thrown out, in order that the distribution should closely agree with that in the population at large. This brought down the group statistically examined to the number of 5,310. The accompanying table[6] gives a comparison of the percentages of distribution, in the respects just noted, of these 5,310 nonvoters with the adult community at large, indicating how representative the investigators were able to make the group studied.

Moreover, as in the case of random sampling, the course of the investigation itself soon indicates pretty clearly whether the sample is representative of the entire field or not. Are the results reached in the early stages of the investigation maintained without substantial change in the later stages? If they are, this is a strong indication that the group selected is genuinely representative, and that if it were enlarged the outcome would vary only slightly from that already reached. This situation was quickly revealed in the inquiry in question. "That the sample taken was fairly representative is further shown by the fact that the characteristics found in the first small samples

[6] This table is quoted by permission of the University of Chicago Press.

Color and Nativity	Adult Citizens in Chicago (1920 Census)		Adult Nonvoters Interviewed (Election of April 3, 1923)	
	Number	Per Cent Distribution	Number	Per Cent Distribution
Total	1,366,515	100.0	5,310	100.0
Native white—native parentage	345,017	25.1	1,323	25.0
Native white—foreign parentage	529,800	39.0	1,308	25.0
Native white—parentage unknown	....	....	331	6.0
Negro	88,620	6.0	350	6.6
Foreign-born white—naturalized	403,078	29.9	1,998	37.4
Country of birth of foreign-born white:				
England and Canada	*25,461*	*1.9*	*99*	*1.8*
Ireland	*41,455*	*3.0*	*123*	*2.3*
Germany and Austria	*99,123*	*7.2*	*368*	*6.9*
Norway, Sweden, and Denmark	*53,939*	*4.0*	*303*	*5.7*
Russia	*39,068*	*2.9*	*277*	*5.2*
Poland	*43,840*	*3.2*	*286*	*5.4*
Czecho-Slovakia, Jugo-Slavia, and Hungary	*39,154*	*2.9*	*239*	*4.5*
Italy	*18,156*	*1.3*	*268*	*5.0*
All other countries	*42,882*	*3.5*	*35*	*0.6*

were maintained in other and larger samples as the inquiry went on. The later material accumulated, for example, in racial analysis, confirmed the earlier material, thus giving consistency and uniformity to the sample, and tending to establish its representative nature."

A relatively small sample is often sufficient

It is very instructive to note that when the method of stratified sampling is intelligently employed, a very small selection from the field may constitute a sufficiently representative sample. Hence a substantial saving in time and expense is made possible. In 1936, the Literary Digest, following a practice of many years' standing, attempted to forecast the result of the November election. Its investigators used the method of random selection, but in a way which did not sufficiently guard against unintentional biases. Their sample was enormous in size—over 3,000,000 citizens—and the expense of send-

ing out and tabulating the ballots was considerable. On the basis of the returns, the Literary Digest predicted Landon's election; but as a matter of fact he only carried two out of the forty-eight states. By contrast, the Gallup polls, using a sample of only 5000 to 10,000 citizens but employing the principle of proportional weighting with scientific care, have been remarkably successful in gauging current opinion and predicting its expression in action, at the ballot box and otherwise.[7]

How, now, does the principle of difference play a part in guiding the statistician's procedure? In causal investigations its role is to secure a systematic comparison between what happens in the appropriate context when a certain supposed cause occurs and what happens when it does not occur, other circumstances being as far as possible the same. The statistician likewise wishes to compare the conjunctions which appear when a phenomenon that he is studying is present with those that appear when it is absent, other circumstances being similar. Otherwise he might unduly emphasize a conjunction which appears prominently with A when as a matter of fact it appears equally frequently with non-A. How does he make such a comparison?

Use of a control group

He does it by setting up what is known as a *control group.* A control group is a collection of instances distinguished by the absence of the phenomenon under investigation, but otherwise as similar as possible to the instances in which that phenomenon is present. Thus, in the Merriam and Gosnell study of nonvoting, the investigators set up a control group consisting of voters—of approximately the same size (5,159) as the sample of nonvoters, drawn from the same areas of the city—and included in it all the main groups which had been included in that sample in order to render it representative. By means of systematic comparisons between these two samples, it was possible to find out what the main differential characteristics of the nonvoters were, as contrasted with the voters. For example, it appeared that a distinctly larger per-

[7] Even the Gallup polls, however, have not been right in all their predictions.

centage of the voters than of the nonvoters fell between the ages of twenty-one and twenty-nine; that while only one eighth of the voters had been residents of the county less than ten years, approximately a third of the nonvoters had been residents less than that period; and the influence of the sex differential was known already. Thus it was shown that whether one voted or not in Chicago at that time was significantly connected with the factor of sex, with that of age, and with the length of his residence in the district. Obviously, these comparisons reflect the same guiding idea that is expressed in the principle of difference when it is employed in situations where the universal cause of a certain effect is being sought.

We turn now to certain general considerations that are important in the recording and organizing of statistical data.

Importance of a careful record of any investigation

In view, first, of the fact that it is always possible to fall into some error in procedure or in the interpretation of one's results, the scientific statistician is always careful in his published report to describe accurately every step of his investigation. This makes it possible for others to tell whether his announced findings are really supported by the relevant facts or not, and to correct any mistake that may have crept in. And if there should be some mistake, the accumulated data will still not need to go for naught; they can be disentangled from the errors and may aid in reaching more objective results in the future. Wherever possible, a scientist will even locate such inadequacies and doubtful steps himself, calling them to the attention of later investigators. Since his aim is truth rather than the buttressing of some prejudice of his own, he forgets personal pride and adopts in such matters the attitude of E. A. Ross in the preface to his *Social Psychology:* "I offer this book with the wish that what in it is sound be promptly absorbed into the growth of the science, and the unsound be as promptly forgotten. . . . Perish the book, if only social psychology may go forward."

Second, we must examine briefly the manner in which any careful statistical study will present the quantitative results

which it has accumulated. For a fuller and more technical treatment of the kinds of table, chart, and graph that may be employed, together with the main problems faced in determining averages, deviations, curves of distribution, etc., the reader should consult a textbook on statistical procedure.

Presentation of statistical results—frequency tables

The primary data gathered by a simple statistical inquiry can often be best summarized in the form of a *frequency table.* Suppose, for example, that in addition to sending in to the university recorder a grade report for each student in a recent class in Introduction to Philosophy, I am asked to indicate the distribution of the various grades in the class as a whole. This can best be done by a frequency table such as the following:

Phil. 1 First term 1941–2	40 to 49	50 to 59	60 to 69	70 to 79	80 to 89	90 or above	Total
Number	2	4	9	24	15	6	60
Per cent	3	7	15	40	25	10	100

And the distribution according to these intervals can be graphically represented by a *frequency polygon,* as shown in

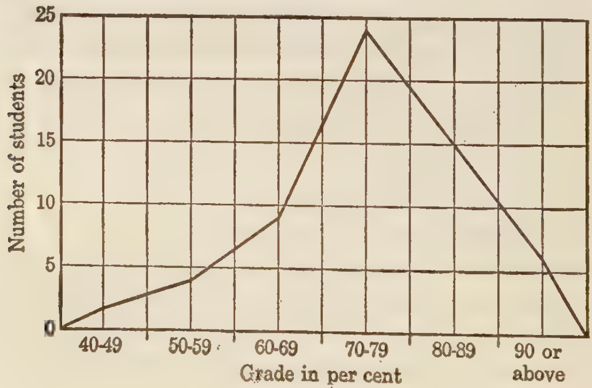

FIG. 18

Fig. 18. Such a table and curve exhibit, in a form aiding ready apprehension, an answer to the question, how many students in the class were given grades falling within each block of ten

percentage points—that is, how frequently each of these blocks was assigned.

The determination of averages

Suppose, now, that I were asked to determine the *average* grade of another class containing thirty members. There are several ways of interpreting the meaning of "average" in dealing with statistical data; each is helpful in regard to some of the further purposes which a statistician may have in mind, and no one of them is best for all such purposes. The most familiar average is the *arithmetic mean* of the various particular items involved. This is determined by adding together the individual measurements and dividing the sum by the number of individual cases. For this purpose I should need to record in a table each member of the class, with his grade, as is done on the next page (the reader will at present neglect all but the first two columns).

The column to the left lists the class members, and the column next to it gives the grade of each. These grades are added together, and their sum (2,284) is divided by the number of class members (30); the result is the arithmetic mean, which in this case is 76.

For some purposes and in some situations the *mode* is the best average; it is that individual item in the collection which occurs most frequently. In a frequency polygon it is graphically exhibited as the peak of the curve. In this table the mode is 75, which is the only item occurring four times. The *median* is in many cases a good average. It is the middle item when the various items are arranged in order of magnitude.[8] In the present table they have been arranged in this order, partly so that the median (and the mode) will be revealed clearly. An arrangement of this kind is called an "array." If the number of items happens to be even, as it is here, the median is defined as the mean of the two middle items. Now the two middle items in this instance are L. F. and T. H., whose grades are 78 and 77 respectively. The median of the series as a whole is, therefore, 77½.

[8] In the case of grouped data the computation of the median is somewhat more complex.

Student	Grade	Deviation from Average	Square of Deviation
T. S.	95	+ 19	361
H. B.	92	+ 16	256
R. L.	90	+ 14	196
R. W.	90	+ 14	196
A. J.	88	+ 12	144
S. S.	87	+ 11	121
C. F.	87	+ 11	121
H. D.	86	+ 10	100
P. A.	85	+ 9	81
M. I.	85	+ 9	81
N. R.	83	+ 7	49
G. E.	83	+ 7	49
F. O.	82	+ 6	36
P. W.	80	+ 4	16
L. F.	78	+ 2	4
T. H.	77	+ 1	1
E. R.	75	− 1	1
S. Y.	75	− 1	1
M. O.	75	− 1	1
H. P.	75	− 1	1
T. S.	72	− 4	16
T. B.	72	− 4	16
L. R.	70	− 6	36
N. P.	67	− 9	81
B. D.	65	− 11	121
C. R.	62	− 14	196
E. T.	60	− 16	256
D. U.	55	− 21	441
V. M.	50	− 26	676
S. W.	43	− 33	1089
Total	2284	300	4644
Arithmetic Mean	76	10.	$\sqrt{154.8}$ = 12.4

To interpret the significance of any statistical table it is necessary to determine, in addition to the average, the range of variation of the individual items—technically called the amount of *dispersion* in the collection. In the table in question the dispersion is rather considerable, the grades ranging irregularly all the way from 95 down to 43. It would obviously be possible to have a list of thirty members where the average is likewise 76, but whose range of variation is much smaller—say from 80 down to 73. We need to be able to record this difference in some summarizing but exact form.

Determination of the degree of dispersion

The two most common indices of the amount of dispersion in a collection are the mean deviation and the standard deviation. The *mean deviation* is calculated by subtracting the arithmetic mean from each of the individual items, adding the various deviations together without regard to plus or minus signs, and dividing the sum by the number of items. In this table, the mean deviation is calculated in the second column from the right, and proves to be 10. The *standard deviation* is found by taking the sum of the squares of the individual deviations from the mean, dividing by the number of items, and then extracting the square root of the quotient. In the present table it is calculated in the column to the right, and turns out to be 12.4. The standard deviation is symbolized by σ, which $= \sqrt{\frac{\Sigma x^2}{n}}$, where Σ means "sum of," x means any of the individual deviations, and n means the number of items. The standard deviation has an important use in determining correlations by the Pearson method, as will be shown in the next chapter.

We often hear it said that "you can prove anything by statistics." This statement is true only if the word "prove" is used very loosely. It is always possible, of course, for errors either intentional or unintentional to appear in any presentation of statistical results, especially in the interpretations which are claimed for them or subtly suggested by them. But these errors can be detected by one who understands statistical method and is therefore on his guard against uncritically

Common statistical errors

accepting interpretations which are not really justified by the data secured. The most frequent intentional error is that of selecting data favorable to the conclusion one is eager to support, and omitting other relevant data which point toward a different conclusion. For example, before an important election each candidate is likely to present popular polls which, uncritically considered, seem to indicate that he is sure of being elected. The implication is that supporters of other candidates would do well to jump on the band wagon. How are such results reached? Well, each poll conveniently leaves out of account data favoring the other candidates, or, perhaps, only gathers data in areas known in advance to be likely to give the candidate, in whose interest the poll is taken, a majority. One of the commonest unintentional errors is to give a greater degree of exactitude to the result of a statistical investigation than is warranted by the way in which the data were gathered. Suppose, for instance, we are told that the circumference of a certain small island, as determined by the average of three individuals' measurements at high tide, is 2,017½ feet. This looks like a precise and, hence, dependable figure. We would know that it is not as accurate as it looks, however, if we should learn later that the method followed by these men consisted simply in pacing the shore of the island and assuming that each pace is approximately equal to three feet. It would be evident then that a really accurate survey might reach a quite different result. Another frequent mistake, sometimes unintentional, sometimes intentional, is to omit some factor which is essential if two contrasted averages are to be rendered genuinely comparable. For instance, the death rate in the United States Navy during the Spanish-American War was 9 per 1,000, while the death rate in New York City for the same period was 16 per 1,000; can we draw the conclusion that it is safer to be a sailor in the navy during a war than a civilian in New York City? Well, an examination of the evidence for this proposed conclusion soon shows that the two death rates do not have the significance which they appear to have. For the New York City death rate includes the mortality of small children, old people, and people in hos-

pitals; and it is a familiar fact that the death rate for the very old and the very young, as well as for the sick, is relatively high. On the other hand, the navy was composed of men between the ages of eighteen and thirty-five, each of whom had passed a rigorous physical examination. Hence the two death rates do not warrant the conclusion that the navy is a safer place than New York City. Adequate evidence for such a conclusion would require the comparison of two groups which are homogeneous with respect to such important factors as age and state of health.

The probable error

The scientific investigator will not only be careful to avoid such fallacies as these; he will even give his readers as definite a measure as he can of the degree of dependability of his result. And at this point we are dealing with the first of the two problems regarding probability listed on pages 428 and 429. Statisticians have invented devices for accomplishing this, one of which is called the *probable error,* and another the *standard error.* The latter is simply the standard deviation, employed in a way not yet discussed, and the probable error can be readily defined in terms of it.

Suppose that we have a frequency polygon which exhibits an approximately "normal" distribution of the data summarized (Fig. 19). This means that the curve of the polygon falls away symmetrically on each side of the peak, which locates the average of the various measurements, and that it is symmetrical in relation to the base. Let us next determine the standard deviation, and mark off a distance equal to it on either side of the average. When this is done, we find that approximately two thirds of the data fall within the area between $-1\,\sigma$ and $+1\,\sigma$.

Now suppose we think of this standard deviation as not only summarizing the dispersion of the data already accumulated, but also as providing a guide to reasonable anticipation about further data belonging to the same class. In other words, on the basis of this sample, let us estimate what we may expect as to where further data of the same kind will fall. In accordance with the postulate of predictive uniformity, it can be said that there are two chances to one that any such datum

will fall within the area between $-1\,\sigma$ and $+1\,\sigma$, and only one chance out of three that it will fall outside those limits. The probable error is determined by a similar procedure, only in such a way that half the measurements fall between -1 P.E. and $+1$ P.E. Thus we may predict that a subsequent datum will have an even chance of falling either within or without those limits, indicated by a probability of one half.

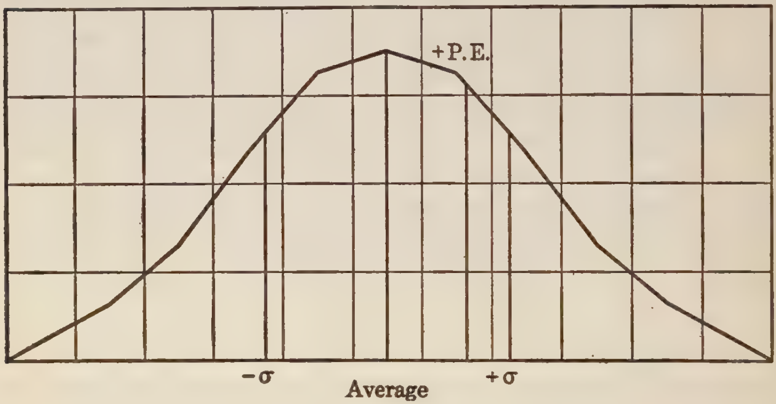

FIG. 19

Measured in terms of the standard error, the probable error is approximately $.6745\,\sigma$. In such ways as these, then, the statistician measures the degree to which an increase in his sample is likely to agree with or vary from the tendency exhibited by that sample.

Once the standard and probable errors of a sample have been determined, they can be used to calculate the standard or probable error of any of the various measures derived from analysis of that sample. For example, we might be interested in determining the error of the average (arithmetic mean) of that sample. The formulae which may be employed for this purpose are as follows:

$$\text{Standard error of the mean} = \frac{\sigma}{\sqrt{n}}$$

$$\text{Probable error of the mean} = \frac{.6745\,\sigma}{\sqrt{n}}$$

EXERCISES

1. Describe accurately the general nature of the problem faced by the statistician.
2. Why does exact description of facts involve counting and measuring?
3. What preliminary problems need to be solved before counting and measuring can be intelligibly engaged in?
4. How does one distinguish between additive and nonadditive properties? Give a good illustration of each that is not mentioned in the chapter.
5. Distinguish clearly each of the three purposes which, according to the chapter, may guide a statistical investigation. Provide your own illustrations. Can you think of any other purposes that might fill this role?
6. In which of the following problems where a representative sample is to be selected would the method of random selection be appropriate? In which would the method of stratified sampling be required? In the latter cases, what main factors would need to be weighted proportionally?
   a. What is the bacteria content of the water in this pond?
   b. What proportion of churches in the Pacific Coast States have Orientals in their membership?
   c. What is the January temperature at the South Pole?
   d. What proportion of powdered shell is mixed with the sand on this beach?
   e. What proportion of books in this library are bound in octavo?
7. Arrange the following data, first in the form of an appropriately chosen frequency table and frequency polygon; then in the form of an array such as will facilitate determining an average. Calculate the mean, the mode, the median, the mean deviation, and the standard deviation.

   Among a group of twenty-six persons, A has an income of \$1,500 per year; B, \$3,200; C, \$2,400; D, \$2,700; E, \$5,000; F, \$4,800; G, \$3,500; H, \$2,000; I, \$3,100; J, \$1,200; K, \$2,200; L, \$2,800; M, \$3,000; N, \$3,100; O, \$1,800; P, \$3,000; Q, \$3,600; R, \$2,800; S, \$4,200; T, \$3,000; U, \$4,400; V, \$3,400; W, \$2,900; X, \$2,800; Y, \$4,500; Z, \$3,000.
8. Construct an illustrative case of your own such as will bring out clearly each of the main errors of interpreting statistical results that are mentioned in the chapter.

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CHAPTER 22

CORRELATION—INDUCTIVE FALLACIES

Let us turn to the second of the two problems mentioned in the preceding chapter. Owing to the nature of the material with which we are now working, it is impossible as yet to hope for the establishment of any causal law; all the conjunctions which catch our attention are too irregular. But if the data have been described as accurately as possible by the statistical methods just discussed, the more frequent conjunctions of the phenomenon under study will be brought to light and clues suggested as to which of them it would be well to investigate carefully. How should this more specific investigation be carried on, and in what form should the results be expressed if the highest degree of exactitude that is feasible may be realized in them?

The problem of discovering correlations

Suppose, for example, that a statistical study of the decline in the value of farm property in various areas in the United States has brought out the fact that this process is often accompanied by a decline in farm population in the same areas. We wish to find out more precisely how closely these two processes are conjoined. The first step will presumably consist in expanding our data so as to obtain information about the conjunction in a larger number of areas—the areas being selected in such a way that, taken together, they will be a fair sample of whatever field (say, upstate New York) we hope to make predictions about as a result of the inquiry. The information gained by this preliminary step, when summarized by counties, can be exhibited in a three-column statistical table like the following:

	Average Decline in Population, Last Ten Years	Average Decline in Value of Property, Same Period
County L	5%	6%
County H	10	14
County P	2	8
County W	1.5 *	3
County F	12	10
County J	7	9
County R	4	4

* Increase

It is evident at once that on the basis of such data no causal or functional law connecting these two processes would be justified. No causal law is evidenced, for in one case (County W) the population has increased while the value of property has declined. No functional law is supported, partly for the same reason, partly because of the further fact that even in the counties where both population and property have declined there is no constant ratio between the two processes; in County L the property decline is just a little higher than the decline in population, while in County P it is very much higher and in County F it is slightly lower. The conjunction, however, is clearly more than accidental; there is an obvious tendency for the occurrence of the one process to be attended by a parallel change in the other phenomenon. These circumstances are revealed more clearly still in what statisticians call a "scatter diagram," presenting the above data in graphic form (see Fig. 20). Each of the dots on the diagram represents one of the counties listed, in its status with respect to these two variables. If the dots all fell on a straight line, it would be obvious that a functional relation obtains between these magnitudes. Then, from any given value of the one, we could predict the corresponding value of the other. But although this cannot be done, it is clear that the dots would cluster around a diagonal line if such a line were drawn from the lower left to the upper right. None of them would be very far from it. This indicates that there is a greater tendency toward regular correspondence between this pair of mag-

nitudes than would obtain if the dots scattered all over the diagram. How is this correlation between them to be accurately measured? How are we to indicate the precise degree of quantitative correspondence between the two variables? Only by some technique of this sort can our present knowledge about the conjunction be accurately summed up.

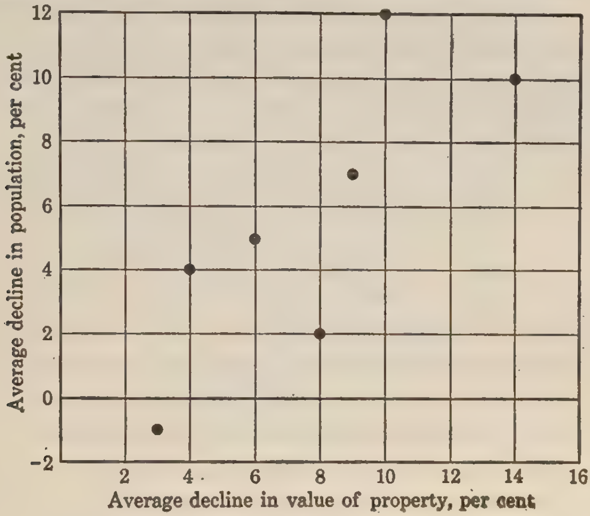

FIG. 20

Correlation as association of attributes

The simplest situation in which correlations can be exactly determined is that in which instances of the relevant phenomena have merely been counted, without being treated as values of a magnitude. This sort of correlation is often referred to by statisticians as "association of attributes." The phenomena are simply taken as events that can either happen or not happen. And the quantity, which is foundational to a measure of correlation of this sort, is a probability fraction, namely, $\frac{ab}{a}$, where a stands for the number of examined instances of one of the phenomena and b stands for the number of accompanying instances of the other. By this fraction we are answering the question: What is the probability of b occurring when a occurs, based on the relative frequency with which b has been conjoined with a in our selected sam-

ple? For example, suppose that in studying suicide among college students, one discovers that of the forty-five cases of suicide noted, twenty-seven were of students failing in their studies. In this case, *b* means the number of students failing in their studies and *a* the number of student suicides, and the probability fraction is then $^{27}/_{45}$.

But, as our discussion of the significance of a control group in the preceding chapter shows, this fraction must be compared with the *a*'s that are not *b*'s, and with the *b*'s that are not *a*'s, before it can be regarded as a measure of the correlation between these two phenomena. The necessity of such comparison becomes quite evident if we should find a person drawing significant conclusions, let us say, from the fact that a very high percentage of coffee drinkers die before reaching the age of ninety. It would be evident that this means nothing, because we are aware that a very high percentage of people who do not drink coffee also die before reaching the age of ninety. Only if this conjunction obtains with a significantly higher percentage of coffee drinkers than of noncoffee drinkers would it prove anything to the purpose. As applied to the preceding illustration, only if a larger percentage of students failing in their studies committed suicide than of students who were not failing in their studies, would the relation above noted indicate anything of consequence. Hence the measure of intensity of association, *i.e.*, of correlation under the conditions now assumed, between any two phenomena must be so devised that it will take proper account of these comparisons.

There is no single coefficient that is universally employed for this purpose. However, American statisticians use very generally Pearson's coefficient of "mean square contingency." The distinctive quantity in this coefficient is χ^2, which indicates whether the relation between two or more phenomena under investigation departs significantly from what would be expected if the phenomena were independent of each other. More precisely, $\chi^2 = \Sigma \frac{(f - f_c)^2}{f_c}$, where f is any of

the numbers reached by counting instances of *ab*, *a′b*, *ab′*, etc., and dividing by the appropriate denominator, while f_c is the corresponding number which would be expected if the factors involved were independent of each other.

Let us consider a simple problem which will reveal more exactly what each of these symbols means. I have a class of fifty freshmen, sixteen of whom come from college preparatory schools and thirty-four from high schools. I want to find out whether those who receive higher-than-average grades are more likely to be among the preparatory school graduates or among the high school graduates, *i.e.*, whether there is any significant correlation between high grades in the subject that I am teaching and the kind of institution in which the students were prepared. My data are summarized in the following table:

	(*b*) From College Preparatory School	(*b′*) From High School	Total
(*a*) Grade of 80 or above	(*ab*) 9	(*ab′*) 13	(*a*) 22
(*a′*) Grade below 80	(*a′b*) 7	(*a′b′*) 21	(*a′*) 28
Total	(*b*) 16	(*b′*) 34	50

From these data it appears that twenty-two of the class received grades of 80 or above, while twenty-eight received grades lower than 80. Now examine one of the cells of the table; let us pick the *ab* cell at the upper left, which tells us that nine of those who received 80 or above came from preparatory schools. But the total of those from preparatory schools is sixteen. So far, then, as this cell is concerned, $f = 9/16$. How, next, is the quantity f_c determined? Well, of the whole class of fifty, twenty-two received grades of 80 or above, *i.e.*, 44 per cent; and if coming from a preparatory school made no difference in one's chances of receiving a

high grade, the same percentage of those who came from preparatory schools would have received 80 or above. But 44% of 16 = 7. Hence f_c the number that would be expected in this cell if the two phenomena were independent of each other, is 7⁄16. For this cell, then, the quantity $\frac{(f - f_c)^2}{f_c}$ is $\frac{(\frac{9}{16} - \frac{7}{16})^2}{\frac{7}{16}}$, which = $\frac{1}{28}$. A similar procedure is followed with each of the other cells, and χ^2 equals the sum of these fractions.

With χ^2 thus determined, the formula for the coefficient of contingency, which gives an indication of the strength of the association[1] between *a* and *b*, is

$$C = \sqrt{\frac{\chi^2}{N + \chi^2}},$$

where C is the coefficient of contingency and N is the number of cases. Use of the coefficient must, however, be guided by certain considerations which are too technical to be adequately explained in the present text.[2]

Coefficients of correlation

When the data include exact measurements of each instance of the phenomena correlated, a summarizing indication of the degree of their tendency toward regular correspondence will take account of these varying values and hence of the fact that the phenomena are now being treated as magnitudes. When it does this briefly and precisely, it takes the form of a *coefficient of correlation* in the narrower and generally recognized sense of this phrase. In such cases one is employing the principle of concomitant variations, modified in such a way as to be applicable to situations where there is no uniform law of corresponding increase or decrease in the one magnitude with given increases or decreases in the other, as well as to situations where such a law can be verified.

[1] Though not an exact measure of it. To secure greater exactitude, more complicated formulae need to be employed.

[2] Consult, for example, M. J. Hagood, *Statistics for Sociologists* (1941 edition), chap. 19, especially pp. 501–514.

Correlation, it will be remembered, covers quantitative correspondences that fall short of complete functional dependence along with those that illustrate it.

Just what, then, is meant by a "coefficient of correlation"? Briefly, it is a mathematical symbol by which is measured the degree of quantitative correspondence between two phenomena that are capable of analysis as magnitudes. The coefficient $+1.00$ stands for a perfect positive correlation; it means that any measurable increase in the one is attended by an increase in the other according to some uniform law. The coefficient -1.00 stands for a perfect negative correlation, which means that any increase in the one is attended by a parallel decrease in the other. Any coefficient actually reached in a correlation inquiry will lie somewhere between these two extremes. A coefficient of 0 means absence of any but an accidental connection; a quantitative increase in the one phenomenon just as frequently accompanies a decrease as an increase in the other. In statistical terminology, the two variables are in this case "independent" of each other. Low positive or negative coefficients may not indicate any consequential association, since in any sample uncontrolled factors may play a part in determining the outcome; but a coefficient of $+.5$ in a carefully selected sample of sufficient size would usually imply a significant positive correlation between two phenomena, while one of $-.7$ under the same conditions would indicate a high negative correlation.

By what formulae are correlation coefficients worked out in any given problem?[3]

Coefficients for correlation by ranks

This question is answered by different formulae according to the form of the data and the degree of exactitude needed in the result. Standard works on statistics should be consulted for the justification of these formulae.

A quite simple formula enables us to calculate the coefficient of correlation when we know the rank of each of

[3] We are confining ourselves in what follows to correlations involving linear relations only.

the cases studied with respect to the factors correlated, but do not know their exact numerical measurements. We are treating them, in short, as nonadditive magnitudes. The result reached by this method only approximates the true correlation, and not very closely then unless the number of cases considered is fairly large and their distribution fairly normal. But the calculation is much easier than with the more exact method, and for this reason it is sometimes preferred to the latter even when the precise measurements are known, if a rough approximation to the correct correlation will do. A concrete application of this method will be given. Suppose, for example, two high school teachers, one of algebra and one of history, make no attempt to give a percentage grade to each student, but have formed the habit of ranking their classes in order from the best student to the poorest. It happens that in the course of their work they both teach the same class of twenty-two students, and wish to correlate achievement in history with achievement in algebra on the showing of this class. How shall they calculate the desired coefficient? The formula for such correlation is

$$\rho = 1 - \frac{6\,\Sigma d^2}{n(n^2 - 1)}$$

Here ρ stands for the coefficient of correlation by ranks, n for the number of cases, d for the difference between the rank of any individual in one of the factors correlated and his rank in the other, and Σ, as indicated in the preceding chapter, means "sum of." The numerator in the fraction of the formula therefore reads: six times the sum of the squares of the differences between the rankings of the individuals in the two variables compared. Let us see how, by the use of this formula, the two teachers would solve their problem. Their data are given in the first three columns on the next page. The fourth and fifth columns are then worked out, which consist of the differences in ranking of each of the individuals studied, and the squares of those differences, respectively. The sum of the items in the last column is then calculated,

Individual in the Class	Rank in History	Rank in Algebra	Difference in Rank	Square of the Difference
T. G.	12	11	1	1
R. L.	19	14	5	25
H. Y.	8	10	2	4
S. B.	5	3	2	4
I. M.	9	9	0	0
Y. P.	2	5	3	9
V. C.	21	19	2	4
P. L.	14	15	1	1
J. T.	6	7	1	1
W. F.	11	16	5	25
A. D.	22	18	4	16
R. R.	17	20	3	9
C. K.	4	1	3	9
O. U.	7	6	1	1
Y. M.	13	8	5	25
A. R.	18	21	3	9
B. O.	16	22	6	36
K. I.	1	2	1	1
P. S.	3	4	1	1
A. L.	10	17	7	49
M. N.	15	12	3	9
F. D.	20	13	7	49
				288

which proves to be 288. Since we know the number of students in the class to be 22, we may now express our equation:

$$\rho = 1 - \frac{6 \times 288}{22(22^2 - 1)}$$

$$\text{Solving, } \rho = 1 - \frac{1,728}{22(484 - 1)} = 1 - \frac{1,728}{22 \times 483} = 1 - \frac{1,728}{10,626}$$

$$= 1 - .162 + \qquad = .837 +$$

There appears thus a high positive correlation between achievement in these two subjects as revealed in the work of these pupils. Of course (to repeat a warning given in the preceding chapter) it would be fallacious for these teachers to assume that this correlation would necessarily hold for any other class working under other conditions than those

obtaining at that school at that time, or that it yielded more than an approximation even then.

The Pearson correlation coefficient

The more exact formula is known as the Pearson correlation coefficient,[4] since it was derived mathematically by one of the great leaders in the field of social measurements, Karl Pearson. It aims to give an exact determination of the degree of association between two phenomena when we have the numerical measurements of a group of individual instances of each of the phenomena compared. The problem on p. 488 illustrates this situation.

The Pearson formula is:

$$r = \frac{xy}{n \cdot \sigma_x \sigma_y}$$

Here $r =$ the coefficient of correlation; $x =$ the deviation of the measured value of any of the various instances of one magnitude from the latter's mean value; $y =$ the deviation of the measured value of the corresponding instance of the other magnitude from the latter's mean value; $n =$ the number of instances studied; and σ_x and $\sigma_y =$ the standard deviations in the case of the one and of the other magnitude respectively.

To apply the formula, accordingly, we need, in addition to the columns giving the numerical values of the two variables, columns each for x, y, x^2, y^2, and xy.

Let us take a very simple problem. Suppose that we are residents of Portland, Oregon, and are interested in the climatic conditions of our locality. We know the average rainfall and also the average percentage of possible sunshine for each of the months of the year, as determined by Weather Bureau records embracing a period of fifty years or more. We wish to determine the correlation between these two phenomena. Applying the formula, we proceed with the calculation as follows:

[4] Or, according to the method of its derivation, the "product moment" coefficient.

Month	Rainfall in Inches	Percentage of Possible Sunshine	x	y	x^2	y^2	xy
Jan.	6.6	26	2.9	— 18	8.41	324	— 52.2
Feb.	5.8	30	2.1	— 14	4.41	196	— 29.4
Mar.	5.0	39	1.3	— 5	1.69	25	— 6.5
Apr.	3.1	48	— .6	4	.36	16	— 2.4
May	2.4	47	— 1.3	3	1.69	9	— 3.9
June	1.6	54	— 2.1	10	4.41	100	— 21.0
July	.6	71	— 3.1	27	9.61	729	— 83.7
Aug.	.6	65	— 3.1	21	9.61	441	— 65.1
Sep.	1.7	53	— 2.0	9	4.00	81	— 18.0
Oct.	3.5	44	— .2	0	.04	0	0
Nov.	6.6	25	2.9	— 19	8.41	361	— 55.1
Dec.	7.1	23	3.4	— 21	11.56	441	— 71.4
	44.6	525			64.20	2,723	— 408.7
Mean =	3.7	44			Divided by 12 =		
					5.35	227	
					Square root =		
					2.31	15.1	

After setting down the weather data we compute the mean value of the rainfall per month and the mean percentage of possible sunshine per month. These we find by adding the totals of each column and dividing by twelve, the number of months. Disregarding further decimals, we find these mean values to be 3.7 inches and 44 per cent, respectively. To build up the x column we subtract the mean value of the rainfall from each of the individual values, being careful to note whether the result is + or — in every case. This supplies us with the deviations of the individual values from their mean. To secure the corresponding deviations of the values of the other magnitude (sunshine) we likewise subtract its mean value, 44, from each individual value. The results give the y column. Next, by squaring each of these deviations we fill up the x^2 and y^2 columns, which will be needed in computing the denominator of our fraction, and by multiplying each x deviation with its corresponding y deviation we work out the xy column. Notice that the squares of the deviations will always be plus in sign, while their products may be either plus or minus. In this case they happen to be all minus, which indicates that we shall find a rather high negative correlation between these variables. We are ready now to add up the xy

column, which yields the numerator of our fraction, -408.7. Next, we add up the x^2 and y^2 columns. Dividing in each case by the number of months and taking the square root, we have the values for σ_x and σ_y. We are then able to solve the equation as a whole.

$$r = \frac{xy}{n \cdot \sigma_x \cdot \sigma_y} = \frac{-408.7}{12 \times 2.31 \times 15.1} = \frac{-408.7}{418.57} = -.97 +$$

There thus proves to be a very high negative correlation between rainfall and sunshine, computed according to their monthly averages, in the locality studied.[5]

To the inexpert in mathematics this formula appears rather complicated. After a little practice, however, one realizes that each of the separate steps involved is quite easy, although the number of them required for the calculation takes time. And statisticians have discovered certain aids which, except in the simpler problems, it is usually best to follow. In the illustration given, it is just about as easy to follow the formula literally as to make use of any simplification of the process. This is because the separate values which need to be considered are not many, and none of the figures involved in the computation is very large. When such happy conditions do not obtain, there are ways to facilitate the calculation. In a general survey such as the present these details need not be elaborated; they are clearly expounded in the larger books on statistics, such as those of Yule and Bowley.

Function as a special case of correlation

When the subject of functional laws was discussed, it was explained how one methodological principle can be regarded as a special case of a more general principle; this notion was illustrated by showing how the principles of agreement and difference may be thought of as special cases falling under the principle of functional concomitance as a more general methodological concept.[6] Now that we have become acquainted with the method of correlation, and with exact measurement of correlations by the aid of coefficients, a

[5] In many localities the correlation would be much lower on account of the months in which there is considerable cloudiness but little rain.

[6] See above, pp. 356 f.

further application of this notion becomes possible. The principle of functional explanation itself may now be seen as a special case of the principle of correlation—it is just a correlation between two variable phenomena under the unusually happy situation where the coefficient comes out as $+1.00$ or -1.00 in every instance. A functional law is simply a perfect positive or negative correlation—or at least it expresses a conjunction between two magnitudes such that any divergences from a perfect correlation can be plausibly explained as due to extraneous factors, *e.g.*, imperfections in the instruments used, or some personal variation in the observer. From this viewpoint, the method of correlation becomes the all-inclusive method of factual science for dealing with conjunctions of two or more phenomena that appear likely to be significant for purposes of helpful prediction or systematic explanation. Such an interpretation has a further virtue; it suggests what may turn out to be an important truth about the world, namely, that all factual conjunctions would reveal the characteristics assumed by a correlational formulation if we could ferret out and take account of the ultimate units of which the subject matter involved is composed. It may be, that is, that causal and functional laws are able to present themselves as unexceptional because, instead of dealing with such units as the individual electron, whose behavior is perhaps even more unpredictable than that of an individual member of human society, they deal with large aggregates of these entities in the form of physical objects. In these aggregates the unpredictable variations of the individual units may cancel each other out, leaving some constant tendency which we apprehend as an unexceptional law, just as the death rate per year of a large community is fairly constant even though one cannot predict which individuals will die in any given year. The concept of correlation may thus be legitimately used in a quite inclusive sense; it serves the student of scientific method as a unifying principle, binding together the various activities of the scientist so far as these reach beyond the mere gathering of data and attempt to pursue their promising interrelationships. Factual science, in its essential intent and

procedure, is the disciplined investigation of the correlations of phenomena.

To be sure, correlation includes the notion of function as a special case in a different way from that in which the concept of functional law includes that of a causal relation. The idea of a dependable causal conjunction is preserved in that of a functional law; the latter merely adds the idea of quantitative analysis of each of the conjoined phenomena as a series of numerical values. Correlation, however, includes function in the sense of retaining the quantitative exactitude which is essential to the latter, while abandoning its insistence on unexceptional constancy in the conjunction of the variables involved. This abandonment is necessary in order to include all significant spatio-temporal contiguities of phenomena, irrespective of the degree of irregularity that they may reveal.

The way in which correlation, thus understood, becomes an all-inclusive methodological concept, uniting within its scope all the major methodological principles previously discussed, may be brought out by the table on p. 501.

Correlational knowledge about individuals

In treating of probability the question was raised: What kind of information does a probability fraction give about any individual member of the class whose likelihood of exemplification under specified conditions has been determined? And the answer was that it gives no *predictive* information about such an individual at all; what we know is that the individual belongs to a certain class, and that that class is exemplified with a certain average frequency under the conditions named. But whether any particular member of the class will exemplify that frequency cannot be known. The same conclusion applies to correlation coefficients, since they simply give an exact measure of the tendency toward regular conjunction, or toward quantitative correspondence, between one phenomenon and another. Suppose that the correlation, for example, between blond hair and blue eyes is found to be + .60. This means that one can predict, with a degree of confidence determined by the extent and care with which the inquiry was conducted, that in any sizable group of blond-

Correlations

- Less than constant
  - Expressed without measurement of instances (coefficients of association of attributes)
  - Expressed on the basis of measurement of instances (coefficients of correlation, in the narrower sense of "correlation")
- Constant
  - Pursued through quantitative analysis of the correlated factors (functional laws)
  - Pursued without quantitative analysis of the correlated factors (causal laws)
    - When cases of occurrence and non-occurrence of the conjunction are systematically compared (one-one causal laws, established by the method of difference)
    - When only cases of occurrence of the conjunction are systematically studied (many-one causal laws, established by the method of agreement)

haired people a large majority will also have blue eyes. But obviously we cannot tell in advance which of the individuals in the group will be the ones without blue eyes. Accordingly, if we know nothing about a girl except that she has blond hair, we can say simply that there is a high probability that she will have blue eyes. Much more exactitude is secured when one is able to say this than when he makes a guess based on hit-or-miss observations, even though in the case of any given individual our expectation may be disappointed.

The human significance of correlation coefficients

If this is so there is great human importance in systematically secured statistics and correlation coefficients. Look at the way, for example, in which most of us at present choose the business or profession to which we intend to devote the major energies of our lives. Even eliminating the large number who simply drift into a calling without any assignable

reason, all but a few of the rest decide this very important question in ways that could hardly be rationally justified. Some enter a calling because it is their father's, others because it expresses their first serious interest, others because it offers an immediately attractive income, still others because it is the career pursued by an admired hero. Need one wonder that there is so much unhappiness in life, or that work and play seem quite impossible to combine for so many people? Suppose that we had exact information showing with what qualities and with which of their various degrees success in different professions is most highly correlated, and likewise a system of tests by which we could tell, in the case of any boy or girl, just how he or she ranks with respect to these qualities, allowing so far as possible for indications of future promise as well as for present achievement. Then, by comparing the chart embodying the results reached in studying any given youngster with these "career specifications," it would be possible to say in just what calling he or she would be most likely to reach the highest attainments that capacity and interest permit. In the case of any particular individual, of course, the guidance thus founded might prove mistaken, but since it would rest on the most accurate quantitative analysis available, it would be a thousandfold more dependable than the unscientific methods now generally used. And this is only one of a host of valuable applications of the concept of correlation.

The discussion of correlation should not be closed without reminding the reader that the only purely mechanical part of a statistical or correlational investigation is that of carrying out the mathematical computations. Every other part depends on the exercise of informed judgment, guided by a clear sense of the problem which one is trying to solve. Such informed judgment itself depends on thorough familiarity with the field of facts relevant to the problem, together with alertness of imagination in discriminating promising clues. This is especially evident in the basic and preliminary task of planning an investigation—deciding what data it will be

worth while to secure, what conjunctions should be systematically followed up, and in what order. Here is a challenging fact whose obvious conjunctions seem rather accidental, for example, a rapid increase in our penitentiary population. Just what detailed records ought to be secured about crimes committed, arrests made, court procedure, penalties imposed, etc., in order that the social significance of this fact may be laid bare? What correlations then ought to be sought between these various circumstances and other measurable social tendencies such as might furnish a helpful clue to a remedy for the phenomenon? No mechanical rules can be adequate in answering these essential questions. Only a thinker widely acquainted with the relevant historical and psychological material, sensitive to the human needs involved, and flexibly hospitable to new and unconventional solutions, will be able to meet them with any likelihood that his decisions point in the right direction.

Fallacies of inductive reasoning

We are now in position to engage in a summarizing survey of the fallacies into which the enterprise of factual science may fall. As in the case of formal fallacies, the main errors have been noted in connection with the rules or principles which they violate. But it is of value in this case as well as in the other to bring these fallacies together in a systematic list and assign to each of them more technical descriptive terms.

The primary and broadest division of these fallacies will distinguish between the fallacies of *irrelevant evidence* and those of *insufficient evidence*. By the former is meant cases of reasoning about facts whose conclusion rests in part upon evidence that does not really support it; by the latter, cases in which the evidence is relevant but is inadequate to justify the conclusion.

Irrelevant evidence

Irrelevant evidence appears in too many different forms to make possible a helpful subdivision of the cases that fall under it. Any serious consideration of suggestions, or appeal to facts, that do not really bring to light significant conjunc-

tions of the phenomena concerned, would exemplify this fallacy. But two prominent forms of it may be mentioned. One is *false analogy.* As has been noted above, the law of association by similarity renders it inevitable that all reasoning about facts will be guided by the similarity of one phenomenon to another, that is, by analogy. But it makes all the difference in the world whether the analogies selected are relevant, *i.e.*, can be justified by objective, empirical data, or are irrelevant, *i.e.*, rest merely on some dramatic or otherwise appealing association. Many, perhaps most, superstitious beliefs rest on false analogy,[7] as we noted at an early stage in our study. That false analogies at times influence even the sophisticated we shall realize if we consider certain beliefs that have seriously obstructed scientific progress in the past. One of Galileo's critics attempted to disprove the existence of Jupiter's moons by showing that there are "seven windows given to animals in the domicile of the head, two nostrils, two eyes, two ears, and a mouth," and that from this "and from many other similarities in nature, such as the seven metals, etc., which it were tedious to enumerate," it follows "that the number of planets [*i.e.*, satellites in the solar system] is necessarily seven."[8] Viewed in terms of the Pythagorean cosmology, this was a very persuasive idea, but when the data were impartially examined it became evident that Nature has no preference for any particular number. Similarly, the long-accepted notion that the celestial bodies must move in circles because the circle is the most perfect curve really rested on a false analogy. Because God embodies our ideals of moral perfection, it was assumed that he must have created the heavens in accordance with our ideals of geometrical perfection. But objective evidence does not support this analogy as a sound one.

A second form of the fallacy of irrelevant evidence was treated in the traditional logic as a pair of related fallacies called "accident" and "converse accident." To commit the

[7] See above, chap. 4, pp. 50–53.

[8] Quoted in W. H. Werkmeister, *A Philosophy of Science*, p. 12.

fallacy of accident is to assume that a general maxim applies under circumstances not taken into account in its formulation and which are therefore irrelevant to its intent. To hold that a man should take no active part in politics just before election time, on the general ground that one should devote his energies to his business, would exemplify this fallacy. A rule is interpreted as applying without question to circumstances which there was no intention to include when it was formulated. The converse fallacy of accident assumes that what is true under special conditions must hold good, for no further reason, in more usual situations. Expressed in broader terms, the essence of these two fallacies is that one fails to take into account the context that is assumed when a law or principle is asserted.

Insufficient evidence

Fallacies of insufficient evidence may also arise in many different forms. What they have in common is that they interpret evidence as warranting a more general, more exact, or more detailed conclusion than the evidence really justifies. A large number of these fallacies are cases of an *unrepresentative sample;* the others are more difficult to classify under good descriptive rubrics.

The unrepresentative-sample fallacies may be helpfully divided into those arising in the pursuit of statistical descriptions and those which appear in the pursuit of causal or functional laws. In either situation the error consists in assuming that a chosen sample is genuinely representative of the field to which it is proposed to apply the results reached, when the investigator has not taken the precautions necessary to insure that it will be representative. Thus we may affirm, or imply, in the presentation of a statistical description, that a calculated average, or a mean deviation, holds for a large class of cases when the sample on which it is based is only typical of a smaller class. Examples of this error have, however, been given in the preceding chapter; it is hardly necessary to add to them here.

Unrepresentative samples that appear in the quest for causal or functional laws will vary in character, of course,

according to whether the investigator is employing the principle of agreement, the principle of difference, or the principle of functional correspondence.

Let us suppose that he is using merely the principle of agreement, but is failing to observe all the cautions required in a sound application of that principle. "I see that all the people in my town who have died this last year have been attended by doctors. When I am sick I shall be careful, therefore, not to call a physician." The author of this reasoning has clearly used the principle in such a way that, except by chance, his sample would be far from representative. He notices that phenomenon B (death) has in every case in the assumed context been preceded by phenomenon A (attendance by a physician), and at once draws the suggested conclusion; he does not pick his sample in such a way that if there is another phenomenon C (such as serious illness) which also preceded B in that context and was perhaps its real cause rather than A, he will find it out. That is, he does not allow for the circumstance that there may be more than one antecedent event conjoined with the event he is attempting to explain, and that unless he looks for all of them and brings out the evidence that is relevant to each, he is very likely to fix upon the wrong one as the cause. This form of the fallacy of insufficient evidence has been known traditionally as *post hoc, ergo propter hoc.*

Let us suppose that he is using the principle of difference, but is not doing so with sufficient care. "This advertisement describes the great enjoyment that many people have found in smoking cigarette D, and also the unfortunate experiences of those who have used other brands of cigarette. I gather that I ought to switch to cigarette D." But, of course, the people referred to in the advertisement may not be a representative sample, and hence there may be other points of difference between the experience of smoking cigarette D and that of using other brands which, if brought out and examined, would lead to a quite different conclusion. There may, in short, be some explanation other than the clear supe-

riority of cigarette D, for the fact that some people have enjoyed it and some have been disappointed in other brands; adequate application of the principle of difference would take account of this possibility and make sure that a sample is secured which will enable this hypothesis to be tested impartially along with other plausible ones. As we have insisted earlier, controlled experiment should be used wherever it is possible; where not, the Joint Method of Agreement and Difference should be employed in as careful and responsible a way as the circumstances permit.

When it is the principle of functional correspondence that is being used, one may mistake a merely *temporary* or *partial concomitance* for complete functional correspondence. Having noted for some days, for example, that the street lights appear in my community just ten minutes after sunset, I might announce a general functional law in terms of that interval of time. But later in the year I observe a different interval. The sample had thus been too meagre to be representative. Further observations prove that my original law was of merely temporary validity, true for only that part of the year when the first observations had been made. A correct functional relation between these variables becomes statable only in more elaborate form. The remedy, of course, is to secure a large enough variety of measurements, under different circumstances, so that an exception to a too simple formulation of the correspondence would be almost certain to have turned up if it exists.

Fallacies of insufficient evidence that are not cases of an unrepresentative sample consist in various other ways of assuming that the evidence proves more than it actually does. One of the most common ways is to assume that all relevant factors were taken into account when some were omitted (often called the fallacy of "neglected aspect"), as for instance in the conclusion drawn in the preceding chapter regarding the comparative death rate in the United States Navy and in New York City. Or one may attempt to draw predictions from a statistical law about individual members of the

classes involved. Other forms are found in the treatment of a many-one causal law as though it were a one-one law; in attributing too great a significance to a low correlation, or to a high correlation when the probable error does not warrant it; or in assuming a greater degree of exactitude in a result than is justified by the data.

All these the responsible inductive reasoner will be careful to avoid.

EXERCISES

1. a. Define: correlation, association of attributes, coefficient of correlation, inductive fallacy.
   b. Explain clearly the relation between correlation and functional correspondence.
   c. The twenty employees in a certain office were measured for their height and weight, with the following results:

Individual	Height	Weight	Individual	Height	Weight
T.E.	6′3″	180 lbs.	E.S.	5′6½″	150 lbs.
M.P.	6′1″	165 lbs.	R. T.	5′5½″	145 lbs.
S.B.	6′0″	172 lbs.	N.C.	5′5″	130 lbs.
S.W.	5′10½″	164 lbs.	W.H.	5′4½″	124 lbs.
G.D.	5′10″	155 lbs.	T.L.	5′4¼″	140 lbs.
P.E.	5′9″	160 lbs.	O.R.	5′3½″	116 lbs.
R.N.	5′8½″	157 lbs.	B.E.	5′3″	128 lbs.
W.J.	5′8″	153 lbs.	C.M.	5′2″	112 lbs.
K.L.	5′7½″	162 lbs.	L.J.	5′1½″	105 lbs.
A.C.	5′7″	143 lbs.	D.S.	4′11″	108 lbs.

   Find out the correlation, for this group, between height and weight, (1) by ranks; (2) in terms of a Pearson coefficient.

2. Describe and criticize the following cases of inductive fallacy. What further evidence would be required to prove the conclusions offered?
   a. My friends Tom and Dick got passing marks in Professor Z's course without doing any work. I must be sure to take that course next semester.
   b. This patent-medicine booklet tells of many people who have been cured of various diseases by taking the medicine advertised, and also of unfortunate people who have died through failure to take it. I gather that it is just what I need to cure me.

c. The tradition arose in the island of St. Hilda that the entrance of a ship in the harbor was likely to cause an epidemic of colds. After a time some one noticed, however, that the harbor was so located that a ship could enter only when a stiff northeast wind was blowing.

d. "Early to bed, and early to rise, makes a man healthy, wealthy, and wise." Hence you had better not stay up any later to finish this final chapter.

e. It is a belief in many seaside communities that babies are born at high tide and deaths occur at low tide. Investigation failed to discover any empirical warrant for the belief. How do you think the belief arose?

f. Mr. A. has been an exemplary man in his family relations, and his friends can always count on his rewarding them for every good turn they do him. He is therefore just the man we need for governor of the state.

g. Last week I got into trouble through imbibing too much brandy and gin. The other day it was ale and gin. And I remember that two months ago I spent a sorry day after an evening with beer and gin. I see, accordingly, that it is the gin that must be responsible. I must give that up and I shall be all right.

h. The present restrictions on sex relations supported by custom grew up in different social conditions, when people did not know how to control the results of such relations. I assume, therefore, that these restrictions have no pertinence at the present time.

i. A spoonful of this medicine cured a light cold that I had last month. Half a cupful, therefore, ought to rid me of this severe one.

j. I notice that when my children are spoken to in a quiet tone of voice they pay no attention, but that when I address them harshly they obey at once. I must, therefore, form the habit of always speaking to them sternly the first time.

k. The more I struggle to improve this book the less does it satisfy me. Therefore it would be better if I erase all my revisions.

l. I observe that the use of brandy does people much harm. It is, therefore, a mistake to use it to revive this man who has just escaped drowning.

m. A graduate student, investigating people's opinions regarding the efficiency of their city government, picked, at random, the same number of persons in each of the city's wards to interrogate.

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CHAPTER 23

ON THE BORDERLINE OF SCIENCE

How much does "science" include?

Attention has been called to the fact that as one passes from the sciences of molecular physics, such as astronomy and mechanics, to the social sciences, he passes from fields in which the verification of causal and functional explanations is to a large extent possible, to fields in which scientific inquiry must content iself, in the main at least, with statistical descriptions and correlations. In terms of the general classification of rational disciplines ventured in Chapter 6, which has provided the outline subsequently followed, the social sciences constitute that area of factual science lying nearest the branches of thought not admitted by many to be scientific at all. They lie, in short, on the borderline of what is generally recognized as science. In fact, not a few thinkers whose own work falls within the field of physical science draw the line between science and what is beyond science so close to the region in which systematic functional explanation is possible that they refuse to accord scientific status to investigations in the social field. For them, "science" becomes an honorific rather than a merely descriptive term.

Probably, however, no student of the factual sciences whose primary interest lies in the methods of reasoning there exemplified would wish to draw the line at this point. There seems to be no coercive reason why "science" should not be defined sufficiently broadly so that it will include all honest and painstaking inquiries into objectively verifiable conjunctions between phenomena. The essence of science may be justifiably interpreted as consisting simply in these general characteristics, and if it is so interpreted, responsible social studies will

clearly become a part of science instead of lying outside it. Historical tradition, in general, supports a broad rather than a narrow definition of "science."

Reasons for the special logical difficulties in social science

Nevertheless, since our project includes a study of the problems of reflective method—of the ways in which we discriminate sound from unsound reasoning—in branches of thought whose scientific status is still more dubious than that of the social sciences, it is desirable to pause before crossing the borderline not far ahead and to consider an important question. What is there about the behavior of men in their social groupings that poses special difficulties in the way of attaining, in its regard, the ends which science always pursues? Why should it be this area rather than some other that lies near the boundary separating science from what is not generally admitted to be science?

The answer in general would seem to be clearly this: The fundamental criteria of evidence in scientific thinking—empiricism and objectivity—cannot be as readily or confidently applied in this area as is possible in dealing with physical and biological phenomena. It is more difficult here to tell the difference between a fact and what might seem to be such but is not, and it is more difficult to discover significant conjunctions which can be objectively verified as quite dependable. But why is this the case? An answer appealing to more specific considerations is needed, and among the considerations that are relevant the five following demand particularly careful examination.

1. Human conduct is affected by a great variety of factors

For one thing, human conduct appears to be a more complex affair than a physical or a simple biological process—that is, it is decisively affected by a larger variety of apparently independent factors. So far as we can tell, physical events are affected only by other physical events, save in the comparatively rare and quite local situations in which organic creatures use them to further their own ends, as when a tree draws moisture from the soil or a man pries out a block of stone and sets it in the foundation of his house. Biological happenings are affected both by physicochemical events and

by other biological processes, but they show no effects of human action, except again in the exceedingly limited sphere in which man exploits them to satisfy his needs. The conduct of a human being, however, is not only affected by the physical and biological conditions in his body and its environment; it is in part conditioned by the fact that he responds to the presence and distinctive behavior of other human beings, in all the manifold and kaleidoscopic forms which such behavior may take. And his responses may be very indirect and delayed as well as immediate and direct. After writing the preceding paragraph, for example, I laid down my pen, stretched my arms, stepped outside my study, and looked in the direction of the down-town section of my city. I had just heard a fire whistle blowing. Some of the physical and physiological factors active in that series of events are obvious, but it is also obvious that I was responding to the whistle not merely as a noise beating upon my ear. It was its social meaning that moved me—its meaning as a sign of some other event, purposively produced as such by an appointed agent of the community. This is clear from my action, which takes into account that social context of the whistle's blow. Any attempted explanation of what I did must, then, also take all these factors into account. But their variety, together with our inability to isolate them one by one and hence to determine once for all how each will be affected by its conjunction with any of the others, renders impossible a causal explanation such as could guide confident anticipation of what I would do when I hear a fire whistle in the future. No one would expect that I will always act as I did on that occasion, whenever a fire whistle blows. As we have seen, one can collect statistical data about what groups of people do when such an event happens, and can make probable predictions from a selected sample. This means, however, that he is neglecting many causal factors really involved in the phenomenon and is focussing attention on the conjunctions that are usually apparent in the behavior of a majority of each group. But without systematic understanding of the other factors, it is

impossible to explain why groups of people behave as these generalizations lead us to expect, while certain individuals belonging to any such group behave differently.

2. The social determinants of action are in rapid change

Have we not here put our finger on the main reason for a circumstance noted in recent chapters—that statistical and correlational knowledge cannot be used to predict the behavior of any individual member of the group studied? But there is another, equally important consideration. The conduct of any human being at any given moment is not only affected by such a variety of factors that it is impossible to examine them all, but also we never know whether the distinctive features of his environment at the next moment may not be so influential that in their presence he will act differently than he had ever done before. This consideration provides an answer to a question that might well have been asked by the reader at the end of the preceding paragraph. Suppose one took time enough for it, could he not examine in turn each of the factors affecting a man's behavior and learn how to allow for them accurately by discovering just what difference each makes? After a while, then, could he not detect in any situation all the relevant conditions that are present, and predict confidently what their combined effect is going to be? Viewed in this light, the backwardness of social science would appear to be merely due to its youth—a temporary incompetence that can be remedied by devoting a larger amount of time and pains to its problems. But the difficulty is more serious than this question assumes. While man's physical, and even, to a large extent, his biological environment changes only very slowly, the social setting of his action (or at least many features of it) changes rather rapidly. A very few years or less may bring significant differences in many parts of that setting. This fact places the attempt to secure dependable predictive knowledge of human behavior under a peculiar handicap. Suppose that a group of scientists were to converge today on a systematic analysis of all the causal factors affecting a given phenomenon of human action; by the time their investigation is completed some of

those factors will have passed out of existence, and will have been replaced by others whose effects are not yet known. Thus the conclusions reached would no longer have any predictive value; they would apply only to phenomena now past. Think, for example, of an economist attempting to study exhaustively all the factors affecting the market price of cabbages in a certain locality. By the time he has tracked each of them down, important changes are bound to have taken place in some among them—changes in climatic conditions, in the constituents of the soil, in the tools used for production and the means of transportation, in the eating habits of the community, in government regulations affecting the markets supplied, in taxes, etc. The general principle that we must recognize here is evidently this: Predictive causal knowledge is only possible when the time required to gain it is less than the time during which the phenomenon studied continues to recur in the context assumed. And this does not seem to be the case with many phenomena of human behavior because of their complex causal involvements and the rapidity of change in many relevant factors.

3. Man's awareness of what he does makes a difference

A third consideration is the fact that human beings, unlike physical objects and the lower organisms, normally act in conscious awareness of what they are doing. This circumstance imposes more than one handicap on the sciences which deal with human conduct, but one of peculiar interest is this: People can be conscious, among other things, of proffered explanations of their behavior, and that consciousness often makes a difference in the behavior explained. Hence, as soon as the explanation is rendered publicly available through articles or books, it tends to falsify itself to a greater or less extent.

Of course, it is not the case in all situations that awareness of an explanation has this effect to any noticeable degree. The wide publication, for instance, of statistics regarding suicide, with the implied prediction that the same ratio of suicides per hundred thousand of population will continue (in whatever locality is in question) during the following

year, would hardly be expected to make any difference in the suicide rate. Even if all who contemplate suicide knew the published figures, whatever effect that knowledge might have on their conduct would be quite insignificant as compared with the powerful emotional forces under which an act of suicide takes place. But suppose that one of the guests at a small dinner party should predict, when the group is being seated at the table, what the others would do and say on that occasion. A prediction which, kept silent, might be fairly well verified would be almost certain to falsify itself if it were made known. For knowledge of it by the members of the party would be sufficient to arouse in them the motive to act differently, and thus prove to the venturesome predictor that they cannot be successfully treated like unconscious entities; this motive would be strong enough, for a time at least, to outweigh the effects of habit which their colleague had relied upon in making his forecast.

A situation illustrating the operation of this principle on a large scale is the movement of average prices on the stock market. Anyone who has observed the course of these prices during periods of prosperity and depression will have noticed that the high point of a "bull" market occurs several months before the period of greatest prosperity which it is "discounting," while the lowest point in a "bear" market comes several months before the period of deepest depression. Why is this? The explanation lies in the fact that many of those who buy or sell securities are conscious of the general law of human behavior that stocks will be regarded as more valuable when their companies are making large profits and less valuable when profits are small or nonexistent. Acting in the light of this awareness, they buy in greatest volume before an anticipated boom period has arrived rather than when it is actually at hand, and sell in greatest volume before an expected business crisis has become acute instead of waiting for that time to appear. The average course of prices, which is the net result of the purchases and sales of many thousands of investors and speculators, thus reflects this anticipatory aware-

ness rather than business conditions as they actually stand.

Since social science, like other branches of science, is not an esoteric discipline but naturally seeks, through publication, objective confirmation of its results as well as the other advantages of a cooperative pursuit, here is a factor which definitely stands in the way of accurate predictive knowledge of many phases of human conduct. Nonhuman phenomena do not occur differently because laws of their occurrence are known and formulated. But phenomena of human action may occur quite differently merely on this account, and there seems to be no way of telling in advance just what the differences will be.[1] Any one among various motives to action may be released when people become aware of how they are expected to act.

4. The method of experiment has only limited uses

A fourth consideration is that in dealing with human behavior the method of experiment, under conditions approximating those of the laboratory, is available only to a very limited degree. To be sure, in the case of a number of relatively simple performances by individuals and small groups, the psychologist has managed to employ the method of systematic experiment, but when more complicated phenomena are in question, or larger groups are studied, the conditions necessary for such analytic testing appear incapable of realization. This is partly because of the very complexity of the facts investigated and the elusiveness of many of the causal factors involved; a procedure by which each can be effectively isolated after the fashion of physical and chemical experimentation seems to lie in the distant future if, indeed, it becomes feasible at all. But the reason is partly that vigorous moral and social attitudes stand in the way. People strongly object to becoming "guinea pigs" in matters where conventional prejudices, personal interests, or ethical ideals offer resistance. Such opposition is so powerful that even possessors of dictatorial power find themselves severely restricted in any attempt

[1] If there were such a way, the statement of any proposed law would, of course, include recognition of the differences; but that inclusion would itself bring about still further differences on account of the operation of the very principle here illustrated.

to impose upon their subjects a pattern of action which the latter are not already well prepared, in idea and attitude, to accept. So far, then, as large scale social phenomena are concerned, there seems to be no way of engaging in scientific experimentation, strictly so-called. Here and there, where a number of individuals voluntarily cooperate to such an end, it becomes practicable in limited ways though still subject to the other difficulties just mentioned; elsewhere its distinctive virtues in securing exact and conclusive explanations apparently must be renounced.

5. Wishful thinking in social matters is hard to avoid

The presence of these vigorous interests and moral conventions leads to a fifth consideration. When we study physical objects and organic processes at the subhuman level, the tendency to assume that their behavior factually coincides with what we wish it might be, or think it ought to be, is relatively slight. This has not always been the case; in prescientific periods and in communities to which the scientific spirit and method have not yet penetrated, many superstitious beliefs about these matters, as well as others, can be traced to precisely this tendency.[2] But competent physicists and biologists in any civilized country have now learned, by their developed techniques and scrupulous care in applying them, to avoid this danger pretty completely. When human phenomena become our subject matter, however, avoiding it is an affair of much greater difficulty. Here, because of the four factors just discussed, we are often unable to establish a single decisive explanation as alone adequate on the evidence; other explanations remain plausible no matter how carefully we endeavor to sift the competing alternatives. In such a situation it is inevitable that the explanation most congenial to accepted convention, personal interest, or moral aspiration will gain an advantage over the others; the objective evidence is unconsciously weighted by the influence of these factors. They are always present in our minds, and here they can be very effective.

Furthermore, as the clever propagandist well knows, a very

[2] See above, chap. 4, pp. 53–57.

successful way of persuading people to act as he wants them to act, or thinks they should act, is to imply that that is the way in which they normally do act—to introduce, that is, in his descriptions of their actual conduct only those features which he wants their conduct to exemplify more fully. This tendency to confuse the factual with what is regarded as desirable often operates unconsciously rather than under such purposive awareness, but it operates nonetheless effectively. It is now fairly evident, for example, that the classical descriptions of economic processes which prevailed in Western Europe during the late eighteenth and the nineteenth centuries reflected not merely an impartial recognition of what actually happened in the activities of production and distribution, but also an eagerness on the part of the authors to help bring about the state of affairs portrayed in their descriptions. They wanted an economic theater free from the various restrictions still frequently laid upon it by governments, guilds, and ecclesiastical bodies, hence they formulated their "laws" of economic behavior in terms implying that these restrictions were at best irrelevant and at worst positively obstructive to sound economic ends. These formulations were historically a real force in bringing about the removal of many such restrictions. Nowadays many economic theories emphasize as essential precisely the factors which these classical descriptions neglected as accidental or worse. Thus the controlling but usually unconscious evaluations of a social scientist make a difference in the analysis and explanation of his chosen subject matter. And since different scientists may be, and often are, dominated by quite different evaluations, this circumstance imposes a serious obstacle in the way of reaching objective conclusions.

These considerations are especially evident in social theories

In discussing the distinctive bearing of these considerations upon social science we have been thinking of the latter as an enterprise naturally seeking causal and functional explanations of its phenomena, and contenting itself with less only because, at present at least, such explanations seem beyond reach. When we think of the further quest, which is also char-

acteristic of all branches of science, to organize whatever detailed results are established under some comprehensive theory or group of theories, it becomes evident that these factors which stand in the way of reaching definite, objective results operate with still greater force. For, in the pursuit of that quest, the criterion of simplicity comes into play in addition to the criteria of empiricism and objectivity. Now if it is often difficult to apply that criterion confidently where the empirical facts and their objective interdependences are pretty clearly determined, it is still more difficult to apply it when they are shrouded in the penumbra of variable complexity that appears to be unavoidable when the factors just listed must be reckoned with. In such a situation there is rarely any single, quite obvious answer to the question, which among the theories that cover all the relevant data is the simplest. Usually, indeed, no theory clearly covers all the data, and investigators differ here, more than in other fields, in judging which data are crucially important and which are of merely minor significance. Hence the choice among theories, which inevitably reflects such judgments, will vary markedly among social scientists dealing with the same subject matter. The illustration just drawn from the field of economics is pertinent here. An economist who regards governmental interference in processes of production and distribution as irrelevant will naturally adopt a unifying theory in economics which implies that such interference can be neglected or recognized merely in hypotheses supplementary to the main strands of his system; while one who regards such controls as inevitable, and therefore essential to any adequate description of what goes on, will adopt a theory explicitly including them and attempting to describe the way in which they operate.

Because theories are guides in the process of further discovery

But can one not tell on objective grounds which data and which patterns of interconnection between them *are* important? The answer would seem to be: No, as yet one cannot. In Chapter 19 it was pointed out that even in the unification of inorganic facts the most comprehensive theories—those which attempt to bind together all the branches of knowledge

under a common set of basic ideas—are adopted not merely because they possess inclusive scope but also because they reflect some widespread and growing interest, *i.e.*, evaluation, characteristic of the age. We must now note an important consequence of this fact. It means that such theories represent more than an effort to unify knowledge already accumulated; they express *proposals* as to how that knowledge may be hopefully expanded through the absorption of new facts. Thus they cannot help indicating what kinds of facts the prevalent temper of mind renders scientists especially eager to find. Adoption, for example, of the theory of atomic mechanism by early modern science not only provided a way of organizing knowledge already available; it also offered a way of gaining and assimilating new knowledge of such a sort that a far larger degree of exact anticipation and control of processes in nature (especially motions in space and time) than had previously been realized became possible. The men who were most active in the scientific inquiries of that period shared the growing interest in increasing man's control over the world and the growing faith that such mastery is possible. They, therefore, wanted a theory of nature which would, among other things, guide them in so selecting the facts and analyzing their explanatory conjunctions that a maximum of confident prediction of the future in terms of the present and the past might be realized. That this interest was not always openly expressed indicates merely that, as is often the case with insistent and widespread human interests, it operated at the subconscious level more than at the level of conscious recognition.

Consequences for social theorizing

Now the same considerations that make this inevitable in natural science render it unavoidable in social science; the social scientist, too, is engaged in the expansion of knowledge, and he seeks theories that will guide him in selecting and understanding important new facts as well as in effectively organizing old facts. This means that his feeling as to what sorts of social fact are important and in terms of what structure of interconnections they should, if possible, be under-

stood can hardly fail to affect profoundly his decision as to which among various alternative theories is most adequate. But when social rather than physical or biological phenomena are in question, the consequence of this is that the fifth of the factors above discussed operates with special force. In this field one's sense of what facts are important and what way of analyzing and correlating them is best can hardly be separated from his conviction as to what pattern of social arrangements is desirable. In short, he cannot wholly detach the unifying social structure that, as a scientist's theory, guides his detailed investigations of human behavior, from the unifying structure which, as a citizen's social ideal, he thinks ought to prevail in human affairs and hopes may sometime be more fully realized.[3] His social theory is thus essentially a program of action along two lines which are kept in some measure of harmony with each other by that theory—action in assimilating social facts for purposes of systematic understanding, and action aiming at progressively molding the social pattern, so far as he can influence it, into what he thinks it ought to be. It should be no marvel that in fascist countries social science tends to reflect the social ends of fascist philosophy, in communist countries the ideals of communism, and in democratic countries the belief in individual freedom and the right of voluntary association for the pursuit of varied group interests.

No wonder social scientists, distressed by these handicaps in the way of reaching objectively verifiable explanations, feel happiest when they confine their inquiries to statistical descriptions and correlational studies, which are objective as far as they go and must obviously be somehow included in any adequate theory and any causal explanation that may be worked out in the future. But even while concentrating on these limited inquiries they cannot help regarding some general pattern of social interaction as more suitable than others, nor can they help being guided by it in the prosecution of

[3] Otherwise either his theory or his social ideal would appear intrinsically incapable of realization; it would be a fantasy merely.

their scientific studies. If the theory toward which they thus incline makes no other difference, it at least makes a difference in the kinds of facts which they select as worth investigating, and the particular correlations between those facts and others which seem to them worth pursuing. An indefinite variety of choices in these matters is always possible, and one's actual selection can hardly fail to be determined in large measure by his social evaluations. Here are serious difficulties in the way of pursuing objective knowledge in the arena of human behavior that must be fully recognized; they, more than any other set of considerations, explain why social science stands near the borderline which separates science from the areas of thinking that at present lie beyond.

It is very important, however, that the considerations thus far emphasized in the present analysis are not misunderstood. They do not justify scepticism in social science or despair as to its capacity to progress. They give no excuse for contentment with any smaller degree of objectivity in social thinking than proves unavoidable. When we seek truth we seek truth, not a mere projection of our personal wishes or group prejudices. Hence the serious inquirer in social science, as elsewhere, is obligated to reduce the personal equation to the minimum, to become as clearly conscious as he can of the factors that stand in the way of reaching objective results, and to find dependable ways of dealing with them as rapidly as proves feasible. But any such responsible quest must be based on a thoroughly realistic understanding of the difficulties if it is to attain whatever degree of success is possible. No problem is solved by blinding oneself in its presence.

Special problems in the field of history

Now these difficulties, and especially the difficulty arising from the varying evaluations of social facts given by different thinkers, assume an especially provocative form in the field of history. Is history a social science, or is it a kind of inquiry that lies beyond science? It is hard to answer this question confidently. The purpose of the ensuing discussion is not so much to answer it as to analyze it critically, in the hope that the analysis will bring out still more sharply the distinctive

problems which arise when human group behavior becomes the subject matter one is endeavoring to understand. It will also prepare the way for fruitful consideration of the special problems attacked in the following part of the book. That these problems take a more challenging form in history than in such studies as economics and sociology seems to be due primarily to two factors: (1) the historian's concern with the uniqueness of the events he seeks to interpret, and (2) the peculiar way in which evaluative considerations play a part in his work.

That any event sufficiently important to make a historian regard it as worth detailed study would be quite complex, is evident without argument. It is usually an occurrence in which large groups of men have participated; their activities in connection with it have been of various sorts, and have been affected by many forces in the total setting of their physical and social environment at the time. Furthermore, in an important sense, it is more complex to the historian than to any sociologist who might be interested in it. In two ways the latter will be apt to simplify that setting, neglecting much that the historian will feel responsible to include. First, he will simplify it because he will approach it as either an economist, a political scientist, a psychologist, etc.—that is, he will select what is relevant to his technical specialty and omit the rest, while the historian will seek an interpretation which cannot be adequate unless all these aspects are dealt with at once. Second, he will simplify it because, as a scientist, he is usually interested in it as a recurrent phenomenon; he wants to make predictions about phenomena belonging to the same class that have not yet been observed, and he therefore tends to neglect those features of its context which he does not expect to see repeated.[4] If he studies nonvoting, for example, he is concerned with it as something which can happen at various times and places, and this circumstance guides his selection of relevant material. The historian, on the other hand, who might investigate nonvoting, would be interested

[4] This is not necessarily the case, of course; see above, pp. 465 f.

in some particular exhibition of it in some particular area during some particular period, and he would seek to understand it in the total network of conjunctions in which it was then and there embedded. Of course in practice he cannot quite realize this ideal, because of limitations of time and available data, but his aim is always to face and interpret the entire event in all its bewildering complexity.

It will hardly be necessary to consider in detail the specific form which the first four of the difficulties above discussed assume in the work of the historian. Because he is trying to understand an event as a unique occurrence which will never be repeated, the writer of history usually does not attempt to make predictions about the future in the same way that the social scientist does; he is too soberly aware that in his field definite predictions, however carefully based, are far more likely to be falsified than confirmed. Nonetheless, one of his motives in studying the past can hardly fail to be an eagerness to throw light on its effects in the present—and to provide such guidance to the anticipations of his contemporaries as will enable them to confront the future with the increased confidence that fuller understanding of man's ways of acting brings. So far as this is the case he, like the social scientist, works under the danger that those features of the contemporary scene which he hopes to clarify may pass away before his study has been completed. In regard to experimentation, it is even more flatly impossible for the historian than it is for the sociologist. In the nature of the case, one can only experiment with happenings in the present, but the historian's material consists entirely of happenings in the past.

Distinctive role of evaluation in historical explanation

By far the most serious difficulty in his case lies in the distinctive way in which evaluation operates in his reasoning, together with the fact that in his field there is no enterprise that satisfactorily plays a role comparable with that of statistical and correlational studies in social science.

To take the latter point first, the task in historical writing that most nearly performs this role is the careful accumulation of records and other materials bearing upon the events

that are to be understood, and their critical sifting to determine the authenticity and reliability of each. This pursuit is indispensable, and must be engaged in under the guidance of canons that have been tested and approved by long experience, if the results are to command assent on the part of competent judges. But the historian thinks of such an enterprise as merely preliminary and instrumental to his main task, which is to offer an interpretation of the events revealed by these materials. The social scientist is permitted to stop with statistical inquiries, if he distrusts his ability to do anything more ambitious; it is part of the historian's job to attempt something more ambitious. After assembling and testing his data, the historian at least assumes the responsibility of tying them together in what he believes to be their proper temporal sequence; the way in which he does this, however, inevitably reflects some theory of how affairs of human behavior are temporally interdependent—some philosophy of history—which may not be acceptable to other historians. Even if the available evidence includes an accurate dating of the events concerned, which is often the case with recent occurrences, different historians will differ in estimating their comparative importance and therefore in the emphasis to be laid upon each. A Marxian historian, for example, cannot help emphasizing the economic changes included in his data, for he believes that they are especially important in throwing light on everything else that occurred.

Particularly in interpreting causality

But most historians are not satisfied to reduce any historical account to a mere series of annals, highlighted at certain points. They wish to make explicit and to justify persuasively an answer to the question: What caused this or that event to happen in the way it did? One finds himself plunged in deep perplexity when he asks what causality means in historical inquiries, and what principles must be respected in formulating and verifying a causal hypothesis in this field. In the work of the natural scientist, what is meant by saying that one thing is the cause of another has happily become fairly clear; at least, scientists agree sufficiently to accept unanimously a

certain procedure as yielding sound causal laws wherever it can be successfully applied. As the student familiar with history knows, and the illustration soon to be given proves, nothing quite comparable has so far been achieved in historical investigation. Why is it that historians find it much more perplexing to reach agreement on what they mean by a causal explanation, and to formulate an objective criterion by whose application they can find the right one for any given historical occurrence?

The considerations just mentioned provide, of course, a part of the answer. A further part becomes clear when one recognizes that evaluation plays the same role in a historian's thinking that it does in social science; the historian, too, can hardly separate his theory of how events in human behavior actually happen from his conviction as to how they ought to happen.[5] Since historians do not agree in their social ideals, this poses a barrier in the way of establishing an objective criterion for tracing historical causation. But there is still another difficulty which gives the variable evaluations of historians a quite distinctive prominence. For, whatever hopes may be encouraged by certain trends in modern psychology, in practice we find it impossible to make headway in interpreting what people do, whether as individuals or in groups, without asking what they *wanted* in the situation in question. As conscious beings, possessed of memory and imagination, men are constantly dealing with desired ends, and adjusting means to their attainment. Especially do we find this true when we neglect their routine activities, and turn to those large-scale shifts in social patterns which draw our special interest and appeal to the historian as weighty happenings to interpret. To such revolutions in the course of human events, one can apparently find no intelligible key without asking what it was that the persons concerned were trying to accomplish in these endeavors; what goals, first of all, were the leaders pursuing; and then what desired ends did their followers expect to gain in supporting their venture? In short, it is largely in terms of

[5] Or, if he is a pessimist, as to how they ought not to happen.

the ends sought by the participants in these historical transformations that we find it necessary to couch our causal explanation.

Now the science of psychology is yielding increasingly accurate information as to what most people want in situations that are not unique. This means, among other things, that the explanatory factors to which it appeals are so selected from the circumstances in which they appear that they can be found to recur in different situations; their conjunctions can then be studied and correlated on this basis. If, in its progress in this direction, psychology continues to speak of wants or desires as explaining what happens, it must try to mean by these wants elements of human nature that are repeated in an objectively verifiable way. But this is not exactly what the historian can mean when he appeals to what people want as an explanation of events in history. For, since such events in all their complexity are unique and unrepeatable, so the wants exhibited in that complexity are unique and unrepeatable, even though in many cases our poverty of language may force us to describe them by the same word. Both the American colonists in their war with George III and the Southern States in the Civil War were fighting for liberty, but it is surely obvious that the precise content of the term "liberty" in these two cases is not the same. Perhaps, indeed, nobody can want exactly the same thing in two different situations; still less can two different groups of people want exactly the same thing in varying historical contexts. But the historian, unlike the sociologist, is attempting nothing less than to explain tangled masses of occurrence, which are always different, largely in terms of human wants which, for that very reason, are also always different.

A novelistic element is thus inevitable in history

If this is a true account of his task, it is evident that his procedure must essentially differ from that of the social psychologist. How shall he determine what people were struggling for in the great crises in human affairs that arouse his interest? If he says that they were struggling for liberty, or for foreign markets, or for national glory, how shall he put pre-

cision of meaning into these concepts in any given historical context? Apparently, only by engaging in a distinctive kind of evaluation himself every time he uses them. He must become absorbed in the records of his period and feel his way sympathetically into the living situation he is seeking to interpret. On the basis of what the participants were doing and saying, he must try to enter directly and intimately into the consciousness that accompanied their deeds. And how can this essentially intuitive, sympathetic insight remain uncolored by his own varying responsiveness to the social values that seem relevant? As he broadens his mastery of the factors conditioning their situation, his own feeling must vibrate appreciatively with their needs and ambitions. He understands what they must have wanted because he realizes in terms of his own present immersion in their purposive activity what it was that they could not avoid wanting. In becoming absorbed in their state of mind, he inevitably assimilates it, in some measure, to his own fundamental evaluations. And is there not palpable evidence on every page of history that something like this occurs in the thinking of the historian? History is a story, as well as a scientific analysis; a novel founded on fact, adding a variable personal contribution of the historian who writes it. Of course it must respect the records of the past. It must make painstaking use of all the tools and results of available social knowledge. Otherwise it will be mere drama, not history. As history, it must transcend scientific abstractions, and in doing so it inevitably enhances the play of personal evaluation in the historian's thinking. We shall then expect that his portrayal of a given event will reveal not only the general assumptions about human nature prevalent in his intellectual environment, but also the peculiarities of his own responsiveness to human wants. These peculiarities will inevitably be conditioned by the range of his individual experience and the distinctive appeal to him of the various satisfactions it has disclosed.

Illustration from contrasting interpretations of the adoption of the American Constitution

The extent to which this situation actually obtains in historical writing is well revealed by a comparison of two widely

different interpretations of the causes of some outstanding historical occurrence, such as the adoption of the Constitution of the United States in 1787.

The contrasting interpretations selected will be that of C. A. Beard, offered in his *Economic Interpretation of the Constitution of the United States,* and that of A. C. McLaughlin, in *The Confederation and the Constitution.*

Let us begin with representative quotations from Beard.[6]

The interpretation of Beard

. . . The inquiry which follows is based upon the political science of James Madison, the father of the Constitution and later President of the Union he had done so much to create. This political science runs through all of his really serious writings and is formulated in its most precise fashion in *The Federalist* as follows: "The diversity in the faculties of men, from which the rights of property originate, is not less an insuperable obstacle to a uniformity of interests. The protection of these faculties is the first object of government. From the protection of different and unequal faculties of acquiring property, the possession of different degrees and kinds of property immediately results; and from the influence of these on the sentiments and views of the respective proprietors, ensues a division of society into different interests and parties. . . . The most common and durable source of factions has been the various and unequal distribution of property. Those who hold and those who are without property have ever formed distinct interests in society. Those who are creditors, and those who are debtors, fall under a like discrimination. A landed interest, a manufacturing interest, a mercantile interest, a moneyed interest, with many lesser interests, grow up of necessity in civilized nations and divide them into different classes, actuated by different sentiments and views. The regulation of these various and interfering interests forms the principal task of modern legislation, and involves the spirit of party and faction in the necessary and ordinary operations of the government."

Here we have a masterly statement of the theory of economic determinism in politics. Different degrees and kinds of property inevitably exist in modern society; party doctrines and "principles" originate in the sentiments and views which the possession of various kinds of property creates in the minds of the possessors; class and group divisions based on property lie at the basis of modern government; and politics and constitutional law are inevitably a reflex of these contending interests. . . .

The requirements for an economic interpretation of the formation and

[6] Pp. 14 ff., 188, 250 ff., 324 ff. Reprinted by permission of The Macmillan Company. (Copyright 1913.)

adoption of the Constitution may be stated in a hypothetical proposition which, although it cannot be verified absolutely from ascertainable data, will at once illustrate the problem and furnish a guide to research and generalization.

It will be admitted without controversy that the Constitution was the creation of a certain number of men, and it was opposed by a certain number of men. . . .

Suppose it could be shown from the classification of the men who supported and opposed the Constitution that there was no line of property division at all; that is, that men owning substantially the same amounts of the same kinds of property were equally divided on the matter of adoption or rejection—it would then become apparent that the Constitution had no ascertainable relation to economic groups or classes, but was the product of some abstract causes remote from the chief business of life—gaining a livelihood.

Suppose, on the other hand, that substantially all of the merchants, money-lenders, security holders, manufacturers, shippers, capitalists, and financiers and their professional associates are to be found on one side in support of the Constitution, and that substantially all or the major portion of the opposition came from the nonslaveholding farmers and the debtors—would it not be pretty conclusively demonstrated that our fundamental law was not the product of an abstraction known as the "whole people," but of a group of economic interests which must have expected beneficial results from its adoption? Obviously all the facts here desired cannot be discovered, but the data presented in the following chapters bear out the latter hypothesis, and thus a reasonable presumption in favor of the theory is created. . . .

In the succeeding chapters, which are too detailed for helpful quotation, Beard attempts to show from an analysis of the events leading up to the Constitutional Convention, the interests represented in the Convention itself, and the factors influencing the ratification of the new Constitution, what the fundamental issue between the Constitutionalists and their opponents was. It involved on the one side the interests of owners of securities, land, and slaves, who sought economic stability, the full repayment of debts, and the adequate protection of their holdings; while on the other side were ranged the interests of small farmers and debtors, who wanted freedom from heavy taxation and generous concessions in the repayment of obligations. As indicating his interpretation with

respect to these matters, we may quote briefly from the conclusion of the chapter describing the ratification of the Constitution. After presenting figures proving that only a very small number of adult males voted in the election of delegates to the state ratifying conventions, Beard says:

> Admitting that these figures are rough guesses, it appears, nevertheless, that the Constitution was not "an expression of the clear and deliberate will of the whole people," nor of a majority of the adult males, nor at the outside of one-fifth of them.
>
> Indeed, it may very well be that a majority of those who voted were against the adoption of the Constitution as it then stood. Such a conjecture can be based on the frank statement of no less an authority than the great Chief Justice Marshall, who took a prominent part in the movement which led to the formation and ratification of the new instrument of government.
>
> At all events, the disfranchisement of the masses through property qualifications and ignorance and apathy contributed largely to the facility with which the personalty-interest representatives carried the day. The latter were alert everywhere, for they knew, not as a matter of theory, but as a practical matter of dollars and cents, the value of the new Constitution. They were well informed. They were conscious of the identity of their interests. They were well organized. They knew for weeks in advance, even before the Constitution was sent to the states for ratification, what the real nature of the contest was. They resided for the most part in the towns, or the more thickly populated areas, and they could marshal their forces quickly and effectively. . . .
>
> Talent, wealth, and professional abilities were, generally speaking, on the side of the Constitutionalists. The money to be spent in the campaign of education was on their side also; and it was spent in considerable sums for pamphleteering, organizing parades and demonstrations, and engaging the interest of the press. A small percentage of the enormous gain to come through the appreciation of securities alone would have financed no mean campaign for those days.
>
> The opposition on the other hand suffered from the difficulties connected with getting a backwoods vote out to the town and county elections. This involved sometimes long journeys in bad weather, for it will be remembered that the elections were held in the late fall and winter. There were no such immediate personal gains to be made through the defeat of the Constitution, as were to be made by the security-holders on the other side. It was true the debtors knew that they would probably have to settle their accounts in full and the small farmers were aware that taxes would have to be paid to discharge the national debt if

the Constitution was adopted; and the debtors everywhere waged war against the Constitution—of this there is plenty of evidence. But they had no money to carry on their campaign—they were poor and uninfluential—the strongest battalions were not on their side. The wonder is that they came so near defeating the Constitution at the polls.

At the end of the volume Beard summarizes the conclusions which, he holds, the facts presented appear to warrant.

The movement for the Constitution of the United States was originated and carried through principally by four groups of personalty interests which had been adversely affected under the Articles of Confederation: money, public securities, manufactures, and trade and shipping.

The first firm steps toward the formation of the Constitution were taken by a small and active group of men immediately interested through their personal possessions in the outcome of their labors.

No popular vote was taken directly or indirectly on the proposition to call the Convention which drafted the Constitution.

A large propertyless mass was, under the prevailing suffrage qualifications, excluded at the outset from participation (through representatives) in the work of framing the Constitution.

The members of the Philadelphia Convention which drafted the Constitution were, with a few exceptions, immediately, directly, and personally interested in, and derived economic advantages from, the establishment of the new system.

The Constitution was essentially an economic document based upon the concept that the fundamental private rights of property are anterior to government and morally beyond the reach of popular majorities.

The major portion of the members of the Convention are on record as recognizing the claim of property to a special and defensive position in the Constitution.

In the ratification of the Constitution, about three-fourths of the adult males failed to vote on the question, having abstained from the elections at which delegates to the state conventions were chosen, either on account of their indifference or their disfranchisement by property qualifications.

The Constitution was ratified by a vote of probably not more than one-sixth of the adult males.

It is questionable whether a majority of the voters participating in the elections for the state conventions in New York, Massachusetts, New Hampshire, Virginia, and South Carolina, actually approved the ratification of the Constitution.

The leaders who supported the Constitution in the ratifying conventions represented the same economic group as the members of the Philadelphia Convention; and in a large number of instances they were also directly and personally interested in the outcome of their efforts.

In the ratification, it became manifest that the line of cleavage for and against the Constitution was between substantial personalty interests on the one hand and the small farming and debtor interests on the other.

The Constitution was not created by "the whole people" as the jurists have said; neither was it created by "the states" as Southern nullifiers long contended; but it was the work of a consolidated group whose interests knew no state boundaries and were truly national in their scope.

The second of the two original hypotheses he thus regards as conclusively verified so far as the available data go.

To carry the theory of the economic interpretation of the Constitution out into its ultimate details would require a monumental commentary, such as lies completely beyond the scope of this volume. But enough has been said to show that the concept of the Constitution as a piece of abstract legislation reflecting no group interests and recognizing no economic antagonisms is entirely false. It was an economic document drawn with superb skill by men whose property interests were immediately at stake; and as such it appealed directly and unerringly to identical interests in the country at large.

The interpretation of McLaughlin

McLaughlin, however, finds the chief cause of the adoption of the Constitution in the persistence, after the Revolution, of individualistic and rebellious tendencies which had gained expression during the war, and particularly in the general impotence of the federal Congress in the face of practically unimpaired sovereignty on the part of the separate states. These matters taught intelligent men (not of any special class economically) that the future existence and strength of the colonies were dependent on replacing the Articles of Confederation by a document establishing some more authoritative central power.[7]

Experience, it is plain, had before 1786 taught the necessity of bestowing on some central authority the power to regulate commerce and

[7] The ensuing quotations are from *The Confederation and the Constitution*, pp. 173 ff., 272 ff., 278 ff., and 316. Published by Harper & Brothers.

the power to obtain revenue without merely begging for it. Every passing year since the adoption of the Articles had shown more clearly that these two powers should have been given to Congress, because without the latter Congress was impotent and ridiculous, and without the former it had no method of meeting the exactions of European nations. It was compelled to look on helplessly while the states working at cross-purposes were angrily trying to retaliate against foreign restrictions, or at the next turn of the wheel of popular caprice were threatening their neighbors with commercial war. The necessity of bestowing such powers was clear, for the lesson had been sharply taught, and though of course it was not learned by the ignorant or the narrow-minded, it was obvious to the intelligent statesmen and men of affairs, who were not yet prepared to give up their country to civil war and ruin. At least this much of the great task of imperial organization had been made clear by the troublesome years of war and the no less anxious years of peace.

But there were plainly other troubles. Congress had been given power to make treaties, but this power could not be properly exercised; and our commissioners, confident and loyal as they were, could not negotiate with assurance or make equitable treaties as long as the states, feeling that they were quite as wise as Congress, were ready to disregard all foreign obligations when they chose. Our relations with Spain and England were fraught with danger. Surely, if commerce was to prosper, if the country was to hold up its head among the nations, the states must be under compulsion to perform their parts, to abide by the promises of Congress, and not wantonly break the plighted faith of their representatives. In the formation of lasting political order in America some method must be discovered for securing the observance of treaties; without assurance of honesty any confederation or any national system would be but sounding brass.

This was clear, and thinking men went further; they saw that, if America was to hold together, more than mere promise and pledge was an absolute necessity. Not for treaties alone, but for other obligations as well, for the satisfactory exercise of the powers given to Congress, there must be some sort of compulsive authority.

Since the movement thus began largely as a means of curtailing the selfishly exercised powers of the separate states, McLaughlin thinks that the most important debates at the Constitutional Convention reflected the struggles between representatives of the larger states, eager for proportional representation in the new Congress, and those of the smaller states, who held out for state equality. These struggles were

finally ironed out in compromise, and the document went forth for ratification to the state conventions. Of the Constitution thus conceived McLaughlin says:

> The Constitution has sometimes been spoken of as if it were in all respects the creature of the men at Philadelphia, or as if it were, as Mr. Gladstone once said, "the most wonderful work ever struck off at a given time by the brain and purpose of man.". . . The idea that they created institutions out of nothingness loses sight of the manner and the conditions of their work. . . . But there is no evidence of borrowing or of slavish copying; for, while they were students and readers of history and knew that their own little experience was not the sum of knowledge, they were practical political workers, had for years studied the problems of forming governments, and had been acquainted with the great process of making state constitutions. The men of the generation who declared independence and formed new states were steeped in political theory as their great-grandfathers had been in theology, and for years they were engaged in the difficult process of adapting old institutions to new ideas, framing governments and laws that suited the economic, social, and moral conditions which the New World had produced.
>
> We might, therefore, expect to find from these experienced craftsmen, not a document hurriedly patched together, nor one taken in part from distant ages or strange climes, but an American document, in its entirety new, but made up of parts that had found their places in the state organizations. If we look, then, for the origin of the Constitution, we find much of it in the failures of the Confederation, in the tribulations of eleven confused years, when the nation was without a proper government and when distress and disorder and incompetence were showing the way to success; and much of it, too, in the state constitutions which had been drawn up by men familiar with colonial governments and administration. . . .

Whence came the opposition to the ratification of the Constitution?

In truth, the Constitution had many foes to meet. There was a little band of irreconcilables who could see no good in making the central authority efficient, who had always opposed the extension of national authority, and knew not how else to act. There were men of wide influence, like Samuel Adams, who had said so much about liberty that they were not conversant with the arguments for government. There were those who had already begun to cherish sectional antagonism,

fearing the development of the west, or disliking the growing power of commercial New England. There were the paper-money men and the discontented needy, who saw in the Constitution a prohibition of bills of credit and of laws impairing the obligation of contracts—a party which had just been successful in controlling the legislatures of seven states. There were those who had been indignant at the proposition to close the Mississippi and were in no mood to see federal power increased and the full right to make treaties bestowed on the central government. There was the body of the people who for a generation had listened to the enchanting oratory of liberty and could be easily aroused to dread. There were those who, living away from the busy sources of trade, saw no need of a central government with wide power of taxation and authority to regulate commerce. No one of these elements was dangerous alone, but together they constituted a party of opposition which was aided, of course, by the big body of hesitants who at such times pause and shake their heads and wonder if it would not be best to let well enough alone. Fortunately, to leave bad enough alone was the alternative, and every day was sure to bring a few thoughtful reluctants to the support of the new Constitution.

Thus in time the Constitution was ratified. The far-seeing statesmen who sponsored it lost no opportunity to press its rational claims upon their hesitant fellows; in states where the opposition was strong they patiently won over yet doubting leaders to their cause, such as Adams and Hancock in Massachusetts, before the matter was brought to a vote; the alternatives to the essential structure of the new document were so clearly discredited that the refractory elements were finally overcome.

In the twelve years that followed the Declaration of Independence the American people had accomplished much. The war was carried to a successful conclusion; the settlements stretching along the Atlantic coast came into the possession of a wide territory extending over the mountains to the Mississippi; state constitutions, laying down broad principles of liberty and justice, were formed on lines of permanence; a new colonial system for the organization and government of the great west was formulated, a system that was to be of incalculable value in the process of occupying the continent and building up a mighty republic; new settlements that showed capacity for self-government and growth were made in the wilderness beyond the Alleghanies. And finally, a federal Constitution was formed, having for its purpose the preservation of local rights, the establishment of national authority, the recon-

ciliation of the particular interests and the general welfare. In solving the problem of imperial organization, America made a momentous contribution to the political knowledge of mankind.

Summary statement of the problem in historiography thus revealed

As explanations of the same sequence of historical events these two accounts are quite irreconcilable. Yet both are written by able men, living in the same period of historiography, and possessing an equal command of the relevant historical materials. To Beard the fundamental goal of men is economic security and prosperity. All other motives, in his opinion, are so far subordinate to this that every great change in history must be viewed as due at bottom to such economic interests and explicable in terms of them. McLaughlin, on the other hand, recognizes a greater diversity of interests as each playing an important role on the pages of history; in determining the adoption of the Constitution there cooperated with the economic motive, as equal in historical importance, such demands as those for stability, for national aggrandizement and effectiveness in dealing with the European nations, and for the equitable adjustment of state jealousies.

And can we fail to sense, behind these two theories of historical explanation and finding expression in them, two different preferences with respect to social ends, each reflecting, in part, a different sensitivity to human needs? On the whole, McLaughlin is convinced that the human interests which were satisfied by the adoption of the new constitution are the ones which ought to have been satisfied; he is quite content that other demands came to be subordinated to these. Beard, on the contrary, is evidently very sympathetic with the needs of those groups that, in general, were given scant consideration in the structure of the new political order; his emphasis on their presence, and their ineffective struggle against the interests of the more powerful, strongly suggests that he would prefer a social pattern which had more fully taken account of their plight, and that in contemporary issues he would side with the groups whose role is analogous to theirs.

Such a situation brings before us in very challenging form the problem of finding and applying an objective criterion for explanations in the social sciences, and especially in history.

If we are to reach beyond what the techniques of statistical description and correlation make possible, a solution to this problem must be discovered before results can be established in these matters which any competent mind will recognize as sound. And science demands such results; it is never satisfied with conclusions that may be no more than an expression of the idiosyncrasies of some individual or some limited group. But if the analysis of this chapter is, in the main, correct, the attempt to reach such objectivity here requires us to face questions that can be left aside by the natural sciences. There, a fairly definite procedure is taken for granted by all investigators. If it cannot be applied successfully at once in any given inquiry, the remedy lies in accumulating further data; or if the data are such that universal laws cannot be verified, then the methods for determining correlations are employed, likewise in quite objective fashion. Here, no procedure possessing these virtues is at hand, and it is not enough to admit modestly that one's conclusions are merely probable, for there is no way to show on objective evidence which conclusions are more probable than others, and how much more probable they are. Especially challenging is this question: Can we find a way of eliminating variable evaluations from theories in social science, and if not, how can we render our evaluations impartial—responsible, that is, to some criterion that all competent investigators will be prepared to accept? In history, indeed, this question seems to take an especially difficult form: How can a historian realize such an alert and flexible sympathy with the interests of all the groups who participated in the course of events he is portraying, that he can be confident that no significant causal factor is left out of his account, and that each factor is given its due weight?

With these perplexities we stand on the borderline of science and look toward the area beyond, which will be examined in Part IV. So far as that area is concerned, matters of evaluation occupy a central position and all problems which arise must be dealt with in full recognition of this circumstance. But before we plunge into that subject, a theme long postponed must be given attention. We are now ready for it.

EXERCISES

1. What is meant by the phrase "borderline of science"?
2. Explain in your own words, and with your own illustrations, each of the five factors which, according to the chapter, render objectivity especially difficult in social science. Can you think of any other factors that should have been mentioned?
3. Just why is one's social theory inevitably a program of action along two lines—assimilating social facts for purposes of systematic understanding, and molding the social pattern, so far as one can, in the direction which he approves?
4. In what respects does history seem to be a social science? In what respects not?
5. As between Beard's and McLaughlin's accounts of the adoption of the Constitution, which do you think is more objective? Why? How can we tell?
6. Read the contrasting interpretations of the events leading up to the Civil War, in J. G. Randall, *Civil War and Reconstruction*, and Avery Craven, *Coming of the War*. Write an essay on the particular way in which the contrast illustrates the points emphasized in the present chapter.

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CHAPTER 24

DEFINITION AND DIVISION

Introduction to the theme of definition

We learned as early as Part I that evaluation is an integral aspect of human reasoning. Any course of reasoning always has a goal, namely, the correct solution of the problem which the reasoner is confronting; he values that goal enough to be unwilling to stay where he is, preferring to engage in the reflective labor necessary if he is to have a chance to attain his aim. This means that there is always a purpose guiding and controlling his reasoning—the purpose of attaining that specific goal. In the next chapter we shall plunge into the more difficult logical problems arising from the presence and nature of evaluation; in the meantime we may make specific use of the fact that reasoning is always guided by some purpose, and that purposes differ from one situation to another. Now there are certain important logical problems, having to do with matters of formal and factual science among others, which cannot be adequately handled without explicitly introducing this factor of varying purposes. Two such problems are those concerned with the definition of words and with the division of general classes into the more specific classes which make them up.

The theme of definition was first introduced in Part I, where distinctions were drawn between the pragmatic, the syntactic, and the semantic meanings of words. In speaking of the pragmatic meaning, we anticipated the point of which we are at present making more systematic use: that the meaning of a word in this dimension is determined by the problem which its user is trying to solve and hence by his controlling purpose.

In Part II it was seen that, syntactically considered, any correct definition establishes a relation of equivalence between definiendum and definiens, so that either can be substituted for the other in any context in which they are appropriate. This property, expressed in terms which have now been introduced, may be thus described: The purpose expressed in any correct definition is such that it can only be attained by establishing a relation of equivalence in this way. Some of the reasons why this is so were seen when that relation was discussed and illustrated; we wish now to consider the subject of definition more fully in a context emphasizing the different purposes which may affect it, and including an examination of semantic meaning as well as syntactic. For none of the distinctions which must be taken into account in dealing with this theme can be understood apart from the purposes which may be expressed in a definition. Two of these distinctions are quite basic to any adequate analysis of the problems of correct definition.

Impartial versus propagandistic definitions

The first such distinction is that between impartial and propagandistic definitions. An impartial definition is one whose purpose is straightforwardly to realize the end which one naturally assumes to be sought when a sentence is put in the form of a definition. This end is that of enabling the hearer or reader of the definition to comprehend clearly what the definer intends by the word defined, and to master by its aid the subject matter to which it refers—the end, in short, of making the definiendum a medium of successful intercommunication and cooperative understanding. The definition of "man" as "rational animal," whatever may be its defects in other respects, is an impartial definition. Its aim is to make "man" a tool of intelligible discourse by specifying exactly the class to which the word is applied, and calling attention to the distinctive property by which one may identify a member of that class. A propagandistic definition, by contrast, assumes the form of a definition, but its real purpose is to create a favorable or hostile attitude toward the thing defined, rather than to make the word which refers to it an instrument

of clear understanding. Consider, for example, the two following definitions of "religion":

> Religion is the opiate of the masses (Feuerbach).
>
> Now and again we must draw a deep breath of relief, and that is religion (Havelock Ellis).

When one views the highly complex personal and institutional realm of fact which is religion, it is evident that neither of these definitions seriously tries to secure the relation of equivalence between definiendum and definiens. Each is really a piece of propaganda rather than an attempt at rational clarification of the subject matter involved. One of them damns religion, by including it within the class of things which produce in people a state of helpless somnolence; the other blesses it, by reducing it to the satisfaction of a simple, universal need.

Accordingly, one must be prepared, when confronting any definition, to distinguish under which of these two types it belongs, and if he finds ground for suspecting it to be propagandistic he must be appropriately on his guard. In that case he will realize that it is futile to expect any helpful clarification of the subject defined, and that the important point on which to focus his attention is rather the question: What attitude is the definer hoping to create by this definition, and what is his aim in trying to create it? In addition to answering this question he will presumably also find it of interest to classify the definition under one or another of the ways in which words can be used for propagandistic purposes, the more obvious of which were listed in Chapter 4. In view of the main line of discussion in the preceding chapter, one will realize that such definitions are much more likely to be met in highly controversial fields than elsewhere, and especially in the subjects with which social science and social policy are occupied. And he will remember that the propaganda may be unconscious instead of conscious, that is, that the definer may not be clearly aware of the emotional bias expressed in his

definition; accordingly, that it is his own responsibility to disentangle that factor and allow properly for its presence. We shall discover soon that when the term defined refers to some value rather than to a given fact there are very serious difficulties in the way of carrying out this task.

Definition as affected by the syntactic, semantic, and pragmatic dimensions of meaning

And this circumstance brings us to the second of the two basic distinctions needed. It can be drawn in the case of both impartial and propagandistic definitions, but we shall now leave the latter out of account, concentrating on the classification and analysis of impartial definitions. This second distinction answers the question: Which, and how many, of the three dimensions of meaning is the definer concerned with, in offering us any definition that is under examination? It has been noted that a definition may have a pragmatic, a syntactic, and a semantic aspect. Now which among these it has, and how they are interrelated, are factors that make an important difference.

Let us begin the analysis suggested by this circumstance with an elementary consideration. In one sense every definition has a pragmatic aspect and a syntactic aspect. When anybody proposes a definition he is always trying to solve some problem and therefore is trying to fulfill some purpose. The presence of such a problem and such a purpose in the definer's mind gives it pragmatic meaning, as the latter was explained in Chapter 2. These factors determine the realm of discourse to which the word defined is relevant, and in what way it is relevant. And since the definition is to be capable of use in any piece of reasoning to which it is relevant, it must have syntactic meaning; one would otherwise be unable to draw deductions from it at the fourth step of the reasoning process. Hence, in this functional sense, every definition has both pragmatic and syntactic meaning.

But in another sense, a definition may have one of these three aspects without the other two, or two of them without the third. This is the sense which we will have in mind when we ask the question: What does the definiendum denote, a formal relation, a fact, or a value?

First, it is evident from our entire study in Part II that certain words are needed to refer specifically to formal relations and nothing else; one could not talk unambiguously about those relations if words of this sort were not available. Illustrations, of course, are found in the technical terms of mathematics and the logical constants, such as "and," "not," "or," "implies"; these, as currently employed, do not refer to any perceived fact, or to any purpose or value, but simply to formal relations that may obtain between the elements of rational discourse. Accordingly, one of the specific purposes which may control a definition is the purpose of employing a certain word or phrase to denote clearly such a formal relation. It is obvious that a definition of this kind will be a syntactic definition, possessing pragmatic meaning only in the general sense explained above, namely, that any definition reflects some problem which the definer is trying to solve.

Second, words are also needed to refer to facts, either facts capable of being perceived or facts which have to be inferred in order to explain those that can be perceived. As such, they have semantic meaning, which the ones just described do not. They also have syntactic meaning but in the sense in which all words must have it, *i.e.*, the sense determined by our need to reason deductively about facts as well as about formal relations, which would be impossible if assertions about them had no syntactic meaning. They likewise have pragmatic meaning, but again in the merely general sense in which all uses of words reflect some purpose. In the case of these words the specific purpose expressed in any impartial definition will obviously be twofold: (1) it will seek to establish a relation of equivalence between definiendum and definiens, which is necessary if the definition is to fulfill its syntactic role properly; and (2) it will seek to specify unambiguously the range of facts covered by the word, which is necessary if its semantic role is to be successfully performed.

Finally, it is clear that some words are needed by which to refer to factors that are distinctively involved when the pragmatic aspect of meaning is given explicit attention. That is,

there are times, *e.g.*, in the present chapter, when we need to talk about such things as purposes themselves, and the goals at which they aim. In other words, while all elements of discourse have pragmatic meaning in the general sense above indicated, some such elements have the specific role of referring to entities which appear only in the context of evaluative analysis. The most interesting words of this sort, from our present viewpoint, are words which refer to the kinds of value that may be pursued by people; there are challenging problems about any attempt to define them accurately which will have to be faced in the following Part of the book. These words, of course, have syntactic meaning; we often need to reason deductively about them, and could not do so if this were not the case. They also have semantic meaning, but only in a limited sense which Chapter 26 will try carefully to indicate.

Analysis of syntactic and semantic definitions

Our immediate task, then, is a further analysis of the syntactic and semantic aspects of definition in the light of the above considerations. Definitions illustrating these were used in Chapter 8 to bring out the formal relation of equivalence between propositions, and the circumstance that this equivalence is either strict or material. We are now in a position to see, among other things, when and why each sort of equivalence must obtain.

When anyone defines a word or phrase, his main purpose will be to accomplish one or another of three things, and in each case it leads to a particular kind of definition, reflecting the nature of that purpose. Hence there may be verbal definitions, conventional definitions, and real definitions. We shall begin with the first of these three.

Since all terms without exception have syntactic meaning, a part of the intent expressed in any impartial definition is to disclose that meaning clearly—that is, to indicate to any reader or hearer what word or phrase he may at any time substitute for the word or phrase defined. If "not-*p*" is defined as meaning "*p* is false," any reader will take it for granted that wherever he comes across "not-*p*" in the definer's

formal system he may substitute for it "p is false." If "man" is defined as "rational animal," one similarly assumes that either of these two terms may properly be substituted for the other.

Verbal definitions

Suppose now that the major purpose controlling the formulation of a definition is simply to provide an appropriate equivalent in this way. This is frequently the case, and when it is so the resulting definition is called a *verbal* definition, *i.e.*, one which merely indicates clearly what linguistic substitutions may be made in interpreting the word defined as the author uses it. And the equivalence established in such definitions is strict equivalence, as examination of appropriate illustrations will quickly show. A scientist, for instance, often needs to introduce a technical word by defining it in terms of words already familiar to his colleagues. Newton defined the "mass" of a body in terms of its quantity of matter, as measured by the product of its density and volume.[1] Such a definition enables the definer to avoid repetition of a lengthy phrase describing some complex fact or operation which he can now briefly indicate by his new symbol. But since he intends to claim nothing more than abbreviated substitution of this sort, he is complete master of how his symbol is to be employed, so far, at least, as his writings are concerned. It becomes a substitute for the longer phrase it replaces merely in virtue of his arbitrary decision thus to use it. And hence no question can arise as to whether his definition is correct or not. If it is asked: How does one know that the definiendum and the definiens are genuinely equivalent? it is evident that no appeal to any circumstance external to his choice needs to be made; a sufficient answer is simply: He has chosen to make the one a substitute for the other. And if he does not adhere to the definition avowed,[2] but proceeds to use the new symbol in a different sense, his readers will charge him with self-contradiction and demand that he provide a definition

[1] *Principia*, Definition I.

[2] As was the case with Newton's definition of mass. He actually measures it by a different procedure from that specified in the above definition.

which he is willing consistently to respect. These circumstances, however—the absence of any external facts which need to be taken into account, and appeal to the principle of noncontradiction as the sole criterion applicable—indicate clearly that the equivalence thus established between definiendum and definiens is strict. The one formally implies the other in virtue of his arbitrary determination that such is to be their relation.

As this illustration shows, it is not necessary that the term rendered strictly equivalent to another in this way be one which has not been used before. It is always permissible for a thinker to propose his own definitions of terms that are already current if he intends to use them as technical media of communication; indeed, it is requisite to do so if the meaning of those terms has previously been vague or ambiguous. The definitions chosen will then be authoritative within the system elaborated by the author on the foundation which they provide.

There are various other situations in which verbal definition seems appropriate and sufficient. Often, in debate cr serious discussion, a speaker or author is challenged to justify his use of some important word, such as "democracy," "communism," "God," "free enterprise"; perhaps he will anticipate that this question is in the minds of his hearers or readers and attempt to answer it in advance. And the answer in many cases will amount to this: "I am making no claim that the word or phrase is generally used in the sense here proposed, or that it should be so used; but this is one of its possible meanings, and my present purpose is to examine what logically follows if it is taken in this sense." If he adheres to this purpose it is clear that in the context of his discussion the definition is to be taken as merely a verbal one. Such a deductive analysis, guided by a verbal definition, will help to show, among other things, whether the proposed definition is on the whole a wise one or not.

When the word defined in a verbal definition has already been employed in another but more or less related sense, con-

fusion is difficult to avoid. In that case habits encouraged by past associations are likely to lead the readers or hearers of the definition seriously astray. When we find ourselves in this situation it is essential, therefore, to be on guard lest other plausible meanings of the term defined are allowed to slip back surreptitiously and to affect our interpretation of the author's subsequent argument.

Although verbal definitions are completely determined by the fiat of the author, it must not be supposed that they are purely arbitrary. This is because they always have pragmatic meaning, as above explained. The definer is concerned about the solution of some problem, and is selecting the definition that he proposes because he believes it will contribute best toward that solution. And, unless he is just engaging in a bit of private play, the problem is one that concerns others too; hence if he is competent in the field they will take his definition seriously as a plausible postulate. This becomes especially clear when we note that definitions in the formal sciences, at least when they are made explicit, are always verbal definitions. Formal concepts can, of course, be defined in various ways, but the author of a formal system is confronted with a definite kind of problem; he wishes to realize in his systematic interrelation of those concepts the virtues of economy, elegance, and comprehensiveness. Some words will have to be left undefined. Which shall he choose for this purpose, and how shall he define others in terms of them? For example, *Principia Mathematica* defines "implication" in terms of the notions of truth, falsity, and weak disjunction. For p to imply q means simply: "Either p is false or q is true." Why this choice instead of defining "either . . . or" in terms of "implication"? Chiefly, because the authors discovered that their chosen definition seemed to secure best these ends of simplicity, elegance, and adequacy in their formal system as a whole. Their purpose was to reveal the interconnections of the defined concepts in such a manner that all the problems to which they are relevant can be solved with maximum ease and lucidity. But since there appears to be no way of showing,

on objective grounds, that any particular choice is rationally superior to others in the light of these ends, a proposed definition must be regarded as the expression of the author's guess as to what would be most convenient. It carries no authority for anyone else, and hence is a verbal definition.

Conventional definitions

We come then to the second purpose which may control the formulation of a definition. A definer's main intention may be, not just to propose what he regards as a convenient substitute for a given word or phrase, but to state how the latter is actually used in some community, broad or narrow, which he has in mind. In this case the definition offered is a *conventional* definition. And it is clear that such a definition is capable of being correct or incorrect. It is correct if current usage actually does employ the term in the sense proposed, otherwise it is incorrect. Dictionary definitions, for example, are conventional definitions, and may properly be judged by this criterion. The dictionary at my elbow defines the verb "cling" as meaning "to hold on to something firmly." This is a conventional definition, and is correct if the verb is currently used as equivalent to this phrase. It defines "infinite" as having six different meanings. One of them is given as follows: "(Math.) A quantity conceived as always increasing so as to exceed any other assignable quantity in value." This is correct if, in the science of mathematics, though not necessarily in other contexts of discourse, the word is employed in this sense.

Conventional definitions exemplify material, not strict, equivalence. This is because the equivalence obtains merely as a matter of fact, not by any logical compulsion. There is an external standard to which it must conform for that equivalence to hold, namely, current usage, and if that usage should change the equivalence would be lost. Thus, if the time should come when mathematicians no longer use "infinite" in the above sense, the equivalence previously obtaining would no longer obtain, and the definition would be incorrect.

Real definitions

Finally, the main intent of a definer may be neither of these, but rather to propose a definition which will truly conform to the actual subject matter to which the definiendum refers,

and will therefore be one which anybody who wishes to deal successfully with that subject matter would do well to adopt. In this case the definition set up is a *real* definition. It claims not merely to offer a convenient equivalent, and not necessarily to agree with presently accepted usage, but to correspond to the actual nature of the thing defined. Hence real definitions are only possible in the case of words referring to facts, while verbal and conventional ones are possible in the case of any words. When "nausea" was defined in Chapter 8, the definition was intended to be a real definition. Here, as with a verbal or a conventional definition, the definer claims logical equivalence between definiendum and definiens; if "sickness of the stomach with a desire to vomit" is a good definition of "nausea," either of these terms can be substituted for the other whenever we wish. There is a relation of mutual implication between "This is a case of nausea" and "This is a sickness of the stomach with a desire to vomit."

But the equivalence here is likewise material, not strict, and for the same general reason as appeared in the case of conventional definitions. Here, too, there is an external standard. But that standard is now not current usage; it is some class of facts. The aim is to identify unambiguously the semantic referent denoted by the word in question. This can only be done by selecting some property or properties that, in the present state of relevant knowledge, seem essential to that class, and stating them in the definiens. However, the nature of any fact is not controlled by the formal implications of the words through whose aid we now happen to interpret it. As scientific investigation of any class of facts advances, other properties may at any time be discovered that prove to be superior, for purposes of correct identification, to the ones now generally chosen. And the words denoting these properties will not have exactly the same implications as the words previously used. Hence the equivalence affirmed in the definition depends on the facts which one is trying to understand, and what they disclose to continued empirical investigation. The most, therefore, that can be claimed for a real definition

is that at present, when appeal is made to the best relevant scientific evidence, we find that the definiendum and the definiens do denote the same class. There is no logical necessity for them to be equivalent; only material equivalence is then possible.

Another way of summarizing these distinctive circumstances is to note that real definitions, like conventional ones, may be incorrect instead of correct. This happens when they fail to give an accurate description of the class denoted, as judged by available scientific knowledge.

We are ready now to list the rules by whose application one can tell whether a given definition is an adequate one or not. There are six rules which are pertinent to all definitions, whether verbal, conventional, or real, being corollaries of the general purpose expressed in any impartial definition. Then besides these there are distinctive procedures which are pertinent in the setting up of conventional and real definitions. Let us begin with the six rules that are relevant to all definitions. Three of them are absolute rules; three are advisory guides.

General Rules of Good Definition

General rules of good definition

1. The definiens must be equivalent to the definiendum. This equivalence is strict in the case of verbal definitions; material in the case of conventional and real definitions.

This rule has already been sufficiently discussed.

2. Whatever definition is given should be adhered to consistently in the context of the same argument or discussion.

If this rule is not obeyed, certain of the definer's assertions will contradict what is affirmed by others, and his readers will not know how to resolve the inconsistency. But nothing prevents him from adopting another definition in a different context if he finds it desirable to do so. If the definition is verbal, he may decide that another definition is more convenient than the one previously used; if it is conventional or

real, he may find that another definition conforms better to the external standard which such definitions must respect.

3. The definition must not be circular.

This rule means that the definiens must not repeat the definiendum, or use any word derived from the same root. Suppose that the following definition of "the devil" were offered: "The devil is the diabolical force in the world." Clearly, nothing has been gained in the way of clarifying to anybody the meaning of "the devil." It is a definition in form but not in substance, since the definiens merely repeats under another guise the word which was to be defined. Hence, anybody who knows what "diabolical force" means can be assumed to know what "devil" means, and the general purpose of a definition cannot be fulfilled in this way. But it should be noted in this connection that sometimes not all of the apparent definiendum really belongs to the definiendum; in these cases there may appear to be circularity when in fact there is none. "A hot summer is a summer in which the mean temperature exceeds that of the average summer by at least three degrees fahrenheit." This definition may be defective, but not because of circularity. For the purpose is not to define "summer" but to indicate what may properly be meant by saying that that season in any particular year has been hot. Hence in this case repetition of the word "summer," far from violating rule 2, is a real aid in providing a suitable definition.

Three rules now follow which are not absolutely coercive, but which, except in unusual circumstances, it is wise to obey if the definition is to accomplish its purpose.

4. The definer should explicitly state whether the definition is intended to be verbal, conventional, or real.

Usually, of course, one can tell from the context which of these sorts of definition is intended. But this is not always clearly evident unless it is specifically stated. Hence, to avoid misinterpretations, it is well to state it.

5. The definiens should be in positive rather than negative terms.

This rule is not an absolute one; there are cases where a

negative definiens is superior to any alternative. No one, for example, would be likely to object to a definition of "darkness" as "absence of light." But it is a helpful guide, since in all but a few cases positive definitions are possible and are likely to be less vague than negative ones. Mere absence of a property usually leaves open many positive possibilities of interpretation, and does not determine precisely which of them is the one intended.

6. The definiens should be more familiar and clearer than the definiendum; at least obscure language should be avoided.

Since it is the purpose of any definition to play a social function, establishing with one's readers or hearers a common understanding of the term defined, the virtue of this rule is obvious. Nothing is gained by a definition, so far as this purpose is concerned, if the definiendum is already as familiar and clear as the definiens. A violation of this rule (cf. Plato's definition of "time" as "the moving image of eternity") is a case of defining *ignotum per ignotius*. Of course, it is a difficult rule to obey unless the audience contemplated is quite limited and its intellectual habits well known, for what is familiar and clear to one person or group may not be so to another. This relativity is especially evident in scientific and technical definitions. What is familiar and clear to a physicist will not necessarily be so to other people. Hence a definition suitable in his realm of discourse might not be suitable in other realms. And the fact that this is not an absolute rule appears from one significant circumstance. A lexicographer is expected to define, among others, the simplest words in daily use; he cannot fairly be asked to find definiens for these that are more familiar. Who could define "give" in terms that are less obscure than the definiendum itself?

We come now to certain procedures which are helpful guides in the formulating of good definitions. These apply to conventional and to real definitions but not to verbal ones, since the latter are freely responsive to whatever special purposes may affect the definer's choice of terminology. About

all that can be said in their regard is that if the term defined is one already employed, it is best to respect, in the definition, the main associations that its use in the thinking and writing of others has accumulated. Failure to obey this rule leads almost inevitably to misinterpretations of statements in which the term is used. Readers will forget the definition and unconsciously insert in its place some meaning of the term with which they were previously familiar. And the definer may unwittingly do this himself. But in the case of conventional and real definitions there are several procedures, each of which in certain circumstances may promise best to realize the purpose of the definition. The main alternative procedures will now be listed.

PROCEDURES IN FORMULATING CONVENTIONAL AND REAL DEFINITIONS

Methods of expressing conventional and real definitions

1. Often the use of a synonym, or a series of partial synonyms, is a good way to formulate a conventional or a real definition. This is especially the case when the definiendum denotes an attribute. One might, for example, define "precarious" as meaning "risky," assuming that the two adjectives are equivalent, and that the latter is more familiar than the former. Or, if he thought that they are not exactly equivalent, he might give a series of partial synonyms—"risky, uncertain, insecure, untrustworthy"—in the expectation that his readers will distill what is common to these words and that it will be the equivalent needed.

2. The method of using a well chosen series of examples of the definiendum often accomplishes the purpose. In this case one enumerates a number of typical instances of the phenomenon to be defined, in the hope that the reader or hearer, comparing those instances, will apprehend the range of fact which the definiendum is intended to cover. Thus one might define "green" as "the color which is observed in fresh grass, in leaves before they have turned in autumn, in an emerald, in the sea when the sky is covered with light clouds."

3. Akin to an exemplary definition is an ostensive one. This

can only be employed in the actual presence of various examples of the definiendum, so that they can be directly pointed to instead of being named. If one were to give an ostensive definition of "green" rather than an exemplary one, he would point to various green objects in the neighborhood, and say: "I mean the color of this, of that, of the other, etc."

The reader will observe that neither an exemplary nor an ostensive definition formally secures complete equivalence between definiendum and definiens, except when the class defined is so limited that it can be exhaustively presented in a few members. Otherwise, the definer can hardly enumerate all members of the class, and will not attempt to do so. Because of this circumstance the instances listed in the definiens will not exhaust the definiendum and therefore do not constitute its full equivalent. The definer can only hope that they will be sufficient to enable the hearer or reader to apply his meaning correctly to instances not included in the enumeration. And, as we have seen, even the use of synonyms may easily fail to provide a genuine equivalent, or may provide it only by trusting the reader's ability to analyze properly a series of synonyms and to apprehend what they have in common. These circumstances suggest that employment of the method of analysis, where it is feasible, offers, except in unusual circumstances, the best way to secure the precise equivalence sought in a conventional or real definition, and to provide the kind of explanation of the phenomenon referred to that people usually expect in a definition.

The method of analysis

Let us examine analysis in general first, and then see what sort of analysis would seem best to serve the purposes of conventional or real definition.

In general, to analyze any subject matter is to specify the elements of which it is composed, and the relations which obtain between them. Thus, analysis of a plane triangle would specify the three straight lines of which it is composed, together with the fact that they are so related as to form a closed figure. The same entity may be analyzed in various ways, depending on the particular purpose which is to be

served by the analysis. Obviously, if one analyzes a certain valley in such terms as would serve the needs of a farmer, his analysis would appeal to somewhat different constituents than those which would suit the purpose of an artist setting up his easel. In any sort of analysis, however, equivalence must obtain between the entity analyzed and the list of elements in their relations, of which it is asserted to be composed. Otherwise the analysis would not be logically complete, and it would be illegitimate to substitute a proposition expressed in terms referring to those elements for a proposition referring to the unanalyzed object. Analysis thus, by its very nature, fulfills one of the main requirements of a correct definition, and does so completely.

But what sort of analysis can be trusted to explain an unfamiliar term in terms that are more familiar? In many cases, if we analyze an object exhaustively into its constituents, we would find that some of those constituents are less familiar to our readers than the object itself, hence such an analysis would not serve the purpose of a definition successfully. The anatomical constituents of a worm, for example, would be less familiar to most people than the worm itself. In cases where this is not so and where the constituents are not too many (as with the triangle), a definition which exhaustively specifies the parts and their relations is a good definition. How shall we proceed elsewhere?

Analysis by genus and differentia

It happens that there is a kind of analysis that is peculiarly adapted to meet this need. This is analysis in terms of *genus* and *differentia.*

By "genus" is meant a larger class including the class to be defined; by "differentia" properties possessed by the latter which are sufficient to distinguish it from other species belonging to the same genus. We have already made use of this kind of analysis. Thus, in the definition of "nausea" given above, "sickness of the stomach" is the genus, and "with a desire to vomit" the differentia which distinguishes nausea from other kinds of stomach ailment. In the definition of "man," "animal" is the genus and "rational" is the differentia.

The genus or differentia which may be appropriately selected for such a definition varies, of course, with the universe of discourse determined by the definer's purpose, *i.e.*, with the range of fact to which the definition is supposed to be relevant. For example, the definition of "problem" given in the first chapter of this book doubtless differs in both genus and differentia from the definition that a high school geometry teacher would be likely to offer. The purpose of the former definition is to identify the term in such a way as will clarify the nature of reasoning wherever and however employed, while the purpose of the latter contemplates merely the assignment of specific tasks in the limited field of formal education—perhaps the still more limited field of elementary geometry. But in any case the assumption, apparently well justified by experience, is that anyone unfamiliar with the class to be defined would be familiar with some larger class of which it is a species, and would be acquainted with the features which distinguish it from other species of that larger class, even though he might not be familiar with all the constituents that an exhaustive analysis would disclose. Thus, one unacquainted with the word "monograph" might also be unacquainted with some of the constituents which a literary analysis of a monograph would bring out; but he could be assumed to know what a "piece of writing" (the genus) is, and also the meaning of "a systematic description of some closely limited subject matter" (the differentia).

The reader might well raise an objection to our emphasis on this procedure for conventional and real definitions. "Does it not presuppose," he might ask, "that the way to understand anything is in terms of the inclusion of one class in another, and its exclusion from others, *i.e.*, after the fashion of the Greek classificatory hierarchy, whereas the preceding chapters have indicated that understanding only becomes exact, and valuable for purposes of prediction, when things are explained in causal, or at least correlational terms?" The answer is that the contradiction is apparent rather than genuine. One can hardly engage hopefully in the search for a correlation or

a causal explanation unless he already knows how to identify clearly the class of things with which he is dealing. In other words, he must already have in mind an adequate real definition of that class. When one needs such a definition he is not yet in a position to seek a correlation or explanation; he is in the preliminary stage of inquiry at which it is not yet clear, to him at least, just how members of that class are to be accurately identified. And definition in terms of genus and differentia is ordinarily the best way to perform this preliminary step. Only when such accurate identification has been secured can one proceed to find out what other phenomenon, if any, is regularly or significantly conjoined with the appearance of a member of this class.

Division

When we were considering the formal relation of equivalence in Chapter 8, the process of dividing a genus into its constituent species was used for purposes of illustration along with that of defining a term. That chapter explained why a proposition affirming something about a genus is equivalent to one making the same affirmation about the species into which it is divided (when the division is correct), and under what conditions the equivalence is strict, under what, material.

But, as with definition, there are certain important questions about division that can only be answered by examining the purposes expressed in any given division. In the first place, it is well to recognize here, as in dealing with definition, that there are propagandistic divisions as well as impartial ones—the former being those whose real purpose is to arouse a favorable or hostile attitude toward some class mentioned in the division. For instance, conservative journals often divide the genus of American citizens, by implication if not explicitly, into the two species of one hundred per cent Americans and communists. Then they proceed to pin the communist label on all who are not (by their definition) one hundred per cent Americans. It is important to be ready to identify any propagandistic division as such, so that one will not be misled by it and will raise the further questions regarding the author's

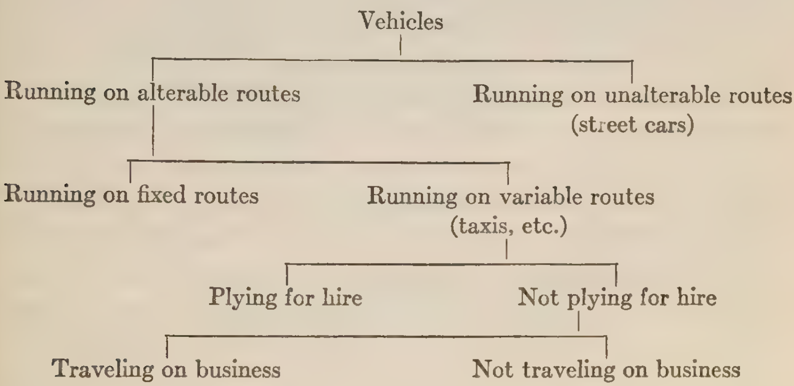
aim that then become appropriate. By contrast, the general purpose expressed in any impartial division is to find a set of species which are mutually exclusive and which, between them, include all the members of the genus divided. When this quest is successful the syntactic relation of equivalence will necessarily be exemplified; if an object is a member of the genus it is also a member of this, that, or the other species, and if it is a member of any of the species it is a member of the genus.

However, an impartial division may exemplify one or the other of two subordinate aims. It may seek formal exhaustiveness as its essential demand, or it may seek factual applicability as its major requirement. The former aim requires strict equivalence between the genus and its species. It is always an appropriate aim when the genus divided is a formal pattern, because in that case a certain set of species is determined merely by the definition of the genus. The genus "foursided equiangular plane figure" is divided into two species—squares, and rectangles with two sides longer than the others. This division can be formally deduced from the concept of the genus, and the equivalence secured is strict; if anything is a foursided equiangular plane figure it must necessarily be either a square or a rectangle that is not a square. The latter aim has to be satisfied with material equivalence. The genus divided in this case is always a genus of facts, and the purpose is to locate each of those facts in one and only one appropriate species of the genus. A division of the genus "bears" into grizzlies, brown bears, black bears, and polar bears would instance this situation. And only material equivalence is possible because, even though this division might be adequate at present, there is no guarantee that the further advance of zoological science would not discover animals that do not belong to any of these species but which will appropriately be classified as bears. The equivalence then depends upon the factual material involved, and is tentative instead of absolutely determined by the nature of the genus as defined.

Dichotomous division

When is it permissible to set up a formally exhaustive divi-

sion if one is dealing with factual material? The answer is: Whenever the division is applicable to that material in such a way as to help in solving real problems that are faced in the attempt to understand its behavior. The procedure employed in this case is called *dichotomous division*. An example which brings out its essential characteristics, and shows that it is sometimes helpful in factual divisions, is the following division of the class of vehicles using the streets:[3]

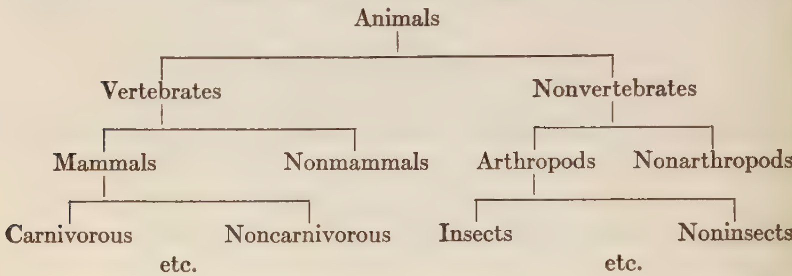


Here formal rigor is secured by dividing each class into two subclasses, of which each is the complement of the other. Thus, also, these subclasses are necessarily mutually exclusive. Clearly, if anything is a street vehicle it must use either an alterable route or an unalterable one, and any route must be one or the other of these and not both. Similarly, any vehicle running on a variable route must either ply for hire or not. The principal scientific problem which arises from the use of this method is the problem of interpreting such a formally rigorous division of a factual genus in such a way that it locates real and important subclasses. In practice this means finding some class with significant positive properties that can be identified with each of the negative classes of the table.[4] In the example at hand this is, to a large extent, pos-

[3] L. S. Stebbing, *A Modern Introduction to Logic*, p. 434.

[4] And such identification secures material equivalence only, between the negative class and the positive one identified with it.

sible. In most cities the class of vehicles running on unalterable routes is identical with the class of street cars, and the class of vehicles not traveling on business is perhaps identical with those used for pleasure. But in many cases this is not possible in any manner that is of scientific value. Suppose that a zoologist should attempt to divide the class of animals in this fashion:



Here it seems impossible to identify the negative classes—nonvertebrates, nonmammals, nonarthropods, etc.—with classes possessing positive characteristics that are scientifically significant. Hence listing them seems to have no constructive value; one must proceed at once to find some positive subclass included within each negative class, before the animals covered by the latter can be dealt with in any way that promotes genuine understanding. In such cases it is better to be content with material equivalence and to set up the best frankly factual division that is possible.

The *fundamentum divisionis*

With one more important consideration in mind, we shall be ready to state the rules of good division. The distinction between impartial and propagandistic purposes expressed in a division has been emphasized, and we have noted why, within the class of impartial divisions, a distinction is needed between divisions which aim essentially at formal exhaustiveness and those which aim primarily at factual applicability. Now any impartial division will also reveal another purpose, more specific than these two. This more specific purpose, when given explicit statement, appears in the guise of what is technically called the *fundamentum divisionis*—the prin-

ciple which guides the division. The nature of such a principle will be brought out by an illustration. If children are divided into obedient and disobedient children, the *fundamentum divisionis* is their attitude toward parents and others in authority, and the specific purpose is to group them in accordance with this attitude. Now it is clear, on the one hand, that divisions of the same genus may legitimately vary in the *fundamentum divisionis* employed; there are many other appropriate ways of classifying children than according to their attitude toward their elders. So there is no single division of any genus that is alone correct while all others are incorrect. For another illustration, consider the several divisions of its patients which a hospital will make. They will be listed according to their disease, to the doctor in charge, to the accommodations given, to the alphabetical position of their last names, etc. Similarly, houses may be classified by location, type of structure, number of rooms, market value, and in many other ways. But on the other hand it is likewise clear that one must not change the principle of division while carrying out any given division. Suppose that one were dividing the genus of living human beings, and began by listing Caucasians and Negroes as species. The *fundamentum divisionis* is evidently the color of the skin. If, now, he should list Australians as another species, this *fundamentum divisionis* would have been abandoned for another; he is now dividing human beings according to the continent which they inhabit. But any continent might have both Caucasian and Negro residents, hence such a change would introduce confusion. This consideration, together with the two noted above when the general purpose expressed in any honest division was described, determines the basic rules which any good division must obey.

RULES OF DIVISION

1. A single *fundamentum divisionis* must be adhered to in the division of each genus.
2. The species into which the genus is divided must be mutually exclusive.

3. The constituent species must, taken together, exhaust the genus.

Main lessons for reasoning in general from factual science

Before finishing Part II, we attempted the difficult task of stating in summary form the main lessons which human habits of reasoning in general can learn from formal science, especially from the distinctive virtues which it emphasizes. Let us now make the same attempt with factual science.

Almost every educated adult in the modern world is aware of factual science as a body of verified knowledge; every student of the subjects discussed in the preceding chapters is also aware of it as a set of tested and approved methods of reaching knowledge. What we are now implying is that in addition to being these things it is the expression of certain attitudes or habits of mind that are typical among members of the scientific fraternity—habits of mind which all who wish to reason clearly and wisely would do well to share. Let us list the more obvious qualities which together make up this attitude of mind.

1. Devotion to truth

In the first place, the habit of mind encouraged by factual science is characterized by devotion to truth. As against every sort of wishful thinking, every kind of intellectual dishonesty, the scientist is eager to discover the real truth about whatever subject he is investigating. His motto in this regard is a famous statement by Bishop Joseph Butler: "Things are what they are; things will be what they will be; why should we wish to be deceived?" This attitude of sincere and eager commitment to truth appears in two main forms. One is respect for evidence. The empirical scientist thoroughly distrusts assertions unsupported by the kind and amount of evidence that long experience has shown necessary if they are to be held trustworthy. An argument which, in any respect, smacks of evading the relevant evidence is to him worse than untrustworthy. It is an overt sin against man's integrity as a rational being, a flagrant violation of the sacred responsibility laid upon every thinker to square whatever he asserts with the facts lying before our attention. He knows that the tempta-

tion to select arbitrarily, to neglect and even to distort evidence, is insidiously seductive—this is the way in which one's emotion or prejudice continually seeks to subordinate his reason to its own irrational demands—and that the temptation can only be successfully met by forming the habit of alert readiness to accept the evidence wherever it leads. The other form is humble submission to an order of nature whose events, outside of man's behavior and the area he has become able to control, take place not because of any relation to human wants or hopes but solely because their natural causes have occurred. The scientist abandons the popular expectation that somehow deserving needs will be providentially cared for, that happy miracles will happen in behalf of persons with good intentions, relieving them of the necessity of wisely adjusting their lives to a world of dependable law. This expectation is not only present in the widespread religious belief that "the Lord will provide for his own"; it is exhibited in the more general Micawberish optimism of those who lazily wait for some lucky chance to turn up and bring them the success that they could not win for themselves. The factual scientist is thoroughly convinced with Francis Bacon that "Nature can only be commanded by being obeyed"—that is, that the effective insertion of man's powers as a causal factor, and the intelligent realization of such human hopes as may be justly entertained, must be based on clear understanding of what happens in nature apart from human interference, and on the modest realization that there is much in the universe that lies beyond man's control. In both of these forms, the attitude of humble truth-seeking fosters a quiet imperturbability in the presence of nature's order that discourages unstable fancies and encourages impartiality, patience, and a penetrating sincerity.

2. Tentativeness

In the second place, the scientific attitude is one of tentativeness. Our natural human longing is for certainty in our beliefs about everything that is important to us, so that we may rest in them with complete confidence, and may be ready to act on them without doubt or hesitation. But we find as a

matter of fact that human experience, and hence all knowledge based on experience, is always changing; many beliefs unquestioningly accepted in the past have had to be modified. Moreover, the world in which we live seems itself to be changing rather than static, and we must constantly readapt ourselves to it. For these reasons the astute of mankind have learned to distrust the feeling of certainty and the tendency to hold fixedly and unalterably to beliefs once adopted. They have found that in general it is far wiser to be prepared to correct their convictions, in the light of whatever new evidence may at any time appear, than to hold fast to them with grim loyalty. This lesson is embodied in the attitude of tentativeness—of complete open-mindedness to further argument and new data—that is characteristic of the scientific mind. Such an attitude demands, for its full expression, a social arena which allows unqualified freedom of thought, of investigation, and of stating any honest opinion, because when these freedoms are absent the discovery and presentation of new evidence are placed under a severe handicap. It bids us welcome new facts, new hypotheses, and new methods of inquiry from any quarter and at any time. The major practical problem arising from adoption of this attitude on the part of any individual is the problem of combining intellectual tentativeness with readiness for fearless decision and action when these are needed.[5] Often one must act even when the hypotheses on which he acts are inadequately verified, as when a surgeon performs an operation while his diagnosis is not yet complete because the diagnosis, so far as it was carried, indicates that an immediate operation is necessary if the patient's life is to be saved. Science bids us use without hesitation the best knowledge available at any given time, even while we are aware that further research will almost surely make it better.

The two most important forms in which this tentative attitude concretely appears are freedom from dogmatism on the one hand, and commitment to continued progress in

[5] Cf. also below, 729 f.

scientific understanding on the other. The tentative mind is always undogmatic in expressing its present beliefs, because it knows that far from being absolute they are subject to modification in the light of new experience. And such a mind expects progress, because only thus can the best knowledge now at hand be gradually replaced by conclusions resting on wider evidence, more critically canvassed.

3. Cooperativeness

In the third place the scientific attitude is a sociable attitude. The knowledge pursued by science is not esoteric but public knowledge. Hence the scientist thinks of himself, implicitly if not explicitly, not as an isolated inquirer but as a member of a far-flung fraternity consisting of all honest seekers after truth in every age and country. This habit of mind is disclosed foremost in the objectivity which is one essential criterion of truth in factual science—the demand that a conclusion of any inquiry must be capable of verification by any normal mind that has undergone the training necessary to understand the problem. Agreement between disciplined investigators is an essential test of truth, because where it is lacking further investigation is always recognized as needed before the verification can be pronounced sufficient. This insistence gives scientific knowledge an impersonal character; it is something that, in the nature of the case, transcends the biased idiosyncrasies of individuals and of groups selected on any other basis than that of intellectual competence. It also affects the range of application that is assumed to be appropriate for scientific knowledge. Governments and corporations may keep items of it under lock and key for a time, but so far as scientists themselves are concerned it is open to anybody, anywhere, to use in the furtherance of any ends to which it is pertinent. Scientific knowledge is, in their eyes, a free addition to the intellectual possessions shared by all humanity; in making his own contribution to it any scientist shows that he cares more for what can be verified and used by mankind in general than for alternative goods that would be satisfying merely to himself or to some limited group. It is no accident that not merely the great discoverers in

medicine but many others have refused to patent their findings or to exploit them in any way for personal ends. Public-mindedness is a vital feature of their scientific attitude.

The same quality is likewise exhibited in other forms of cooperation that are prominent in scientific work. The most important of these arises from the circumstance that a large number of scientific researches are sufficiently complex so that no single individual can carry them through to completion. He must look for the aid of other investigators, in other institutions and often in other countries, if he is to have any hope that his contribution will not be lost. Chapter 18 has vividly illustrated the fact that cooperation of several generations of scientists is frequently needed to complete a promising line of attack and to systematize the results in a few general laws. Realizing this, the scientist adopts the frame of mind appropriate to his modest role in such a process—that is, a willingness to sink himself in a cause whose satisfying outcome may require collaboration on a world-wide scale and may fail to appear till long after he is dead. Dedication in this spirit is, of course, not limited to the enterprise of factual science, but the very nature of many scientific pursuits enforces it on those who participate.

Because of the fact that objective verifiability is an essential criterion of valid scientific conclusions, in an important sense this last quality of sociability underlies and finds expression in each of the two attitudes earlier discussed. Devotion to truth is devotion to truth that can approve itself as such to other thinkers, not merely to oneself or the limited group that happens to share one's own prejudices. The habit of tentativeness proscribes dogmatism because the experience of others may include, now or in the future, facts which require correction of one's own present beliefs. In this sense, then, respect for the honest judgments and reflective capacity of others is the basic attitude encouraged by the nature and methodology of science; wholehearted intellectual cooperativeness is the culminating mark of a mature scientific mentality. This circumstance will provide us a helpful clue in

threading our way through the tangled problems to be faced in Part IV.

Summary of Part III

As in the case of Parts I and II, this Part will conclude with a summary of its main themes. After outlining the distinctive task of factual science, and discussing the postulates on which it rests, we explained and illustrated in Chapters 16 and 17 the principles of verification employed in this field, and clarified the notions of causal and functional law which are exhibited in their employment. Chapter 18 portrayed on a larger scale the way in which factual science historically develops under the guidance of these principles; it was followed by a description of the criteria of evidence applied in scientific procedure, and reference was made to the special problems arising in the endeavor to unify knowledge under comprehensive theories. The nature and scientific significance of probability were next treated, succeeded in Chapters 21 and 22 by a description of the techniques of statistics and of correlation which, in their predictive use, concretely employ the theory of probability. Chapter 23 observed that these techniques are especially characteristic of the social sciences and listed the unusual difficulties which the latter face; it emphasized the seemingly inevitable presence of variable evaluations in social inquiry, notably in history. This completed the foundation necessary to discuss the problems of correct definition and division.

EXERCISES

1. State the genus and the differentia in the case of each of the following definitions:
   a. Man is the animal with the power of articulate speech and the capacity of abstract reasoning.
   b. Reasoning is thinking which systematically attempts to solve a problem.
   c. Research is careful, critical inquiry in seeking facts or principles.
   d. Temperature is a condition with respect to heat or cold.
   e. Virgo is a zodiacal constellation, containing the bright star Spica, situated on the celestial equator, due south of the handle of the Dipper.

f. Gas is an aeriform fluid, having neither independent shape nor volume, but tending to expand indefinitely.

2. Criticize the following definitions in the light of the rules of good definition. Also indicate which of them are propagandistic definitions.
   a. A net is a reticulated texture with small interstices.
   b. Oats are a grain which in England is given to horses, but in Scotland supports the people.
   c. A good man is a man who can be counted upon to do the right thing.
   d. Man is a two-legged animal without feathers.
   e. The king is the voice of God on earth.
   f. The king is the self-appointed oppressor of his country.
   g. God is the Divine Being.
   h. Fun is frolicsome amusement or merriment.
3. Criticize the following divisions:
   a. Books into quartos, octavos, duodecimos, and paper-covered.
   b. Men into good and bad.
   c. Animals into male and female.
   d. Students into industrious, lazy, brilliant, and dull.
   e. Philosophies into true and false.
   f. Murders into premeditated, atrocious, and justifiable.
   g. Europeans into Germans, English, Russians, Latins, Slavs, and Bolshevists.
   h. Political leaders into statesmen and mere politicians.

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PART IV

REASONING AS EVALUATION

CHAPTER 25

THE NATURE OF EVALUATION

Evaluation as a primary theme

The two preceding chapters have acquainted us with some very challenging facts. Not only does thinking unavoidably involve an element of evaluation; this we had realized earlier. But the same evaluation may not always be made by different thinkers dealing with the same subject matter, and they may not be clearly conscious of what these differences involve; where this is the case, special and serious difficulty haunts the effort to reach satisfactory results. We shall now face this factor of variable evaluations in human reasoning as a primary and central theme. And in this role it cannot be evaded, for all of us often make assertions whose obvious intent is neither to draw an implication from some premise nor to describe or explain any factual occurrence, but simply to express an evaluation. "Democracy is better than fascism"; "I like Concord grapes more than Tokays"; "You ought to report that windowpeeping to the police"; "The Golden Rule is the right principle by which to guide one's conduct"—these are instances of such assertions, any of which one might utter or hear in the course of an hour's conversation. The key phrases in these statements are phrases that would never appear in a scientific account of any observable fact, or any formal relation—"better than," "like . . . more than," "ought to," "right"—and of course there are others. Phrases of this sort denote no perceptible properties or verifiable conjunctions of the events or objects mentioned, but appraisals of them. Their use indicates that we find values in our world, which have their own distinctive kind of reality, different from the kinds thus far treated.

What is the logical status of evaluative assertions?

What is the status of these phrases, and of the statements in which they occur, from the viewpoint of one who is eager to direct his thinking aright? A possible answer of course is: "The question is inappropriate; there is neither right nor wrong reasoning about these matters. No criterion exists by which one can tell which statements concerning them ought to be accepted and which to be rejected." Now there is much to support this answer. They cannot be formally deduced, except from premises which also express an evaluation. And on consideration it seems clear that the criteria of empiricism and of objectivity, which provide the primary court of appeal in reasoning about factual matters, cannot be applied to these statements in the manner there appropriate. The criterion of empiricism is not thus applicable, because the affairs denoted by "better than," "ought to," "right," etc., are not capable of being observed. To be sure, if introspection is admitted as a process of empirical observation, "like" is perhaps in a different case—it might seem that by turning our attention inward any of us can tell by direct inspection what his likes and dislikes are—but the reader will remember that there are serious obstacles in the way of reaching coercive results by introspection. The value pointed to by these phrases is in each case something that seems to escape the instruments of observation, natural as well as artificial, on which we everywhere rely in our factual investigations. As for the criterion of objectivity, it was pointed out in Chapter 23 that even when valuing enters merely as an extraneous factor in attempts to explain physical or social facts, it increases the difficulty of making the explanations objective. The natural conclusion from these considerations is that when assertions express nothing but an evaluation, the difficulty would be rendered even more serious. And this conclusion appears to be fully borne out. Such illustrative appraisals as were mentioned in the preceding paragraph are matters of hot dispute—obviously, not everyone agrees that democracy is a better social order than fascism; all persons do not reach the same convictions as to what one's duty as a member of his local community

involves, nor as to what are the proper ethical principles by which to live. And these differences can hardly be ascribed merely to incompetence on the part of some who reason about these matters as contrasted with greater intellectual power on the part of others—at least, any claim of this sort would be as vigorously disputed as the evaluations themselves. Such considerations strongly intimate that perhaps one is pursuing a will-o'-the-wisp in supposing that anything capable of playing the role of criterion can be found here at all.

Some of them appear to be more than expressions of taste

But if one is tempted to adopt this conclusion, he soon finds himself in situations which distinctly point toward a contrary position. Certain kinds of evaluation, as we shall soon see, are demonstrably correct or incorrect, and these kinds must not be forgotten amid the difficulties met elsewhere. Moreover, even in quite different situations, when one's evaluations conflict with those of others he does not always end the discussion at once with the agreement to disagree. Where the matter is one of mere liking, to be sure, this is normally the outcome; as soon as it becomes evident that that is the case we usually regard as appropriate and satisfactory the rule, *de gustibus non disputandum.* There are many cases, however, in which one ordinarily takes it for granted that further discussion, in quest of some standard that might adjudicate the dispute, is in order. An enthusiastic supporter of democracy, meeting in some neutral country a convinced champion of fascism, would be apt to ask: "Why do you believe in such a social order as that?" It would be his hope that argument, guided by this question, might lead to a basic principle acceptable to both, which would thereupon fill the role, in certain respects at least, of a criterion in deciding between the conflicting evaluations. Similarly with convictions expressed in sentences employing the words "ought" or "right." In all these situations people often proceed on the assumption that even though no commonly accepted standard is at once evident, the search for it is not futile. In many cases the success that they achieve in following this assumption is sufficient to render them unwilling to abandon it as an appropriate

guide. They find their own evaluations clarified, more dependably grounded, and broadened to include considerations that they had previously neglected; sometimes, as a result of participating in such discussion, one or both of the disputants is led to a radical change of appraisal which appears in his eyes to be a sadly needed correction of previous error—a less wise conviction has been replaced by what is clearly a wiser one. The expectation that other people's experience and reflection can give us genuine aid in progressing from blinder preferences to evaluations grounded in a more inclusive range of awareness is thus confirmed in practice; disagreement does not seem to be necessarily final.

In the light of these considerations the sensible conclusion to draw would seem to be that even though the criteria of evidence applicable elsewhere are inadequate here, something may yet be discovered that plays an analogous role—the role of a standard transcending our variable initial evaluations and guiding them toward readjustment and self-correction. We are not left completely helpless when we seek closer approximation to the kind of goal that reason by its very nature demands; there is a distinction in this field, too, between more rational and less rational belief, which challenges us as thinkers to respect it here as elsewhere and to use to the full whatever guidance is available. In short, while the principles supplying that guidance are evidently different in detail from the implicative patterns that fill this role in the formal sciences, and likewise from the rules of evidence that function analogously in the factual sciences, there seems reason to believe that they are not entirely absent. In the following chapter we shall try to identify and describe them as accurately as we can; the present chapter will be confined to preliminary clarification.

Hence, the problems and method of Part IV

In any case, the question must obviously not be foreclosed in advance. It is our task to analyze as thoroughly as we can what is essentially involved in any evaluation; then to discover exactly why demonstrable conclusions here are hard to attain, just what kind of criterion seems to be available, and

how it can be hopefully applied in dealing with the different sorts of values which the analysis will require us to recognize.

And what shall be our method in pursuing this analysis? Heretofore the themes discussed have been treated with no special diffidence about the conclusions reached. Dogmatism has, I trust, been successfully shunned, but it has been assumed that serious disagreement on matters of consequence is not likely to arise. Here such a method is hardly appropriate. For one may not justifiably take it for granted that any conclusions about these matters are beyond dispute. The only proper procedure would seem to be one of utter candor, modesty, and realism. Emphasis should be laid on the challenging problems which remain as well as on the procedures which will be approved. The obstacles impeding the quest for anything analogous to an objective standard should be faced with complete honesty. And if a constructive theory to open a path through the tangle is suggested—as would seem to be necessary if the analysis is to introduce any kind of order into the chaos and lead to any profitable outcome—it must be held subject to especially searching criticism.

The nature of value, and its difference from fact

In Part III we were discussing the realm of facts, and the approved procedures for their description and explanation. Appraising is evidently a different process; it is choosing among values rather than portraying facts. Just what is a value, and how does it differ from a fact?

Well, let us take a concrete case involving both. Suppose we wish to compare Chicago and New York in various ways. Which is west of the other? Which is higher in altitude? Which is more northerly in latitude? Which is colder in the winter? Which has the larger population? To these and a host of similar questions definite and objective answers can be found; whatever our partisan feelings about the two cities, the method by which answers to these problems are to be reached, and the correctness of the results, are matters on which we easily agree. Such questions are, in short, questions of fact. But suppose we raise the further query: Which is the better city to live in? You or I might be able to answer at

once which city we individually prefer, but we should recognize that the factors rated highly in our comparison would not be rated so highly by everybody; in a given discussion about this question every fact introduced might be agreed upon without dispute, and yet diverse answers be given to the question itself. In short, this is a question of value, not of fact. And in the light of this illustration the difference between the two would appear to be essentially this: a fact, merely as such, is anything that occurs or is the case irrespective of anyone's attitude about it—irrespective of whether it is liked or disliked, is desired or shunned, arouses interest or aversion, is favorably or unfavorably judged—while a value is always correlated with some such attitude. It is an object of attraction or of repulsion, a thing welcomed or met with hostility, an act approved or disapproved. This is not to say, of course, that it is something separable from facts. The presence of values rather attests the circumstance that we are not passively indifferent to our world, but actively preferential in its regard. We respond selectively to the facts with which we are confronted, some being objects of positive, others of negative concern.

Consider another instance. A politician, elected by the aid of funds from a certain corporation, and now in position to sway the vote of a city council, is approached by an agent of that corporation with the request that he secure the award of a profitable franchise. Now his relation to the corporation, his membership on the council, the offer of the agent, and his subsequent action, are all matters of fact; each is capable of being described and explained in the same way that would be appropriate in dealing with any observable fact. But if he is a man of conscientious scruple he will surely raise a question in this situation which is not a question of fact. "Ought I accede to this demand?" And again, not all persons who would agree about the facts involved would agree on the proper answer to this question. Many would hold that his obligation is solely to the interest of the city as a whole, and therefore that he ought to say no; some would maintain that he has a special

obligation to those who aided his election, provided at least that the city's interest does not seriously suffer. The same facts, but a different value is found in them. In this case, moreover, the value involved does not seem to be merely an object of preference. It bespeaks, rather, some rule of action by which men determine what is proper in their relations with each other—some law of conscience, obedience to which renders their conduct "right" and disobedience to which makes it "wrong." In its presence men appear to be members of a moral order which is different from the factual order of cause and effect; but membership in that order is incapable of verification by empirical methods—it is an attribution of a certain worth to men, not a fact about them, and determination in any given situation of what that membership requires is likewise evaluation, not the description of a fact.

A fact, then, is one thing; the value of that fact, whether approached in the former or in the latter of these two illustrated ways, is another thing, whose analysis compels us to raise different questions, employ different basic concepts, and pursue distinctive methods.

What different kinds of values, now, do we need to distinguish?

Well, values may be divided into subclasses in a number of different ways, depending on the purpose which guides the division. Most of these divisions cut across each other, and hence fit into no single logical pattern. Several of them have already been referred to, directly or by implication. In the following analysis only those modes of division will be mentioned which are fundamental to any adequate discussion of the topics we shall need to consider.

Classification of values: 1. As spontaneously prized or reflectively approved

In the first place, values may be divided according to whether they are *prized spontaneously*, apart from the exercise of any reflective judgment, or whether their worth depends on their being *reflectively approved*.[1] For example, I read in the morning paper an announcement of the murder

[1] Many contemporary writers, *e.g.*, Dewey, restrict the term "value" so that

of a prominent person whom I regard as politically dangerous. An impulsive feeling of satisfaction in this event seizes me; here is a case of spontaneous positive evaluation. I relish the thought that this individual is now removed from the opportunity to bring further harm to the community. But on reflection, my positive evaluation changes into a negative one. I see that the removal of hated figures by the method of assassination is apt to be far more dangerous to orderly political progress than their continued presence in the arena. Here is a case of evaluation reached by critical judgment. It depends on reflection, not mere unconsidered impulse. Of course, not all spontaneous evaluations are thus radically transformed by reflective consideration; the latter may approve them as they stand. But this illustration shows that there may be a serious divergence between the two sorts of evaluation as applied to a given event.

2. As merely variable or in some sense common

Second, values may be divided according to whether they are accepted as properly *varying from individual to individual,* or whether they are felt to be appropriately *assessible in terms of some common standard.* This mode of division has already been employed. In the former case, the conventional phrase by which to describe them is to call them, in part or in whole, matters of "taste."

Cases of spontaneous liking are always also cases of someone's taste, but a person's taste may have gained his reflective approval instead of being an affair of mere unreflective liking, even when no other person is expected to share it. When it is discovered that a threatened dispute about values is merely concerned with such individually relative preferences, the dispute is dropped, because all recognize that any one may properly have his own tastes and that no one else is under obligation to share them. In the latter case the differences are thought of as subject, or at least possibly subject, to some public criterion in the light of which it can be decided which values are objectively appropriate and which

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it applies only to the second of these two classes. Of course, spontaneous prizings may be affected by *past* reflection.

inappropriate, which are superior and which inferior to an impartial judgment. These phrases are admittedly irrelevant where nothing but tastes is involved.[2]

3. In accordance with the subject matter involved

In the third place, values may be classified according to the subject matter involved, and the human attitude naturally called forth in the presence of that subject matter. Where objects in nature or creations of art that appeal to disinterested appreciation embody the values concerned, one refers to them by such terms as "beauty," "grandeur," "charm," "loveliness," and their opposites. These are *esthetic* values, and by many the concept "beauty" is employed as a general concept to include all the positive values in this field; similarly, "ugliness" is often used to cover all the negative ones. Where human conduct or character is concerned, the values distinctively involved are *moral* or *ethical*, and the fundamental denoting terms are "goodness," "rightness," and their opposites. There are also many subordinate concepts employed here, which refer to the goodness appropriate in particular kinds of situation; "sincerity," "temperance," "prudence," "benevolence," are illustrations. A very important subdivision of the ethical field is that between individual and social ethics. Courage is a virtue belonging to individual ethics, because it can be defined without reference to the presence of other persons; justice belongs obviously to social ethics. Where propositions that might be asserted or denied are concerned, and the beliefs expressed in them, "truth" and "validity" with their opposites are the relevant values. They are generally referred to as *logical* values to distinguish them from esthetic and ethical ones. Since Parts II and III have been occupied in detail with problems concerning their criteria, no special attention will be given these values here, except to consider the question of their relation to the other values with which we shall primarily deal.

[2] We do discriminate between "cultivated" and "uncultivated" tastes. But this constitutes no exception to the principle above stated. A cultivated taste is one into which reflection enters and is hence judged to be superior to an uncultivated one. But no one is under obligation to share it in preference to other tastes that might be cultivated.

It is very important to realize, however (as we are now in a position to do), that when truth was being discussed in the above analysis of the methodology of science, and validity as defined in terms of truth, we were dealing with values. It was only because of the distinctive features of these values in contrast with those to which we shall now give major attention, that it was unnecessary to mention this circumstance at that time. Truth is a value, not a fact, but it consists (as other values do not) in properly justified beliefs, assertions, or propositions about facts. This is what distinguishes it from other values, and from guesses, opinions, and hypotheses, which are likewise statements about fact but have a different value than that of truth. Truth, as consisting of verified assertions, is something prized; and it is the distinctive value which science is always trying to realize as fully as it may. From the logical viewpoint, its special status in comparison with other values arises from the circumstance that the criteria that are applied in the search for it—empiricism, objectivity, and simplicity—have, in the civilized world, now gained tolerable clarity and, among intellectual leaders at least, general acceptance. Hence the methodology of its progressive attainment can be formulated in terms of such attested canons as have been explained in Part III.

4. As homogeneous or heterogeneous

In the fourth place, values may be classified according to whether they fall entirely within one of the three realms just distinguished, or whether they overlap two or more of those fields of value. In the former case the values will be called *homogeneous;* in the latter case, *heterogeneous.*[3] From this definition it is obvious that the values referred to in the preceding paragraphs are all homogeneous values. Other homogeneous values falling within the sphere of ethics are "happiness," "virtue," "honesty." The first two are values that are quite general in scope, while the third refers to the ethical virtue that is appropriate under certain particular circumstances. Instances of homogeneous values from the field of esthetics are "harmony," "balance," "charm," "elegance."

[3] These words are long and awkward, but I have discovered no other satisfactory way of describing the distinction in question.

Illustrations of heterogeneous values are "order," "freedom of contract," "vision of God." Order is a heterogeneous value because, on the one hand, it implies an esthetically appealing rhythm of some sort, while on the other it implies some serial regularity in terms of which any true explanation of relevant facts must be couched or any transitive relation understood. Freedom of contract is such a value, because it combines ethical meaning with truth reference. In democratic countries it is prized and given legal protection as a guide to business conduct; but judgments employing it also claim truth, because that protection assumes concrete form in the constitutional provisions and judicial precedents which must be respected when its implications are in dispute. The vision of God is an experience that has intrinsic reference to all three spheres of value. Religious persons think of God as embodying supreme moral goodness and as revealed in the beauty of nature; but He is also for them a source of truth, since He is believed to be the ultimate creator of the world of facts. We shall find later that the discovery of standards when one is concerned with heterogeneous values involves, on account of their greater complexity, a certain further difficulty besides those met in dealing with homogeneous values.

5. As means or as ends

With regard, however, to certain major problems which must be investigated—especially problems concerning the possibility of finding rational criteria for judging values—a fifth principle of division must be mentioned. This divides values according to whether they function in a piece of reasoning as *means* or as *ends*. A means is a value whose worth depends not upon its own nature but upon its causal relation to something else. I sit for an hour in a dentist's chair, for example; this experience is not a positive value in itself but it acquires such value because of its relation to the end of having good teeth, which I am seeking to realize through its instrumentality. An end is a value whose worth is intrinsic; it is good in itself, capable of arousing desire or appreciation for its own sake. Health, just referred to, is in many situations such a value. And it is important to note that what is end in one situation may function as means in another, and vice

versa. I might engage in daily exercise, for instance, because I want to be healthy and have no further end in mind; in this case health is clearly playing the role of end. I might, on the other hand, entertain an ambition to win outstanding success in my profession, and consciously think of more vigorous health as necessary if I am to have any chance of gaining this end; here health is as clearly playing the role of means. And nothing prevents certain values from sometimes playing both roles at once; most likely in the case just mentioned health was thought of as a good in itself as well as being the means to a more ultimate value. Means that are intrinsically painful, however, rarely if ever function as ends, while the highest and most inclusive values such as happiness, virtue, or love, rarely if ever function as means.

Before pursuing the bearing of this division of values on the various problems which it affects, we may pause to note that it reminds us, more vividly than the other modes of division, of the basic circumstances which are reflected in the presence of values in our world. Man is not indifferent to what happens around him; he responds with varying interest to the objects and events which he confronts. Hence, as his experience unfolds, ends and means take form in his thinking. The objects which positively intrigue him suggest ends in which he would find still fuller satisfaction; and he discovers that the only way in which that fuller satisfaction can be dependably realized is through selecting means that promise to bring those ends into being. Moreover, since oftentimes the alternative to pursuing ends of our own choice is not just peaceful waiting for something good to turn up, but an experience which we would definitely avoid, such as hunger or suffering from cold, we do not postpone their pursuit till we have demonstrable proof that the quest will be successful. The mere chance of success is preferable to the pains into which we would sometimes fall through failure to embark on the action which best promises to attain the goal that we desire.

Specific and general ends

For the purposes which this distinction will serve in our subsequent investigations, no further analysis of means will be

necessary. But it will be necessary to classify ends, and to note two of the main relations which the classes listed may have to each other. First, there are specific ends, capable of definite description in terms understandable by any normal person. For example, I am digging a trench beside my driveway; my end is to provide drainage for rain water that might otherwise flow into the garage. Here is a specific end of this sort. Then there are general ends, not capable of such definite and precise description. Health, security, freedom, comfort, will serve as illustrations of these. They are general because they are capable of being pursued in a considerable variety of situations instead of in one sort of situation only, as is the case with specific ends. And among general ends it is important to draw a further distinction between such ends as have just been mentioned and the most inclusive ends of all, such as goodness, happiness, satisfaction. The latter are distinctive in that, in some sense, they are being pursued in every situation to which the notions of purpose and value are relevant. This is not true of the less general ends. Health, for instance, is an end of broad scope, but it is not always sought; I might deliberately prejudice my health for the sake of saving the life of another person.

Now one of the important relations between these classes of ends appears when we note what sort of appeal would naturally be made if one were asked to justify the pursuit of any end which he is trying to realize. Why do I want to prevent rain water from flowing into my garage? Well, I might reply by mentioning another equally specific end; I do not want the garage to be flooded. But if the questioner persists, sooner or later I shall appeal to some more general end, such as one of those just listed among general ends of limited scope. It is for the sake of the safety of my house that I do not want the garage flooded. Here, safety is invoked to provide the justification asked for. And suppose that the questioner still continues. Why do I want safety? The only answer that is relevant now will be expressed in terms of one of the most general ends that are pursued in every situation. I want my

house to be safe because safety is essential to my happiness and that of my family. And no further question of the same kind seems to be appropriate when this point has been reached. Thus specific ends are justified in terms of general ends of limited scope, while the latter are justified in terms of general ends of unlimited scope.

Another important relation between these classes of ends appears when we ask how ends are syntactically defined. Specific ends are normally defined in terms of the concrete situation envisaged, *e.g.*, keeping my garage free from rain water. But general ends of limited scope are defined by using as genus a more inclusive end. "Security," for example, might be defined as "the satisfying state in which one is free from fear and anxiety." The problem, how to define such all-inclusive ends as "satisfaction," becomes the focus of serious logical difficulties, to which we shall in due course give careful attention.

How evaluation takes place

This analysis of values as means and as ends will now be used to help us in examining the psychology of evaluation—that is, in discovering just what happens when a course of evaluation is engaged in by anybody.

For such a process must be dissected rather differently according to whether it is a case of selecting the best means to the realization of some end taken for granted, or whether it is a case of determining the end which one wants to realize. Let us recall here the five-step sequence in any piece of reasoning, discussed in Chapter 2. Now when the reasoning is a case of evaluation, in either of these two forms, no special comment is needed so far as concerns steps one or two. But steps three, four, and five show distinctive features which have not previously been noticed, likewise some features which vary according to which of the two sorts of evaluation is taking place.

When it is selection between alternative means

Suppose that one's problem is to select the best means to some unquestioned end that can be described in quite specific terms. I want, let us say, to go to England and, being in penurious circumstances, I decide that I must go the cheapest

way. Getting to England as inexpensively as possible hence constitutes my end, and the problem is: What means best promises to accomplish this end? Step three will then consist of appropriate suggestions of various means of transportation to England, such as airplane, passenger boat, and freighter; step four will deduce the implications which have a bearing on the probable cost of each route. In step five I will determine these costs in detail; on comparing the figures I can quickly tell which is the least expensive way of getting to England. The result thus reached will constitute the answer to my inquiry, and in this case it will be an objectively demonstrable answer; I can prove to anybody who understands the problem that it is the correct solution. Its objectivity is due to the fact that the end here proposed is quite definite, and is capable of being readily appreciated by any normal person. Understanding fully what I want, he can see just as clearly as I can which among various alternative means is the appropriate one to select. When the end is a more general one, even if it is unquestioned, the conclusion will not be so readily demonstrable. My end might be, for example, a broad social goal—say, the fuller realization of democracy—and comparison of various political and economic proposals might lead me to the conviction that a certain combination of policies promises to be, under present conditions, the best means to further it. But since democracy is a more general and less precise end than getting to England as cheaply as possible, and since it is harder to predict with assurance the effects of any social policy than the effect of boarding a steamer for England, my proposed solution of this problem cannot be rendered as objectively demonstrable as the solution of the other.

When it is determination of ends

Suppose, however, that the problem is not to select means for attaining some end already clearly in view, but is rather to determine one's end—to find out what one really wants to accomplish. I receive, let us say, an attractive offer to take a presumably permanent position of a different kind than the one I am at present filling. Step three in this case will consist

of two suggestions—to take the offer, and to stay where I am. How, now, do I decide between these alternatives? Here, no fixed and definite purpose controls my thinking; accordingly no comparison of given facts or proposed policies in the light of such a purpose can guide me to a conclusion. What I am trying to do is to envision the end at which I shall aim, since at present it has not yet taken form. Now, step four provides the foundation out of which it can emerge. That step, in this case, consists in developing a pair of pictures of my possible future life, each picture composed of the entire set of imagined consequences spreading out before me as likely to be realized by choosing this or that alternative. I say to myself: If I accept the offer—then larger salary, with all that this would mean for my family, together with a more healthful climate and environment for my children (both of which matters I elaborate further in imagination); but on the other hand the work will not be nearly so congenial, nor will I have the same boon of a lengthy period during the year devoted entirely to study and writing along the lines of my greatest interest. If I remain—of course the converse of these deductions, together with some plausible hopes for larger financial return from my present work in the future, etc. These pictures I elaborate in imagination, striving to expand them as fully as possible and to control my forecast of consequences rigorously by what I have observed of similar causes and effects in the past, until one of the two pictures definitely appeals to my entire nature more than the other. In committing myself, then, to one anticipated outcome rather than the other, I have decided that it is the better alternative of the two—and it is only after this careful elaboration and comparison have been performed that the controlling end, the purpose to live a certain kind of life rather than another, emerges into clarity.

What is the fundamental difference between these two courses of evaluation? It is that, in the former, a specific purpose governs the entire piece of thinking; the reasoning proceeds by comparing different means in the light of that purpose and terminates in selecting the one that most nearly fits.

In the latter case, a specific purpose is born only as a result of the evaluation itself; its task is just to bring that purpose into being. In the first case, one knows what he wants all through the process of reasoning—a definite aim is taken for granted; while in the second case he is trying to determine by the reasoning what he wants, and no aim is taken for granted except the very general one of making the best decision. I know that I want to go to England by the cheapest route; what I want to do about the offer of this new position is exactly what my reasoning tries to determine.

There are interesting kinds of evaluation which lie between these two extremes. Of one kind we become aware when we realize that there is no guarantee that an end to which one seems quite committed when he commences to select appropriate means will remain unchanged throughout the evaluation. In the course of his reasoning considerations may turn up which will lead him to revise it. To take a now familiar example, I might raise the question of the best means of transportation for meeting a business engagement, with the purpose in mind at the outset of using the route carrying me to my appointment most quickly. This would, let us say, be the suburban train. But before I decide, it occurs to me that it is a fine balmy day in spring, just suited to a ride on the top of the bus; moreover, that in this case no one will be inconvenienced if I arrive ten minutes late. Accordingly, I choose the bus. Now here the controlling end has changed in the course of the thinking. At the start it was the end of meeting the appointment as quickly as possible. Later it was the end of meeting the appointment by as enjoyable a ride as possible, so long as not much time was wasted and no one was inconvenienced. This forms an intermediate stage between the case of finding out how to get to England most cheaply, and the case of deciding about the new position; I thought I knew just what I wanted when I began the evaluation, but found reason to change my mind. Hence I revised my end before finally choosing between the available means.

Evaluation by the application of some rule

In the illustrations thus far employed we have been think-

ing of situations in which the appropriate value-concepts are those of "good" and "bad," or "better" and "worse," as applied to alternative courses of action. The same contrast appears, however, in cases where the natural concepts to use are those of "right" and "wrong." Here, the controlling end is replaced by some rule of conscience or of duty. Ought I to tell the full truth to a friend of mine when the truth is bound to be exceedingly disturbing to him, or should I compromise on a half-truth which he might be able to take without anxiety? Either of two possibilities may be exhibited in this situation, too. I might have some moral principle confidently in mind right through the process of evaluation, the only question being just how to apply it to this particular perplexity. For instance, I might take it for granted that the Golden Rule should be the guide in any case of this sort; my problem then is simply to decide what I would wish my friend to do to me were I in his plight. On the other hand, I might find myself in a situation analogous to that of the bus trip. I might begin with some such guiding principle as the Golden Rule in mind, but discover reasons for doubting its adequacy when I came to realize what it would imply in this instance. In that case, then, I might have to revise the rule before being able to decide what is the right thing to do. For example, it might seem clear that following the Golden Rule would mean telling him the full truth, but it might also seem highly likely that he would not take that truth in the way I would take it if our positions were reversed. Then my problem would include the task of clarifying and reformulating the rule of duty in such a way that it would give adequate guidance in matters of this kind. It would be theoretically possible that one might have no definite principle in mind at all at the beginning, but in perplexities to which the value-concepts "right" and "wrong" appropriately pertain this extreme probably is not actually exemplified.

Now a critical comparison of the differences between evaluation as selection of means and evaluation as determination of ends throws much needed light on the most serious

logical problems about evaluation, and especially on the problem of securing rationality in the conclusions to which the process leads.

When the problem is that of selecting means to some specific end, describable in precise terms and hence readily sharable by others, the attainment of results which can properly claim objectivity poses no fundamental difficulty. Anyone else, appreciating my end, can tell as well as I can whether, among the alternative means that are available, the one that I choose is the most appropriate one or not. The field of such problems is, in brief, the field of applied science; conclusions there established can be demonstrated to anyone who possesses the relevant knowledge and understands what the inquiry is about. When the problem is that of determining an end, objectivity, as we are familiar with it in science, seems to be completely lost. Whichever decision I reach with regard to such a problem as the acceptance or refusal of a new position, how could I hope to prove to anybody else that it is the correct decision to make? With the same facts before him, he might easily reach a contrary conclusion as to which choice is best.

Evaluation as determination of ends seems conspicuously to lack objectivity, because

Why is this so? What are the main reasons for this apparently inevitable lack of objective demonstrability in the results reached by any determination of ends?

Let us answer this question first in general terms, relating the answer to points that emerged in our discussion of the criterion of objectivity in Chapter 19 and our preliminary consideration of problems of evaluation in Chapter 23. In the former context it was observed that factors which are relative to the individual thinker, such as wishes, emotions, and their projections, prevent the attainment of objective results only if their presence and subjective status are not recognized and clearly understood. If demons, for example, are thought of as existing in the external world after the fashion of stones and oceans, objective results about them cannot be established, because not all people find them present in any such form, and even those who do cannot, apparently, agree on their

spatial location or on the properties to be assigned them. Thus it is impossible to discover dependably verifiable laws about their existence and behavior, correlating their supposed performances with other objects and events, when they are studied on this assumption. But if these entities are regarded as relative to individuals or groups of individuals, being projections of their fears and frustrations, verifiable knowledge about them is capable of being reached. For this approach takes full account of the obvious facts that the nature attributed to these beings and the sphere of their operations varies from individual to individual and is dependent on each individual's emotional condition. By examining them in this orientation, one is thus able to discover many truths about them and their regular relationships which can be agreed upon by all who are willing to face these facts and are competent in the use of scientific methods.

Now the same principle applies to the study of purposes and values. The presence of these factors leads inevitably to confusion and contradiction only when there is disagreement about the real nature of such entities as they function in any problem under consideration. So far as they are understood with sufficient clarity, and in the same manner by different thinkers, their presence need not bring us into any difficulty. This is clear from the evaluations implicitly assumed in the earlier parts of the present book. In the case of the purposes and values which are reflected in the inquiries of formal science, and factual science in its exact branches, such agreement has been already so completely achieved that ordinarily it is unnecessary even to mention their presence, and that is why, until the theme of the preceding chapter was broached, our discussion of the principles and methods of science did not refer to them.[4] The basic purpose of formal reasoning is to realize the value of consistency; every competent worker in the field of formal science is aware of this and understands in the same way what consistency requires. The basic purpose

[4] Except for noting the influence of broad human interests, varying from epoch to epoch, on the cosmic theories that are adopted.

of factual science is establishment of the most exact explanatory laws and correlations that the subject matter makes possible; in the natural sciences this ideal of truth, and the procedures necessary for attaining it, are universally understood in the same terms. When we come to the social sciences this is no longer quite the case, and in the context where they were discussed we sought the reasons for this circumstance. The main reason seemed to be that, here, purposes make a difference in each individual's reasoning, of which he is not clearly conscious; hence he cannot disentangle them from his main aim of reaching objective truth and cannot accurately allow for their presence. Especially in the field of social theories, his wishes and hopes in favor of one kind of social ideal rather than another affect his pursuit of truth so intimately and pervasively that he cannot see these forces from the outside; they are part of what he thinks *with* instead of being something that he can easily think *of*. We are now in a position to describe his plight more adequately. In terms of the distinction which has now been introduced, he is not merely selecting means to a clearly understood end; he is influenced by ends not yet clarified, even in his own mind, and hence not yet capable of being seen by different thinkers in the same way.

In the light of this analysis we can formulate the general answer to our question. In situations where we are selecting means to an end capable of specific and definite description, we know how to allow properly for the presence of the values involved; since the end can be described objectively and the means derive their value from the end, all the values that are operative can be taken into account by different thinkers in common terms. But in situations where ends are being clarified and determined anew, it is impossible to tell how to allow for them in this fashion; they have not yet taken form but are only in process of doing so. The distinctive difficulty here becomes, then, readily understandable.

This general answer, however, is not sufficient for our present needs; we shall want a more detailed analysis of this lack of objectivity in dealing with ends. The further answer sought

is provided by three major considerations, when seen in their significant interrelation. Let us examine them in their proper order, and in each case with sufficient elaboration to bring out the corollaries important for our subsequent discussion.

1. People evaluate the same situation differently

The first consideration may be stated quite briefly. Different people are not moved in the same way by any given pattern of life. Confronted with the demand for choice between two alternative ways of living, some persons will make one choice and others another. A plan of life, for example, which involves many risks will positively appeal to a person who loves adventure, but will arouse anxiety and fear in one who desperately needs a maximum of security. Such individual differences between people just seem to be an ultimate fact. This point has been already stressed and need not be elaborated. But let us now note one of its bearings. These differences mean that probably no two people ever envision exactly the same pattern of future consequences as implied by a given suggestion; when they engage in a set of deductions, it is very easy to select under the guidance of their hopes and fears, and for each to emphasize the particular consequences that loom large to his individual interest or emotion. No matter how much we try to guide predictions in the sober light of similar past experience, our wishes or dreads will creep in and play some part in the expectations formed. One can even divide people, in recognition of this circumstance, into optimists and pessimists—that is, into those whose expectations are mainly determined by hope, and those in whom they are mainly determined by fear. Now this factor may lead to no troublesome outcome when one is selecting means to a definite and clearly understood end. Implications which reflect such personal feelings are simply neglected, attention being paid only to those which would seem relevant to anyone else who is pursuing the same end. But when one is determining an end, the situation is different in this respect, and because of a circumstance which must be carefully weighed.

2. The implications of suggestions are lacking in dependability

This circumstance (which constitutes the second of these considerations) is the low dependability of the deductions

drawn from the alternative suggestions that come to mind. Their vast and indefinite scope makes it impossible for them to be very dependable. When one is selecting between various means to a given end, he develops the implications of such suggestions as occur to him only to the point where he can compare them clearly in the light of that end. In finding out how to reach England most cheaply, for example, he will only pursue those deductions from the suggested modes of travel which bear upon their comparative cost. The process is limited, simple, and usually brief. But in problems where the desired end is to emerge from the evaluation itself, one has to continue elaborating the implications of the suggestions into as comprehensive a picture as he can, in order to reach an adequate basis for choosing between them. What he tries to envision, in short, is his whole future life as lived in one way rather than in another way, so that he may decide which life on the whole appeals to him most. Doubtless it would be an exaggeration to intimate that there are many occasions on which we find it impossible to decide until the implications of a pair of suggestions have been carried out on this grand and inclusive scale. Yet the more important the issue between them, the more do we attempt exactly this comprehensive forecast; to select intelligently one's vocation surely demands as broad a prediction of the future as it is possible to make. And in less important situations where we try to decide what we really want to do, it becomes necessary to develop far more extensive pictures of what is bound up with our suggestions than when we are quite sure just what we want. Often it is only when the forecast has run into a high degree of complexity that one picture is able clearly to overbalance the other so that we can say with confidence which we prefer.

But there is a serious difficulty in such long-run forecasts. It is evident that the more complex these imaginative portrayals become, and the farther in the future the events we are trying to anticipate, the less dependable do the expectations become as measured by actual results. The main reason for this is that a larger number of unexpected events will have

an opportunity to affect the anticipated outcome. Things do happen which we did not and could not prophesy. Use our present knowledge in the most careful way we may, many unexpected occurrences will enter our experience long before the sequence of anticipated consequences which supplied the basis of our present decision is entirely unrolled, and some of these occurrences, had we known that they were coming, would have affected our choice between the alternatives. They would have affected it because each such occurrence, when it comes, is something that we welcome or regret; in the former case it will add unexpected confirmation to our decision, while in the latter case it may force us to reconsider whether the choice was wise or not. To return in the light of this latter possibility to my choice of a career—I might decide, on the basis of the best information and clearest deductions at present possible, to take the attractive offer given me, and yet find, after settling in my new position, that the work became much more uncongenial than I had feared and that the larger salary was not quite as prompt and assured as my present income. These unanticipated circumstances might make me deeply rue the change.

Now experience compels us to recognize this lack of dependability in distant forecasts. As a result it becomes easy, in dealing with problems of this kind, for anticipations to be seriously affected by our hopes and fears, which vary greatly from individual to individual. When it is practically certain just what is coming next, any two honest thinkers will have the same expectations so far as it is concerned; but when much uncertainty is justified, the anticipations of all thinkers will be strongly influenced by their personal feelings. It is impossible to control them in any commonly sharable way.

3. End concepts pose serious problems in regard to their meaning

The third consideration is found in the conclusions to which we are forced, in view of these circumstances, about the meaning of the words by which we refer to ends when they are determined in this fashion.

How shall I refer to the end which is being determined by my choice between two vocations? Well, it is certainly no

specific end, but rather a way of life in general, and hence no phrase by which specific ends are denoted would be appropriate.[5] I might, perhaps, describe it in terms of one of the most inclusive ends that are pursued in every situation—it is "happiness" for my family and for myself that I am seeking. Or I might describe it as one of the general ends of more limited scope—"security," let us say, for my children.

What, now, is the meaning of such a word as "happiness" or "security" when employed in this context? In discussing the significance of the words which occupied us in Parts I, II, and III, we noted three dimensions of meaning—syntactic, semantic, and pragmatic. It will be remembered that by the first of these is intended the meaning of a word in terms of its formal equivalences with other words; by the second its meaning in terms of the empirical phenomena which it may denote; and by the third its meaning as affected by the problem its user is trying to solve, or the purpose which thus controls its employment.

Let us examine, in the light of this threefold analysis, the meaning of words referring to ends when in process of determination. Clearly, they can have semantic meaning only in the sense that they call our attention to the present state of some factual process whose later outcome, we hope, will constitute a fuller realization of the end in question. This distinguishes them from fantasies which, so far as one can now tell, are quite incapable of any realization. If my end is security, for instance, this word reminds me of the more or less unstable social institutions now surrounding me and the particular anxieties confronting my family which provide the material for realizing that aim—the material which, transformed in a certain way, would more fully exemplify the end of security as I envision it. But in other respects an end-word has no semantic meaning. For the goal toward which it points is something which does not yet exist. It is something that we wish might exist, or believe ought to exist; and these words

[5] If it were merely the specific activities involved in one of the two vocations, I could decide the question at once; there would be no serious problem.

imply that it does not yet exist as a given fact. This circumstance, to be sure, is sometimes obscured. I desire the contentment that I see exhibited in the life of a friend; is not that contentment my end, and does it not now clearly exist? No, because what I desire is not the contentment that he already enjoys, but my sharing of it which does not yet exist; as my end, it is a future possibility, not a given fact. And that contentment might be an end for him, too; in that case, however, what is meant would be, not the contentment which he now enjoys, but its continuance in the future or a greater degree of it than he has yet enjoyed. Again, as an end it clearly reaches beyond present existence. This is a crucial consideration with regard to the semantic meaning of end-concepts.

It follows from this circumstance that no special logical problem arises in connection with the semantic meaning of end-words. The nature of the factual materials, as science discloses it, must, as a matter of course, be respected in any pursuit of an end. But since different ends may be pursued through use of the same facts, and the same ends through use of different facts, the semantic meaning is of no distinctive consequence when we are dealing with ends.

In particular, their syntactic and pragmatic meanings

The syntactic and pragmatic meanings of end-words, however, do involve serious problems. Let us consider their syntactic meaning. Like other words, any end-word can be syntactically defined; and, as we have seen, an end of limited generality such as security is naturally defined by employing as genus one of the terms by which we refer to ends of quite inclusive scope. The logical difficulty which lies ahead in this direction consists in the fact that different people have different views as to what these ends of inclusive scope consist in. When we try to define them, we find that no such agreement obtains here as we found among different scientists when dealing with the most general concepts that are relevant to their field. And when these different views are reflected upon, and carefully formulated, they become different theories of value, each championed by some philosophical school. What are we to do in the presence of these competing alternatives

as to what syntactic meaning should be given to our most general end-words?

Let us now consider their pragmatic meaning. This is the dimension of meaning most directly involved in our recent analysis. Whenever one is determining an end, his problem lies in the fact that he is confronted by two or more courses of action between which he is trying to choose, and his purpose is to decide which of them he will commit himself to as superior to the others. Thus, if in such a process he refers by such a word as "security" to the end which is being determined he must mean by that word, pragmatically, whatever line of action he decides to embark upon in the hope that thereby the end will be attained, and the specific consequences which he anticipates as likely to occur as a result of pursuing it. In brief, its pragmatic meaning consists precisely in the factors which emerge and become dominant when his evaluation passes through the third and fourth steps of the reasoning process as above illustrated. The same principle applies to any general end-concept; if democracy becomes, for me, an end, its pragmatic meaning must consist in the particular things that I do or propose to do in endeaving to promote it, together with the specific social consequences that I expect to come into being because of those activities.

How, then, can the pragmatic meaning of such words avoid being very ambiguous and vague? We shall list the reasons for this outcome that have been clarified in the preceding discussion. The meaning is ambiguous, for one thing, because when I use the word "security" I will not be anticipating exactly the same results that anyone else would anticipate when he uses it to denote the end he is pursuing. For another thing, it is ambiguous because, in each new situation in which I employ the word, I will have to determine its meaning over again in the light of the available means and the possibilities which they exhibit in that situation. These will not be exactly the same as in any previous situation, hence the concrete action that will be appropriate and the specific consequences expected will not be the same. Both the American colonists

in their struggle with George III and the opponents of the Rooseveltian New Deal were pursuing "liberty," but the pragmatic meaning of the word is far from identical in the two cases.

The meaning is vague as well as ambiguous because, since any imaginative forecast of future events is uncertain, and the more so the farther in the future those events are supposed to lie, the words which derive their pragmatic meaning primarily from our present anticipation of them can hardly avoid being shrouded in vagueness. What actually happens from year to year, in my efforts to further democracy, will inevitably be affected by many factors at present unknown and incalculable; my anticipation of consequences is at every point subject to this incalculability. Experience compels me to become aware of this margin of uncertainty, and to allow for it in my present expectations. I know that when unexpected happenings occur I will, so far as I am reasonable, modify my activities and revise my further expectations in the light of the lessons which they teach. These lessons may be and sometimes are very radical. But unless they are so radical as to persuade me to abandon democracy in favor of some contrasting social end, it will still be democracy that I am pursuing throughout those modifications and revisions. When we are dealing with general rather than specific ends, it is impossible to introduce a new end-word for each shift in the pragmatic meaning that takes place. Both our flexibility and our resources of language are insufficient to permit this. The result is that the pragmatic significance of the word "democracy" will be unavoidably in flux, and vague instead of definite. It cannot be pinned down to any precise, systematic description.[6]

Words whose distinctive role is to refer to action guiding us into the uncertain future thus impose logical difficulties which do not confront us in the case of words referring to some formal relation or some class of facts. Most vague and

[6] Hence the concept of probability does not apply to the realization of general ends. At best we may regard such a realization, under the conditions assumed, as "likely." See above, pp. 451 f.

ambiguous of all, pragmatically, are the very general concepts "good," "satisfaction," "happiness," etc., under which we try to include all other ends and which in some sense we are always pursuing. Next in order come such aims of limited scope as "liberty," "democracy," "health," "justice," "security," and the like—which are almost as elusive but not quite, inasmuch as we restrict their meaning to certain areas of action, excluding it from others.

Illustrated by the concept of health

To illustrate in fuller detail the basic difficulty arising from the distinctive role of these words, consider the concept of "health." It is obvious that there is considerable vagueness and ambiguity in the meaning of this term. We may do our best to provide it definite and dependable meaning. It may be defined, syntactically, as that condition marked by the normal functioning of all the organs of the body, and one can pursue endless research in physiology in the effort to determine just what such normal functioning involves. But as long as the term is used to denote an end and not merely a set of present facts, such research can hardly succeed. We may learn, for example, precisely what physiological changes go on in the body when a man smokes a cigarette of a certain brand. Some of these will doubtless be deleterious. But this does not prove that smoking is unhealthy unless we prove also that the disadvantage resulting from these changes overbalances the relaxation and other positive values which, one anticipates, will be secured by the smoke. For if we mean by health an end—the organic condition sought—it is clear that nothing becomes unhealthy merely in terms of the tissue changes involved; it must also be decided whether, on the whole, those changes are wanted or not. Our notion of what is "normal functioning" will have to take account of our wants. And since our wants in these respects are undergoing change and are not identical with those of other people in the same circumstances, "health," as thus functioning, vacillates in its pragmatic meaning and depends, in its syntactic meaning, upon varying conceptions of ultimate value.

Moreover, we could not circumvent these handicaps by

arbitrarily defining "health" in terms merely of physiological processes, and by persuading other people to join us in using the word in that sense. We might then, to be sure, establish a quite definite meaning for this word, but we should have to invent another term to indicate the total physicomental condition which we regard as desirable and wish to realize. And the pragmatic and syntactic meanings of that term will exhibit all the logical weaknesses that "health" had exhibited in its previous use. Thus the attempt to deprive end-concepts of their elusiveness helps not a whit in dealing with the basic logical problem; a new term must be invented to take over the character as end which has been surrendered by the old one captured for more precise usage. If a word denotes an end, it apparently cannot escape being revised anew in its pragmatic meaning whenever and wherever it functions; its vagueness and fickleness appear therefore to be ineradicable, and its syntactic dependence on some partisan theory of value seems to be unavoidable.

Yet it is not feasible to abandon the determination of ends

Impressed by the difficulties which beset us in endeavoring to discover something like a rational standard here, shall we conclude that it would be best to forswear any attempt to determine ends, confining ourselves to such selection between means as can be rendered demonstrable to any competent thinker?

No, that program is hardly feasible. To follow it would mean to commit ourselves to whatever ends we now happen to seek, or which later come to seize us, pursuing them with blind fanaticism. But this is both impossible and undesirable. Is it not clear that in many puzzling situations we do not know just what we want to seek, and that oftentimes when we think we know, we learn in the course of further experience that we did not know so clearly, after all? Moreover, as we survey our previous experiences we see that many times when we were confident of what was desirable we were really most mistaken as judged by our present ends, and this awareness leads us to take our wants yet more tentatively and to be readier to raise the question, in any novel situation, whether

our notions about what is worth aiming at do not need to be corrected. In short, the lesson of experience, intelligently considered, is not to abandon thinking about ends, however difficult it may be, but to think about them more seriously and persistently. We see, to put the point still more emphatically, that it is far more important to reach correct results about what is really worth seeking than to reach correct results about what to do in order to attain what we seek; the mistakes into which we fall in thinking about the latter are far less crucial and affect our lives less radically than mistakes about the former. If I am in error, for example, in the methods I pursue for promoting democracy, this error is less fundamental than the one I should be committing if mistaken in supposing it a good thing to promote democracy; in that event my controlling purpose itself would be wrong.

Reasoning, then, in the form of determining ends, is the most important kind of reasoning open to human beings. If it is desirable to think correctly about means, about facts available for our use, it is a yet more crucial and equally inescapable responsibility to think wisely about ends, about what is best to be done with these facts. As one passes, in brief, from reflective selection of means to the thoughtful determination of ends, he passes not only from a kind of reasoning which is relatively easy to one very difficult and imposing a baffling challenge, but also from a kind of reasoning which is relatively less important to one absolutely basic if human living is to be made intelligent. And this means that instead of giving up the search for whatever comes nearest to filling the role of dependable standard here, it is our business to pursue the quest with even greater care and patience, if possible, than are appropriate in the study of forms of implication or the criteria applicable to factual evidence.

Especially in face of the present world scene

This truth is poignantly emphasized when we survey the contemporary human scene and consider the tragic urgency of the social problems now confronting the world. These problems are problems of evaluation, and the most difficult ones are problems of determining the ends which ought to be

pursued in the complex and tangled interrelationships which now bind together all who inhabit the surface of our planet. The insistent problem that most perplexed man in primitive times has now been solved; scientific knowledge has been gained, and technical tools invented, which are sufficient to supply the physical needs of food, clothing, and shelter for a far larger population than is struggling to maintain itself at present. But how should these confused interrelationships be organized so that men can live together in happiness and mutual well-being? This is a problem of selecting means and, even more, of clarifying ends. The ends most persistently emphasized in the epoch of Occidental history now reaching its close—individual liberty and group autonomy—are clearly exhibiting the defects which are inevitable when they are not sufficiently balanced by other factors, and of course no believer in democracy can suppose for a minute that the ends accepted by totalitarian nations will bring lasting satisfaction or even prove capable of being steadily and intelligently pursued. What ends can be envisioned, under the guidance of which men may move hopefully and constructively forward toward the creation of a world community that will endure? No question is more pressing, more sobering than this.

But in our quest for a solution we shall forget at our peril the distinctive differences that characterize reasoning when seeking to determine ends as contrasted with any other of its major tasks. Let us remind ourselves again of the more crucial differences.

How may end-judgments be rendered rational?

Man is not so made as to be indifferent to everything; he is selectively responsive to the various occurrences that take place around him. Some he shuns; in others he finds a positive interest, acting toward them in ways which express that interest. Thus he is a creature who pursues ends. And not only are we unable to avoid pursuing some ends rather than others; we do not wait, before pursuing them, till reason is convinced beyond the possibility of justifiable doubt that their pursuit is appropriate. This circumstance shows that there is an unavoidable element of practical decision in one's commitment

to any end. It enters significantly into the logical status of judgments regarding ends, and distinguishes their adoption from acceptance of the truths of science. Whenever the evidence in favor of a scientific hypothesis seems insufficient for us to regard it as verified, we can refuse to believe it as anything more than a hypothesis, even though we may be aware of no other explanation that is better. To hold our minds open, not committing ourselves to any explanation till adequate evidence is forthcoming, is always an available alternative. But such agnosticism is impossible in the arena of action. We act, without being able to demonstrate that the act is wise, and any act implies that some end is taken to be more choiceworthy than other possible ends.

But people may not be moved to pursue the same end or to adopt the same theory of value, and all ends are vague and ambiguous when viewed in their pragmatic meaning. To determine an end requires us to forecast the future, and the future is largely incapable of accurate prediction. Some few things we can predict with confidence—those which follow from the fully established laws of inorganic change and take place on so grand a scale that other and less predictable happenings cannot distort their behavior much. The behavior of the stars and the rise of the tides exemplify these. But dependable predictive knowledge of this sort is not all-embracing. Unanticipated events constantly wash in upon our experience, and every such event forces us in greater or less detail to revise our imaginative picture of the future and the activities which are guided by it. It may even change our preferences as between alternative general ends to which we naturally refer by different end-words or phrases. And so far as specific ends are concerned, it frequently happens that an event of this kind will radically modify our purposes after we have devoted long effort to realize them and see their attainment near. A business man, for example, may have planned for years to organize his affairs so that he could take his family to Florida for the winter, only to find himself promoted to a position of such challenging responsibility that

he no longer wishes to take a vacation at that time of the year. Our controlling ends thus mark the growing point of our experience. Life is a continual process of reconstructing our dominant aims, some features of them being shoved into subordinate places by newly discovered goods that now seem much more appealing. This kaleidoscopic transformation of our imaginative picture of what we seek constantly goes on, and must go on as long as our capacity to respond does not die and our experience remains incomplete. We see the process in its simplest form in the way in which a child passes in interest from one toy to another, as former enjoyable experiences begin to pall and newly exciting possibilities gleam on the horizon. But the same process continues in the most mature mind, and it infects our conceptions of ends, so far as their pragmatic and ultimately also their syntactic meaning is concerned, with a variability and elusiveness that plunge us into tantalizing logical bafflement.

In order to deal with this bafflement constructively we shall need two further technical terms. The statements with which we were occupied when studying formal and factual science were called "propositions." In order to recognize the distinctive character of statements whose purpose it is to express an evaluation, let us call them "value-judgments." This is appropriate because the word "judgment," as ordinarily employed, implies that the thing judged is weighed or assessed, *i.e.*, evaluated, in some manner. And value-judgments whose purpose it is to express the adoption of an end will be called "end-judgments."

This terminology provides a convenient way in which to describe the enterprise upon which we have now embarked. It is this: In view of the serious difficulties intrinsically involved in discriminating between wise and unwise end-judgments, how should one go about it to make such judgments as rational as possible? What kind of criterion is appropriate here, and why? How is it to be applied to cases where we must choose between one end-judgment and an-

other? The attempt to answer these questions will provide the guiding thread throughout the subsequent chapters.

EXERCISES

1. Explain the contrast between value and fact. In what sense is a value also a fact? When is a fact also a value?
2. List and illustrate the main differences between reflective selection of means and reflective determination of ends.
3. Just why, according to the chapter, does the pragmatic meaning of words denoting ends tend to be vague?
4. Read *Introduction to Reflective Thinking*, Chapter 9, and make a preliminary analysis of the logical problems concerned with thinking about ends.
5. Why are such problems important?

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CHAPTER 26

THE RATIONALITY OF END-JUDGMENTS

In what ways are scientific criteria applicable to end-judgments?

Our first task is to determine more fully and exactly just what the word "rationality" might properly mean when applied to end-judgments. What sort of rationality is possible and appropriate here? How do we tell which judgments conform to the reality of the relevant values? And the main questions which need to be answered in carrying out this task are: What criteria does a reasonable person employ in deciding between alternative end-judgments? Why are they reasonable? In what way are they interrelated? And how can they be formulated in such a way as to give the most complete and dependable guidance that is possible in dealing with situations in which a choice between ends is required?

In discovering these criteria we shall follow the order suggested by the course of the preceding discussions. When the criteria sought have been identified and described, they will dictate a logical reorganization of that order, so that the significance of the result reached for the following chapters will become clear.

Let us begin by considering how far and in what form the criteria that are applicable in scientific reasoning are relevant in this field, passing then to the further norms that are needed by the distinctive nature of end-judgments.

Consistency

The formal criterion of consistency is relevant here, and obedience to it is as definitely demanded as it is in reasoning about any other subject matter. No one can make progress in thinking about anything unless he avoids contradicting himself. And an essential prerequisite to the maintenance of con-

sistency is an explicit syntactic definition of the concepts employed, so that one can tell in each case whether the chosen equivalence is rigorously adhered to or not. Now in principle this is always possible. Though ambiguous and vague in its pragmatic meaning, any end-concept can be defined in terms of other concepts, and the definition can be respected. It is clear that no rational test of two proposed acts by application of the Golden Rule could be made if, when testing one of them, a different definition of some important word in that rule were assumed than when testing the other. The two evaluations would be quite incomparable were that the case, and no progress toward a decision would have been made. Consistency is therefore a necessary feature of the total criterion that is here appropriate.

Factual responsibility

How about the criteria with which we became familiar when studying the factual sciences—those that are pertinent whenever the semantic meaning of words is involved? Well, in this context the fundamental criteria are empiricism, or respect for the relevant observable facts, objectivity, and simplicity. Leaving the last two of these for later consideration, in what sense must the process of determining ends have regard for observable facts? In one very important sense, which the preceding chapter has briefly indicated. An end is not a utopian fantasy, subsisting in unattached isolation from the present facts which provide the materials out of which, if at all, it is to win its realization. It is an envisioned value *of* those facts, and cannot be intelligently thought about in separation from them. Accordingly, it is an essential feature of any appropriate and dependable total criterion here that realistic account be taken of the nature of the facts involved, and especially of their causal relationships. In choosing my future career, for example, suppose that I decide to solve the problem by staying in my present position, the assumption being that as a result of doing so certain future events will take place, *e.g.*, the completion of a book I am trying to write, and that those events will contribute a certain value to the entire set of values that I anticipate. But perhaps the assumption

is mistaken, and another set of events will actually result with quite different values; in that case the value-total that is really relevant—the end, that is, in the light of which I ought to have chosen and would have chosen had I been able to predict more successfully—would be quite different.

The facts which must be taken into account in this way fall into two groups. One consists of the external objects or social institutions which are involved. A person cannot, at present, build houses out of sea water, and if he builds them of bricks he must accept the limitations imposed by the sort of thing that a brick is. He cannot, for example, as he might with some other materials, build a house with more than a certain amount of window space; the walls above the windows would be in danger of collapse. Hence the nature of the facts which constitute one's material, to be transformed under the guidance of one's end, must be respected. They have certain possibilities, but not all things are possible with any given set of materials.[1]

The other group of facts, of which one needs to be intelligently aware when determining his end in a given situation, consists of his own wishes and emotions so far as they affect the process of reaching a decision. It was observed in Chapter 4 that because of the presence of these factors in human nature the exercise of reason easily, and often quite unconsciously, becomes rationalization—it is really a process of finding ways to justify some strong wish instead of impartially seeking to uncover all the relevant implications of the various suggestions that seem worth considering. This tendency appears just as readily and insidiously when reason is trying to clarify ends as in any other mental operation, and its presence is very hard to detect, since the deeper feelings affecting a course of thinking are not ordinarily noticed unless something happens to make us suspect them. The student of right thinking, realizing this fact clearly, must compel himself to be

[1] It is well to remember at the same time, of course, that science is continually discovering new uses for available materials. This is one of the ways in which science progresses.

suspicious, to be constantly on his guard against deception by them. Whenever he finds any ground for believing that he may be rationalizing in his choice of ends, it is essential to drag to light and critically examine the motives that are thus playing a part. If this cannot be done in the time available before the decision must be made, it is better to follow some guiding maxim approved by broad human experience than to run the risk of deciding under the distorting influence of some unrecognized emotion. I might, for example, in the situation where I had to choose between two careers, see warrant for suspicion that the real reason why I was attracted to consider the offer of another position might be dislike of some person with whom my present duties require me to associate. If so, and the time available for decision is not sufficient to allow me clearly to understand that feeling and give it its reasonable weight, I would be wiser to decide by such a maxim as "A bird in the hand is worth two in the bush." This will not guarantee a thinker against the possibility of mistakes, but it will enable him to avoid the more serious danger of confusing anticipations weighted by emotional irritation with anticipations formed by impartial judgment.

In these two ways, then, wise reasoning about ends will be empirically responsible.

Criteria required by the distinctive nature of ends—1. In cases of mere liking

So we are ready now to face the most difficult problem about the appropriate methods for distinguishing between more and less rational end-judgments. What criteria are specifically relevant because of the unique nature of ends, and how are they related to the criteria which have just been examined? In answering these questions we shall also discover in what form the criteria of objectivity and of simplicity play a part in the rational determination of ends.

To deal adequately with this problem, it will be necessary to divide the situations in which people decide between alternative ends into three classes. In this division we shall be putting to practical use the first two ways of classifying values listed in the preceding chapter.

First, there are the situations in which no distinction be-

tween a more rational and a less rational end-judgment seems to be needed. They are the cases where it appears entirely appropriate to decide between ends merely on the basis of one's spontaneous preference. If, consulting the menu card in a restaurant, I find that the first course gives me a choice between vegetable soup and tomato juice, and I like the former while disliking the latter, it would ordinarily be taken for granted that I may properly decide merely in accordance with this immediate preference. Evaluation is direct and simple; deductive elaboration is unnecessary. The judgment, then, that in this context the end attainable by eating vegetable soup is better than the end attainable by drinking tomato juice, can be reached at once; it does not need to conform to any rational requirements.

What changes would transform this situation into one where it would be insufficient to guide one's choice by his immediate liking alone? In anticipation of the subsequent discussion let us briefly state them. My doctor, it may be, has told me that I have not been getting enough of the vitamins that tomato juice contains in large quantity. Now, clearly, it would be irrational to decide merely by spontaneous preference. It is wise to take into account the further consequences of each of the alternatives; an end-judgment which disregards them is now irrational.[2] Or suppose that I am in a party of five who are dining together, and there is not enough vegetable soup for all those who would prefer it. Now, too, it would be irrational to choose merely in accordance with my own likes and dislikes. An appropriate end-judgment will take account of the preferences of others as well as mine, when we are all affected by any choice that is made.

2. In cases where future consequences are important

Second, there are the situations illustrated by the former of these hypothetical changes. Here, considerations arising from my immediate likes and dislikes are complicated by the need of taking into account the future consequences to be reasonably expected from each possible choice, but the inter-

[2] Unless I expect my life to end before the specific virtues of the tomato juice could do me any good.

ests of other people are not sufficiently affected so that they need to enter the picture. I prefer the vegetable soup to the tomato juice, but I also want to be healthy, and health promises to be realized better by taking the tomato juice. It will be well to analyze a typical situation of this kind with some care so that we may see clearly what distinctive criteria are present, and why an end-judgment reached under their guidance is more rational than one which disregards them.

The instance of the reflective determination of an end that was employed in the last chapter and briefly mentioned in this, describes such a situation. Let us examine it again from the point of view of our present problem. The choice, it will be remembered, was between one professional career and another. Now it is clear that a decision might have been made in much simpler fashion than was actually exemplified. I might have stopped tracing the anticipated consequences of each alternative at some earlier point than I did, impulsively committing myself to one of them without any further attempt to see what else was bound up with it. Indeed, I might have done what is usually appropriate in the case of the vegetable soup and tomato juice, *i.e.*, followed the immediate appeal of one of the alternatives without engaging in that process of reflective anticipation at all, or I might have allowed a quite irrelevant circumstance to decide the matter, such as the arrival by a given day of some expected letter, or even the toss of a coin. Usually, of course, in a situation of this sort such decisions by mere chance are resorted to only after an unsuccessful effort to determine what we want has led us in despair to feel that the one important need is to end the suspense.

With these possibilities in mind, two points become clear. One is this. All would agree that in such cases the way of reaching a choice described in the preceding chapter is more rational than any of these impulsive ways just mentioned. And why? Because it is a choice made in fuller awareness of the network of consequences in which one sees each alternative to be embedded, and because it is made under the guid-

ance of that awareness. And why is it more rational to be guided by such awareness than not to be? Because what we have called the more rational way is, other things being equal, not so likely to lead to disappointments in subsequent experience as any less rational way. We have seen that our choices and acts, like other events, are causes which have their natural effects; some of those effects will augment our happiness while others will spell misery or ruin. To decide in the clearest consciousness of the effects that may be reasonably anticipated enables us, in whatever degree is possible, to realize the effects that we want and to avoid those that we do not want, while to decide without such consciousness places us at the mercy of chance so far as the future results of our action are concerned.[3] Of course, even here nonrational ways of deciding are sometimes justifiable when the issue is unimportant, and often rather important decisions have to be made in too short a time to permit complete performance of the rational evaluation that would otherwise be possible. We can, however, usually make some forecast of consequences before acting in such a situation, and action under its guidance is clearly preferable to action unenlightened by any conscious anticipation of effects at all. To be rational here thus means simply to be prudent. In brief, while the more rational way takes account of the immediate appeal of each of the competing alternatives, without which they would never have been suggested as live possibilities, it enables one before deciding between them to supplement this spontaneous like or dislike by awareness of the values, good and bad, of the consequences to which each alternative is likely to lead. Thus we may choose that plan of action which promises to realize the greatest possible value-total in the entire situation affected by our choice.

Criteria that are relevant in such cases

Well, what distinctive criteria are operative here, which on examination appear to be relevant to any case of this kind? There are two, one specific and one general.

[3] A less rational way would instance the reaching of an end-judgment intuitively rather than rationally, in the sense in which intuition was contrasted with reason in Chapter 3.

The specific criterion is just the most comprehensive value-total that looms before one's imagination as a result of systematically developing and comparing the alternative choices that are available—the largest complex of value that can be envisioned as capable of realization in the situation confronted. Once this greatest feasible good becomes clearly present to one's mind, the various courses of action that might be pursued can be judged in relation to it, for it was by tracing the aggregate of values involved if this or that course of action were to be followed that the most inclusive attainable value was able to emerge into clarity. Thus the suggested modes of action can be measured in the metaphorical sense in which measurement is possible in matters of this kind; one can see which among them is the appropriate one for pursuing the supreme value-total that now looms ahead as possible. This criterion obviously varies from situation to situation, since the greatest feasible good realizable in one situation will usually not be identical with the greatest feasible good in another.

But another criterion is operative in this process. It is a general one, applicable to any situation. In order for such a supreme attainable good to emerge in definite form, so that one can assess different courses of action in its light, he must be guided by some implicit or explicit theory as to how we are to identify what is good, *i.e.*, just where in human experience it is dependably found. In the terms used in the preceding chapter, he must have a syntactic definition of "good," as an all-inclusive end.[4] Suppose, for example, that in picturing the two sets of consequences which seem to be promised by the alternative choices in the situation just referred to, I conclude that staying in my present position would be most likely to bring me contentment, while accepting the new offer would be most likely to realize potentialities in my nature which now lie dormant. Obviously, I cannot decide which set

[4] Such a definition prescribes that "good" is to be regarded as equivalent to this or that concept which points to some identifiable area of human experience, such as the concepts now to be used as illustrations.

of consequences is the greatest good without some theory as to how to identify "good" in the presence of these differences. If I identify it with "happiness," and believe that it is most dependably found where steady contentment is attained, I shall presumably decide in one way; if I identify it with "self-realization," I shall presumably decide in the other way. It has been noted already that a serious logical problem arises from the fact that different thinkers may disagree on how "good" is to be defined; the definitions just mentioned are two of the alternatives that compete for acceptance in the arena of contemporary discussion.

Whether one is aware of it or not, whenever he decides between conflicting ends he is assuming some theory as to how a definition of "good" is to be properly made. Such a theory constitutes a unifying principle, binding together in a consistent whole all his choices of ends in the various problems which are decided by employing it as the court of appeal. Hence the role it plays in the determination of ends is analogous to that played in factual science by the criterion of simplicity, under whose guidance specific laws are bound together in a coherent system.

3. In cases where the interests of others are affected

Now for the third class of situations in which ends are determined—the class in which the preferences and interests of others are definitely affected as well as one's own. Consider again the last hypothetical change which was suggested above when the choice between vegetable soup and tomato juice was analyzed. When I am dining as a member of a group rather than alone, the situation becomes one of this class—at least it does so if the dishes available are not sufficient to satisfy the preferences of each member of the group as they initially stand. Then the determination of my end involves not merely the question which food I would choose in terms of my own likes and needs, but also the question how this preference is to be related to the likes and needs of other persons. And it is further clear, in the light of this illustration, what the two main alternatives are with respect to the way in which this aspect of the situation can be met. On the one

hand, I can meet it by insisting on the fullest possible opportunity and freedom to determine my own end, imposing the result of that determination on the other members of the group. That is, my choice would become for them a *fait accompli*, limiting their opportunities for choice in ways in which I was unwilling to be limited myself, and thus, in some degree or other, determining their ends for them instead of allowing them the freedom that I demanded. What this would mean in the particular situation described is that I would give my order to the waiter first, if I could, so that I would be sure to get what I wanted, and the others would then have to choose among the dishes left after my decision had been made.

In such a situation, one who did this would of course be universally branded as an unmannerly boor, and for this reason it reveals with exceptional clearness the nature of the criterion which we feel to be relevant and why it is felt to be so. The other members of the group naturally assume that they have just as much right to choose their ends as I have to choose mine, and yet I have insisted on exercising my right to choose freely even though by doing so I have limited their opportunity to choose. Why should one member have any greater opportunity to determine his end than the others have to determine theirs? The preferences and needs of the whole group are concerned, and are affected by the choice of any individual member; should they not then be equally taken into account in any decision that each member makes? Hence it becomes clear what the other main alternative is. I can meet the situation by assuming that, since the right of other people to determine their ends is inescapably involved here as well as mine, I will determine my end in such a way as will make sure that the others affected are impartially accorded an equal opportunity to realize what they prefer or need. In the situation contemplated, the method by which this principle would be followed is simple and obvious. The group would find out how serious the needs or how strong the preferences of each member are on the matter in question, and

would apportion the available servings accordingly; if this were impossible, or appeared to attribute undue importance to the matter, some general rule of courtesy would be respected, such as that elders should have preference over those who are younger, and that ladies should have preference over men.

Criteria that are relevant in these cases

When we examine these alternatives, it is clear that a criterion is operative here which did not need to be introduced when the interests of other people were not vitally concerned. Where one's determination of an end affects what other people regard as their right to determine their own ends, the criterion that must be employed to meet this aspect of the situation is some rule concerning the comparative privileges of the persons affected. What my decision will reveal, in this regard, is whether the rule guiding me is that of assuming the right to determine my ends in my own way and then to impose the results upon others, restricting their choice of ends in whatever fashion such imposition inevitably would, or whether my rule is that of recognizing an equal right on their part to determine their ends—to find out and pursue what they hold to be good with the same freedom and opportunity that I wish for myself.[5] In the latter case the criterion that I am employing, so far as its central idea is concerned, is the Golden Rule as applied to the process of determining ends; in guiding my decision in this way I am doing unto others what I would wish them to do to me under circumstances where I am affected by their acts. In such a situation one measures different courses of conduct by seeing which of them are consistent with the rule he regards as relevant and which are not. Those that are not are eliminated from further consideration.

It was noted above that in cases of this kind we usually refer to the proper act as "right" rather than merely as "good," and any improper one as not merely "bad" but "wrong." Hence whatever rule is followed in such matters provides our method

[5] There are other alternatives also. But these two are the ones of major practical significance.

for determining what these two ethical terms mean. A right act will be any act that conforms to the rule, while any act that fails to conform will of course be wrong.[6]

After those courses of conduct that appear inconsistent with our adopted rule are eliminated, it may be that only one course remains. Then we can decide without needing any further criterion. If more than one remains, as will sometimes be the case, selection between them will be made by the procedure above described when the second class of situations was analyzed, and the criteria appropriate there will of course be used. Thus, in this third class of cases, analysis must recognize three distinguishable criteria as operative besides those of consistency and responsibility to the relevant facts—some rule governing our relation to others, some theory as to how our ultimate terms are to be identified in experience, and some envisioned value-total by which to make comparisons of better and worse in the specific situation confronted.

How would a reasonable person who is eager to form his end-judgments aright go about it to decide which of these three classes any given instance of end-determination falls under? Such a decision must be made before one can know which, if any, of these criteria are applicable. A moment's reflection will indicate how the decision should be made; our conclusion on this point will give us needed guidance later. Clearly, if one were to classify the situation by his spontaneous intuition, he might regard a value problem as merely a matter of personal liking when in the light of later developments this classification turns out to be quite erroneous. Hence the reasonable way to decide is to consider reflectively the three possibilities in the reverse order in which they have just been examined. Are other people's concerns sufficiently involved so that the situation belongs in the third group? If so, then the criteria for that group are applicable. If not, are

[6] However, those who follow the former of these rules do not use these terms in quite the same way as do those who follow the latter. An act consistent with the rules they would justify as "all right," and perhaps they would hesitate to call acts inconsistent with it positively wrong. Rather, such acts would be, in their eyes, cases of simple-minded folly.

future consequences to myself likely to be of sufficient moment so that it belongs in the second group? If so, then the criteria for that group are applicable. Only if both these questions are answered clearly in the negative can we properly assign the case to the first group and treat it as a matter of immediate like or dislike merely.

What does "rationality" mean in this third group of cases?

Now an especially important question confronts us. The concept of rationality, we said, is not applicable to the first group of cases; and as applied to the second group it becomes essentially identical with what is meant by "prudence." Can we say which of the rules that may be adopted for guidance in dealing with the third class of cases is the reasonable one to accept, so that the end-judgment finally reached by applying it is more rational than any alternative? If so, which rule would it be, and what ground can we give for insisting that it is the reasonable one to follow? Yes, we can. And in order to justify this answer clearly let us examine these alternatives in their broader social implications.

I am a member of a community containing many other people besides myself. Suppose that I have followed, in various cases of determining my ends, the former of the two rules just discussed, and explicitly consider the problem: What relation would need to be established between myself and others so that I would be in a position always to follow that rule with maximum ease and efficiency? The answer is clear. I would need to make them my slaves. Because only then would I be able to prevent them from determining their ends whenever I wished to; they would become entirely subservient to my ends. A slave is the servant of his master's purposes; he no longer has freedom to pursue his own. And if it should prove impossible to subject them to me in this complete fashion, still I would seek to realize as large a measure of control over their ways of thinking and of acting as I can, by whatever methods of propaganda or terrorism are available. Suppose, on the contrary, that I have adopted the other rule, and explicitly face the corresponding problem in regard to it. The answer is again clear, in general terms. I would seek to

realize such a social order as will protect the equal right of all people to determine their ends. But we must analyze these alternatives sufficiently to show a little more fully what each involves, and it will be best to do so in reverse order. What opportunities must be assured, what concrete procedures are required, if the equalitarian freedom just mentioned is to become actual?

First, full opportunity to discuss the policies proposed by any member of the group, in terms of their bearing upon the interests of others. If one person objects to the ends advanced or pursued by another, feeling that they will interfere with what he regards as his rights, he must be able to present his case before the community and have it reasonably weighed. So far as a society does maintain such freedom, every would-be participant has an equal chance to throw his bit into the hopper of debate, laying his end-judgments before others, together with the considerations that make them persuasive to him, and being confronted in turn by attempts to render contrary ends convincing to him. Second, full opportunity to participate in choosing the community's leaders, and to help decide, directly or indirectly, what policies are most worthy of common adherence when they need cooperative support to become effective. For this rule implies that the goals to be thus pursued are simply the ones that succeed in making themselves persuasive to a majority of citizens as a result of such a free and equalitarian process of discussion.

Now, what opportunities will be sought, and what methods practiced, by one whose aim is to dominate others as completely as possible? Well, he will probably not attempt their total enslavement, which is no longer feasible in the modern world, but he will wish to achieve as complete intellectual and practical authority over them as he can. Not only will he not permit the door of free discussion to be held open in the fashion just described; he will wish definitely to close the door to all presentation of ends and policies that differ from his own. From his viewpoint, the ends he has adopted are so obviously good that these absolute values ought not to be

endangered by running the gantlet of argumentative competition with other ends. They should rather be imposed on everybody, by whatever means of art or force promise to be most successful. He has adopted them not tentatively but quite dogmatically—that is, their assertion reflects the assumption that his judgment which has established them is infallible, never in need of correction by the judgment of others. So he will seize whatever social power he can, and proceed to exercise it vigorously in this fashion. Like any would-be master, he is claiming a privileged status in relation to his fellows. He assumes the right to determine what is good for them as well as for himself, irrespective of the fact that they may approve different ends, and are often just as deeply convinced of the rightfulness of their choices as he is of the rightfulness of his.

The social issue involved

There are accepted terms by which to denote the two kinds of social order which result when each of these alternatives is followed in any community. The social order created when the equal right of all to participate in the adoption of ends is protected is democracy, while the contrary order, when achieved in any successful fashion, is a form of dictatorship. A dictatorial regime allows full freedom to the dictator and the henchmen who at the moment enjoy his favor to choose the ends that seem wise to them, and full opportunity to use the tremendous power of the state to impose these ends upon others. In seeking truth, they have full access to relevant facts and may employ among themselves the way of free discussion; others, however, are denied on principle these opportunities, being enticed by propaganda or coerced by terrorism to accept the decisions made by their leaders. Democracy on the other hand, in intent, allows equal opportunity for all citizens to get at the facts on which sound decisions must be based, and to participate in the discussion of diverse courses of action. Hence, in spite of the circumstance that democratic institutions often fall lamentably short of the ideals which they are supposed to express, they nonetheless do exhibit a serious effort to provide the conditions necessary for the maintenance

of equal freedom, as well as for seeking voluntary agreement on community ends and on the broader lines of policy to be followed for their realization. The attempt is made to assure a generous measure of freedom of speech, and the press, except in time of emergency, is guarded against interference by the government or intimidation by powerful groups.[7] So far as is possible in view of the force of human passion and prejudice, individuals are guaranteed the free exercise of the ballot, the protection of impartial justice in courts of law, and the right to worship according to the dictates of conscience—the right, that is, to choose their ultimate objects of religious devotion and to be loyal to them in their own way. Democracy holds that a basic responsibility of every social institution (and especially the state) is to serve and protect the individual citizen in his equal right to participate in the determination of ends, while the belief of dictatorship is that the mass of people exist to serve the aims of the social institutions to which they belong, as decided by the group in control and in the last analysis by its leader.

It means democratic cooperation

Well, then, which of these guiding rules and contrasting procedures is more rational than the other? Is reason, as exhibited in the pursuit of community agreement through free discussion, more reasonable than reason as exhibited in the dictator's authoritarian manipulation of the minds of others, imposing upon them the ends which he has adopted? And, if so, on what grounds can this assertion be made?

Earlier in the chapter it appeared to be the case that when an individual is choosing ends without any special reference to how his choice might affect others, that way of doing so is most rational which uses to the full whatever chance there is of forecasting the probable consequences of each alternative plan of action. It thus insures that the choice will be made in the most comprehensive awareness of all the goods and bads that are actually involved—in the light, that is, of the most inclusive value-total that emerges as relevant. This way

[7] Except those groups which own the influential organs of opinion. Their special position constitutes one of the most serious problems of democracy.

of choosing is most rational, we said, because it gives reason the freest and widest scope in envisioning and bringing into account all the values at stake before the choice is made. On the same grounds it may be asserted that when the problem is how to determine ends that vitally concern more than a single individual, the way of free and equal discussion, on the part of all whose welfare is affected, is more rational than the way of imposing the ends of one person or a privileged few upon the rest. For the former procedure likewise assures reason the fullest and widest scope in taking account of all the values at stake in the lives of all the individuals to whom the decision makes a difference. Only if it could be shown that the dictator and his clique have a monopoly of evaluative wisdom in the community, so that no promising suggestion as to what goals are worth pursuing could occur to any but their minds, would the latter procedure be able to claim plausibly that it is the reasonable one. If such is not the case, it is clear that dictatorship rests on power, not on reason, and the method of respect for the equal opportunity and freedom of all to participate in making the choice is more reasonable. We should have in factual science a situation analogous to dictatorship if scientists were to limit themselves arbitrarily to certain hypotheses only, excluding others that are relevant.

The distinctive difficulty of reasoning as evaluation, we saw, lies in the fact that when an end is being determined that end has not yet taken definite shape; hence there is no way of telling how to allow for it, and no reason to expect that different individuals will allow for it in the same way. But when divergent, and possibly conflicting, ends of different people are at stake, at least these ends can be given a chance to emerge freely, to gain effective expression, and to receive equal consideration on their merits, even though the form they will assume cannot be predicted in advance. In this way, while their specific nature cannot be allowed for beforehand, a social procedure can be followed which makes sure that they *will* be allowed for on an equalitarian basis, as soon as they have taken definite form. Such a procedure expresses

the demand of reason in this situation. It permits and encourages bringing into the open all the values actually affected, in the life of any member of the community, and it insures for them a chance of impartial consideration. Wherever it is possible to combine harmoniously the values precious to one group and those precious to others, it provides the fullest opportunity for that synthesis to be worked out and to gain persuasive formulation. Moreover, if all groups accept the preservation of this procedure of free and equalitarian debate as itself constituting a value superior to the private ends of each—an appropriate superend for the community as a whole—such differences as remain after any particular decision has been made can always be kept within the limits of social harmony and mutual tolerance. Every minority is protected in its rights, and always has its fair chance. Given such acceptance, no conflict of ends within the community will become hopeless, or need be settled by violence.

To be sure, this conclusion in favor of the superior rationality of democracy is itself a value-judgment, and involves an element of practical decision just as any other value-judgment does. There is no pretense that its correctness can be demonstrated in the sense in which a formal inference, or a causal law, can be objectively established. It can be demonstrated only in the way an ultimate value can be demonstrated, namely, by achieving persuasiveness as good and right when it is clearly seen in relation to its alternatives. And so far as it succeeds in winning such persuasiveness, those who accept it are able to answer a question for which all our preceding analyses have helped prepare the way.

What, then, in the end, is reason?

Just what is it to be rational? Each part of this book has considered one of reason's distinctive roles or characteristics; now that the final and most important role has been introduced, a summarizing description of the nature of reason in the light of all these roles would seem to be appropriate.

Part I presented reason as exhibited in the persistent and clear-headed performance of the entire sequence of operations which the occurrence of a perplexity initiates, in face of the

emotional and impulsive factors in human nature which work subtly or overtly to abbreviate, confuse, or distort this often lengthy and difficult process. In that context its contrasting alternative is mere intuition. The subsequent parts have brought out one by one the major demands that such performance must meet, and the functions which reason fills in meeting them. Part II discovered reason as seeking formal consistency, directly in those portions of the process where the reasoner engages in inference from premises to conclusions, and indirectly wherever language is employed in its syntactic meanings. The opposite of rationality here is contentment with ambiguity or self-contradiction. In Part III, reason appeared as insisting on respect for relevant evidence, again primarily in any investigation where verification of hypotheses is sought, but also wherever the semantic reference of words is involved. When one is irrational in this field he betrays it by lack of scruple in dealing with the pertinent facts; he neglects evidence, or fails to be thorough or critical in examining it. Now, in Part IV, reason becomes our guide in the enterprise of democratic cooperation; it is the dynamic intermediary through which the ends sought by one individual are kept in equalitarian and hence impartial relation to the ends that seem good to other members of the community. Its opposite in this context is dictatorial authoritarianism—readiness to impose one's own ends upon one's fellows.

An answer has been implicitly assumed

This role, in virtue of which reason appears as an interpersonal creator of mutual understanding and sincere agreement rather than a merely individual faculty, is evidently its pervasive and culminating role. That it fills such a function has really been assumed in our earlier analyses, although until now specific mention of it in these terms was not needed. In discussing formal science we took it for granted that the syntactic meaning of a word cannot be something which varies from one individual to another, because words are supposed to serve a social purpose—to be the medium of intercommunication, usable in argument, discussion, illustration, debate. But if a word is to fill this role, its implications as between the

one who employs it and his contemplated audience must be the same; and explicit definition of key words is important just because only thus can it be made reasonably sure that this identity of meaning will obtain. To insist, however, that syntactic meanings must be capable of such sharing indicates that one prizes as supremely valuable the opportunity for discussion with others on a basis of mutual understanding. Their possession of such a property will seem essential only if he is eager to make sure that the conditions requisite to demonstrate the validity of inferences to others will always be present.

Still more clearly is this culminating role presupposed in the criteria of factual science. As we have seen, factual evidence is not evidence that can appear such to this or that individual merely; it is evidence that can approve itself as coercive to the entire body of competent inquirers. A scientist recognizes that what he calls the "personal equation" may obtrude at any time, and that when a certain body of data appears to some particular thinker or thinkers to justify a given conclusion it may do so because of the distorting influence of this subjective factor. But he recognizes also that it is the responsibility of science to find ways of overcoming such idiosyncrasies and of reaching results capable of winning the assent of all who understand the problem and are familiar with the procedures appropriate to solving it. The main reason, in fact, why the principles employed in factual science for guiding description and explanation are as solidly established as they are, lies in the fact that anyone sincerely interested in the subject can see without difficulty the probative force of evidence gathered under the guidance of these principles. Hence science insists that satisfactory results must be objective. Now this insistence on objectivity implies that investigations pursued with the aim of winning the approval of others are believed to be intrinsically superior in value to investigations that might reflect the presence of some personal vagary. Furthermore, it implies something else that needed no emphasis when we were examining the criteria of factual

science, because scientists take it so completely for granted. This is, that the agreement with other competent inquirers that is sought when one is guided by the criterion of objectivity is not agreement anyhow secured, by hook or crook; it must be without compulsion, won by free persuasion in the presence of the relevant facts. Well, the attainment of this sort of agreement is obviously precluded if any attempt is made to discourage any inquirer from seeking or presenting evidence which seems to him to bear on the truth of a proposed conclusion. For this reason, one who is committed to scientific objectivity will always wish to keep the field of data that are relevant to any assertion claiming truth, open to continued investigation on the part of all who might be interested. He will want anyone who disagrees with that assertion to have a free opportunity to produce whatever evidence can be produced in support of his dissent, and to argue for whatever contrary conclusion seems to him more reasonable. The dissent of any sincere and competent dissenter, for him, throws doubt on the truth of the proposition in question, and he wants that doubt raised whenever anyone honestly feels it. Only when the dissent has been removed, either by the persuasion of the one who expresses it or by his ability to convince other interested inquirers that he is right, is the proposition freed from that shadow. In brief, he is implicitly committed to the social end of democracy; it is clear that only in a community practicing democracy can the criterion of objectivity be faithfully respected. Reason as guide in the process of democratic cooperation has thus really been presupposed throughout our entire study; this role underlies each of its other roles and finds partial expression in them. Wherever the democratic way of determining ends that concern others is rejected on principle, and the rejection is carried through consistently, all functions of reason will be radically transformed; we will have a different kind of factual science, and a different conception of syntactic meaning than those portrayed in the preceding chapters. This is evident today in the totalitarian countries. Science there, in political intent, at

least, is subordinated to the aims of the state, as interpreted by the dictator who happens to be in power. What can be discovered as truth is determined in the end, not by the objective evidence, but by its harmony with whatever general theory of the world he proclaims to be orthodox. Similarly with consistency and validity of inference, although here the totalitarian ideal is more difficult to realize. The dictator's conclusions are valid whatever their logical relation to his premises, and any *argumentum ad populum* is consistent provided it is successful as propaganda—that is, provided it persuades the masses to accept the decisions prepared for them.

It is the faculty of socially responsible thinking

What is essentially in common, then, between the methodology of science as it has historically developed and the procedure cherished by democracy? It is just friendly cooperation in the pursuit of the varied goods that appeal to men as worth pursuing. Valid deduction and verifiable truth are good, among other things, because they are media of equalitarian cooperation among men. Reason, in the last analysis, is that part of our minds which is expressed in *socially responsible thinking*, thinking which accepts responsibility to all the considerations really involved—all the formal implications that are relevant, all the factual evidence that has any appropriate bearing, and in every situation all the human values affected. And the last-named responsibility means, among other things, responsibility to all the individuals in whose experience these values appear, without bias or partiality.

Summary of the criteria of end-judgments

We now need a summary statement of the criteria which have been found applicable to judgments expressing a choice of ends. (1) The criterion of formal consistency applies to them in the same way that it does to any set of statements. (2) The criterion of empirical responsibility applies to them, inasmuch as the end chosen depends upon certain factual materials for whatever realization it may gain. And three further criteria have been discovered. If the end-judgment concerned is a judgment of prudence rather than one of immediate liking, it appeals to (3) the specific criterion provided by the commanding value-total envisioned in that situation, and to

(4) the general criterion provided by a theory as to how to identify in experience the referents of the ultimate value-concepts employed or implied in the judgment. If it is a judgment expressing the choice of an end where the interests of others are affected by the choice, (5) a further relevant criterion is the rule of impartiality—the rule that one is obligated to respect the equal right of all others to choose their ends on the same basis as oneself. This rule, as we shall see later, applies without qualification in situations where others respond with the same attitude. Where they do not, certain qualifications must be recognized; one may not, in every respect, treat equally those who are unwilling to treat others equally, lest the social foundations of impartial co-operation be lost.[8]

But since this fifth criterion has been established, and its significance in the entire enterprise of reason clarified, it is possible to show the logical interrelations of these criteria as they perform their proper function in guiding end-judgments. They were introduced in the order that seemed most natural when one endeavors clearly to disclose each of them in the light of the material available from our preceding discussions. But now we need to know how they are interrelated as guides in the rational determination of ends.

They are unified under control of the fifth

From the above analysis, the fifth criterion is evidently the basic one. This means that while it does not depend on the others, it plays a part in determining the real nature and mode of functioning of the remaining four criteria; they fill their role properly only when they accept its authority and operate in full harmony with its demands.

We have just seen how this is the case with the first and second criteria, namely, formal consistency and factual responsibility. It is reasonable to respect these criteria not only because of the persuasiveness that they can command in their own right, but because, in the end, when one does not respect them he is violating the right of other thinkers to have access to validity and truth on the same basis as himself. Objectivity

[8] This important point will be dealt with in detail below. See pp. 716 f.

in factual science, for example, has now revealed its underlying character as commitment to the value of equalitarian cooperation. Whether one feels that he ought not to accept any proposition as fully demonstrated until it shows that it can win uncoerced agreement on the part of other competent persons, or regards this kind of appeal as irrelevant, is a question of his controlling ends, and what they imply with respect to the proper way of defining "demonstration" and "verification." Shall verified truth, for him, be something sharable, so that unless it is persuasive to other intelligent minds he will refuse to accord it the status of truth; or shall he be willing to give it this status even though it is unable to approve itself to other competent thinkers as it does to him? The decision of the scientist is that the former answer is the correct one, and in making it he is, so far forth, committing himself to the criterion of social impartiality, of which it is the main expression in his field.

We saw earlier, by implication, that this is likewise the case with the other two criteria. It became clear that whenever the fifth criterion is relevant, it applies without restriction by any considerations to which one might be led under the guidance of the third or the fourth, and that they have their freedom only in problems to which it is not relevant. This implies that in situations in which they and it are alike involved, the fifth is the controlling one, and that any question concerning the other two which arises in such situations should be answered in the manner which it would prescribe. What does this principle concretely mean?

Let us consider its bearing on the fourth criterion—the implicit theory as to how our ultimate value-concepts should be defined. We have seen that the major logical difficulty arising in this connection lies in the fact that serious thinkers differ in the theory which they adopt and hence disagree in their interpretation of this criterion. Moreover, their rival views, as must now be noted, not only clash theoretically but sometimes lead to social conflict as well. My neighbor, resting upon his theory as to how "good" or "happiness" or "satisfaction" is

to be identified, may commit himself to an end whose pursuit in action will violate what I, resting upon my theory of these concepts, regard as my rights. How is this difficulty to be met? Well, if the fifth criterion is the controlling one, the obvious answer is that such disagreements are to be resolved by the method of democratic cooperation—the method of allowing a free field for all proposed definitions of end-concepts to reveal their merits, and of reconciling them on an impartial basis. One who sincerely accepts that method will want the theory to which he is predisposed, and the notion that he now has about his rights, to enjoy no special privilege merely because they are his. If, for example, he now favors the view that the good is to be defined as happiness, he will want that view to be critically assessed along with others, and to undergo whatever revision such critical assessment shows to be needed. His guiding assumption is that the right definition is not one that could be prescribed in advance by him or by anyone else; it will emerge only in the course of this impartial, cooperative process. In the two final chapters we shall analyze in some detail a conflict of this sort between ultimate value-theories, and show how the method of democratic cooperation would be applied to its resolution.

Let us consider its bearing on the third criterion—the specific value-total envisioned as commanding in any attempt to determine one's end. We have seen that this criterion depends on the fourth, in the sense that when one has to choose between two or more suggested modes of action, each with its own set of forecasted consequences, he cannot tell which set contains the value-total that *is* commanding without being guided by some theory as to what, ultimately, distinguishes good from bad—how good is to be identified in experience. Hence, whatever procedure he accepts for determining that theory will also indirectly determine, so far as he is consistent, the specific criteria as they function in the various problems which he attempts to answer.

One who is sincerely committed, then, to the way of democratic impartiality will inevitably find that these other criteria

take their form under its guidance and are determined in accordance with its demands. It becomes, in effect, a supercriterion, in the light of which all other evaluational criteria gain their meaning; they constitute, with it, a coherent, unified system.

Viewed thus, the conception of democratic cooperation plays, in the field of end-judgments, a role analogous to that of a cosmic theory in giving complete unity to the body of factual science. All end-judgments, when determined aright, fall into a pattern, each rendered harmonious with the others by their common acceptance of its authority. It provides at present the rational standard for filling this comprehensive role. In time it will be superseded, but appropriately so only by a supercriterion which includes its distinctive principle and adds something more that in the course of experience proves to be needed.

Two cautions regarding the way of democracy

But since it has been said boldly that democratic cooperation is rational while any procedure inconsistent with it is irrational, two cautions must be added; if they are forgotten we may easily deceive ourselves or be deceived by others when attempting to tell where democracy is actually at work, and where it might seem to be but is not.

The first is that democracy is present, and is being furthered, only where cooperative procedure is at work. Any method for adjudicating the determination of ends can succeed, and can secure wider acceptance than it now enjoys, in the long run, only through means that are harmonious with the way of social life which it betokens. Democracy is no exception. It can temporarily preserve its existence against external threat by nondemocratic means, but it can expand in the number of persons genuinely committed to it only through democratic means—only, that is, through its power gradually by the persuasion of its actual exemplification to make itself accepted in competition with rivals that are always aggressively asserting themselves against it. Where other means are used, except as temporary expedients, something else is being promoted instead of democracy and its coopera-

tive method. The second is reflected in the adverb "genuinely," which was employed with set purpose. Not only is none of us immune to the subtle solicitation of authoritarian methods; we may succumb to them, and still call ourselves democratic when our actual ideal has become a quite nondemocratic one. We may do this without being clearly aware of the difference. But it may be done on purpose for, like any appealing word, "democracy" can be consciously used as an instrument of propaganda. Just as advertisers can exploit the thought of friendship for the sake of private profit, and missionaries can preach the gospel of love in a most hateful manner, so the appeal to respect for the equality and freedom of all men can become a spellbinding technique, hypocritically covering the most appalling violations of its spirit. And democracy must constantly struggle against inertia as well as against temptations to exploitation. In each new human situation novel values may be at stake, fair consideration of which demands an altered perspective on old values; in such cases all that is needed for the principle of friendly cooperation to be violated is blindness to the new goods that have become possible —persistence in the attitudes and evaluations become habitual in the past.

Democracy is as democracy does; and friendly cooperation is realized where friendly cooperation is going on.

EXERCISES

1. Explain clearly why reasoning about ends must avoid self-contradiction, and in just what sense it must respect factual evidence.
2. Just what is the relation between the two criteria that are distinctively applicable to judgments of prudence?
3. Why, when determining under which of the three classes mentioned any case of choosing an end falls, is it rational to consider them in the opposite order from that followed in their analysis?
4. Exactly what is meant by "democratic cooperation in choosing ends"? Why is it more rational than any alternative?
5. Explain fully the following statement in the chapter: "Reason

appears as an interpersonal creator of mutual understanding and sincere agreement rather than as a merely individual faculty."

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CHAPTER 27

HOMOGENEOUS ENDS AND HETEROGENEOUS ENDS

Introductory considerations

We have seen that the special logical difficulties which arise in connection with end-judgments are revealed most vividly when one considers the pragmatic and syntactic meaning of the concepts in terms of which those judgments are expressed. But the possibility of bringing the various criteria which apply to such judgments into harmony under control of the principle of democratic impartiality gives a clue to the rational resolution of these difficulties.

As for the pragmatic meaning of end-concepts, there is apparently no way of freeing them from vagueness and ambiguity. These characteristics arise from their distinctive function. But they lead to no irrationality in the pursuit of ends, provided that all persons stand on an equalitarian basis in the process, none receiving any special privilege for himself as against others. Then the ends which emerge from this vagueness and ambiguity sufficiently to become concretely realized in the course of social living will be such as take into impartial account the needs and interests of all members of the community. And the crucial test, revealing whether or not individuals are willing to accept this equalitarian relation with others, appears when we consider the syntactic meaning of end-concepts. Any individual's way of choosing ends implies some theory as to how the most general end-concepts are to be defined, *i.e.*, how to identify their referents in experience. But different thinkers adopt, explicitly or implicitly, different theories on this vital matter, and this circumstance not only creates theoretical disagreement but sometimes leads to

social conflicts too. We clearly need, therefore, a way of rationally adjudicating these theories when they come into conflict in this fashion, and the proper way will consist in using the method of democratic cooperation to resolve any such antagonism. Even, indeed, when social conflict is not threatened, a thinker can hardly help but feel a challenge in the fact that other people seem sure of quite different theories than his own, and some concern to find the proper basis for reconciling them. If he wholeheartedly accepts that method for dealing with these differences, we may be confident that he is sincerely content with the status of equality, and that his procedure for determining ends will throughout be kept in impartial relation with the determination of ends on the part of others.

What are the more influential theories of value that compete with each other in the Western world today? Several of them have already been mentioned for purposes of illustration; now let us look at the main types under which they may fall more systematically. A good way to classify them for purposes of guidance in such a survey is to do so under three main heads: there are the psychological theories, the realistic theories, and the relational theories.

The psychological theories of value

The psychological theories locate the essence of value in some characteristic of the individual or individuals who find an object valuable—some factor in his or their psychology, in short. The most influential current theory of this type maintains that the essential ground of value is the factor of interest; value is "any object (of) any interest."[1] Thus, if I find value in a glass of beer, the value according to this view is no quality of the beer itself, comparable to its golden brown color and its fluidity, but is constituted by the fact that when thirsty I want it—seek it when it is absent and drink it with zest when it is present. Similarly, if after comparing the career of a university teacher with that of a doctor I choose the former, its superior value is constituted by the fact that when I see as clearly as I can all that is involved in each alternative, I de-

[1] R. B. Perry, *A General Theory of Value*, p. 115 f.

sire the kind of experience that promises to be realized by that choice more than the experience to which the latter would be a means; it is nothing attaching to the activities of a university teacher as such. Thus, from the standpoint of this theory, value is something radically different from properties of objects like size, weight, color, hardness, sweetness, etc. Those belong to the object, while value is to be located in the individuals who find the object interesting.

But "interest" and "desire" are very general terms; moreover, people who agree on the legitimacy of defining value by their aid may disagree on the question: What are the dependable characteristics of this aspect of our experience that we refer to by the words "interest" and "desire"? There are psychological theories that attempt a very specific answer to this question. Two such theories which have exerted considerable influence in modern times are hedonism and self-realizationism. The hedonist holds that we always desire pleasure and shun pain. Hence, for him, "goodness" is ultimately to be identified with happiness, conceived as enjoyment of pleasure and avoidance of pain, while "badness" is to be identified with pain, and absence of pleasure. Utilitarianism is a prominent modern school which espouses hedonism, and also insists that all human beings are equal in their right to enjoy pleasure and freedom from pain. Self-realizationism finds the essence of value in the kind of experience indicated by this phrase. Its champions insist that in the end a man's interest is in growth, in expanding his selfhood to include the realization of as many as possible of the potentialities hidden in his nature, and that he is quite willing to accept pain provided that it contributes to this process of expanding growth.

So, when confronted by any problem of end-determination, one who holds any of these psychological views will naturally attempt to solve it by turning his attention to the human interests or feelings which, as he sees it, constitute the values in question—provide the material whose examination will yield the needed clue to a wise choice. His criterion tells him that he must find it there.

According to the realistic theories, the value of an object does not depend on desire or any other mental factor, but is a feature of the object itself. An influential current theory of this kind regards value as an unanalyzable and hence indefinable quality of an object or event, belonging to it in just the same way that a simple attribute like redness does. Thus, for such a theory, the goodness of beer is a simple property of the beer, comparable to its golden brownness. The fact that some people desire it and some do not is irrelevant, he thinks, to its goodness, just as the fact that a color-blind individual cannot discriminate redness as others do, does not prejudice our conviction that redness is a real quality of the red object. The realistic theories

A holder of this theory, when dealing with any problem involving the choice of ends, will of course turn for his clue to the valued object rather than to the desires or aversions that he and others feel in its presence. He will expect to solve the problem by a careful examination of the object, locating more precisely this property of value and discriminating it more clearly from its context.

In the field of ethics there are important realistic theories which regard "right" and "wrong" as the fundamental ethical concepts rather than "practical goodness" and "practical badness." For them, a certain rule of conduct has absolute authority over our actions; any act that is consistent with it is thereby shown to be right, and any act inconsistent with it is thereby proved wrong, irrespective of what our psychological attitude toward these acts might happen to be. The most influential theory of this kind during the last century and a half is that of the German philosopher Kant. According to him, the Golden Rule, in a rather abstract formulation of it, provides the criterion by which conduct can be confidently judged in this way. The view that beauty is "unity in variety" is an instance of a realistic theory in esthetics. It tells us that we are to find the essence of esthetic quality in that feature of the beautiful object in virtue of which its distinguishable parts are bound together and harmonized into a coherent whole.

It is evident that the main strength of the psychological

theories lies in the fact that they can take account without difficulty of the circumstance emphasized in Chapters 23 and 25, namely, that values appear to vary as between different individuals and different groups. By locating in these individuals any value that presents itself, such variations seem capable of accurate explanation. The main strength of the realistic theories lies in their ability to respect the fact that a value spontaneously appears to belong to the object valued, just as its color or hardness does. It seems to be a feature of whatever our interest is focused on, rather than something dependent on our interest. It also respects the fact that many objects seem to be valued in the same way by all normal people, at least after mature discriminations have been developed or trained.

The relational theories

Because of these elements of strength on both sides, a third type of theory, which attempts to take account of the virtues of psychological and realistic theories alike, appeals to many thinkers. This type consists in the relational theories of value. According to them, value is not to be located exclusively in the psychology of the interested individual, or in the object or event that is valued. It lies in the network of relations by which the individual and the object are bound together in the value-situation. Upholders of these theories believe that an important consideration has been given insufficient weight by both the psychological and the realistic viewpoints, namely, that the value of any object depends on other properties that it possesses and also on specifiable needs of those who find the object valuable. When this consideration has been duly taken into account, they think, it becomes evident that value is a function of some (usually rather complex) relation between certain features of the object and certain characteristics of the responding individual. The value of beer may again serve as a helpful illustration. If one finds beer good, he does so in part because he is thirsty, or at least capable of thirst—this is the essential factor on his side of the relation, since beer would obviously lack value (the value of a drink, at any rate) were there no such thing as a thirsty organism in the world.

On the other hand, he does so in part because beer has certain qualities which make it capable of satisfying thirst—its liquidity, its smoothness, its mellowness, its power to nourish, and the rest. Besides these there is, of course, the simple taste quality or tang which is immediately liked, but the value of the beer clearly depends on these other properties too, and their relation to the organic demands which they can meet. Eliminate these features of the object, and its value would disappear just as completely as it would if the subjective need were gone.

Since these relational theories are of quite recent development, it is hardly necessary in their case to dwell upon differences between one relational theory and another. The general characterization just given of the group as a whole will be sufficient for our purposes.

Practical differences resulting from adoption of varying theories

The important circumstance that when one engages in a process of choosing between competing ends he will often reach a different result by following one of these ultimate theories than he will by taking the guidance of another, has already been illustrated. But a further instance, in the present context, will be appropriate. Consider the problem that one might face when visiting a very sick friend. The sufferer does not know, let us say, how serious the illness is, but the visitor knows; shall he speak the truth in response to the sick man's insistent question, or shall he tell a lie? Well, those who believe that value is determined by the interests or desires present might easily be led to the conclusion that the decisive factor in this situation is the mutual desire for the friend's recovery, hence that the recovery hoped for is the essential value to be considered. If, then, it seemed likely that a lie would give him a better chance to recover than the discouraging truth, there would be no hesitation about telling the lie; it would seem the act of highest value under the circumstances. On the other hand, a person who believes that value is a real property quite independent of any desire would probably turn his attention to the contrasting assertions that might be made in this case—the true one, and the lie which suggests

itself as an alternative. Examining them, it might seem to him clear that the decisive consideration is just the fact that one statement is true and the other false; the criterion that then appears valid would emphasize respect for truth as more crucial than any other value at stake. If so, he would tell the unvarnished truth, and let the consequences be what they may. He would say to himself that truth is always good, while the attempt to forecast the desirable or undesirable effects of telling a lie is much more uncertain. As for believers in the relational theory of value, they would, of course, in this situation, pursue their inquiry in two directions. They would note those characteristics of a true statement that make it ordinarily superior in value to a false one; they would also note the human interests that are at stake in the particular circumstances now at hand. Clarifying the relation between these, they would try to see what value emerges as the commanding one from that whole relational pattern. This would be the decisive value for them. It might, of course, bid them tell the truth; but if so, the truth would be told for a different reason than the one which would be given by a proponent of the realistic viewpoint. It might be a lie; but if so it would very likely be a different lie than would be told by a follower of one of the psychological theories, for it would be told under the guidance of a different set of controlling considerations.

In the next chapter we shall examine, for the purpose of providing a detailed illustration of the clash of value-theories, and of how the cooperative method can be used to resolve it, a conflict of views which is of special contemporary importance. It is between a psychological theory, namely, the view that finds the essence of good in individual happiness, conceived as implying a maximum of freedom to do what one likes, and a relational theory, which finds the highest good in a pattern of interdependence by which individuals are functionally related to the social institutions of which they are members. These two theories justify social policies and programs that are radically different and, at certain crucial points, incompatible.

The distinction between homogeneous and heterogeneous values

But we have not yet faced the difficulty arising from the distinctive nature of end-judgments in their most complex form. In order to remedy this defect, and to complete our preparation for the detailed illustration soon to be considered, we must turn to a division of values mentioned in Chapter 25 which has not yet been put to use. This is the division between homogeneous and heterogeneous values. It will be necessary to identify more fully typical cases which fall under each of these heads; the logical nature of end-judgments in these two fields will have to be analyzed; and the special problem about the rationality of end-judgments which thus comes to light will need to be clarified.

First, let us remind ourselves of the nature of this distinction between homogeneous and heterogeneous values, with sufficient elaboration so that it can serve as a guide in our present enterprise. On what does this distinction rest—that is, what does it assume about the values with which we find ourselves from time to time dealing?

Well, according to classical value-theory the three ultimate, intrinsically distinguishable values are truth, practical goodness, and beauty. And without committing ourselves to any particular theory about these, it is possible to give a helpful brief description of the distinctive context in which each of these three sorts of value presents itself. For people find themselves standing, in any given situation, in one or another of three relations to whatever objects they happen to confront, and each of these relations is reflected in a characteristic attitude toward those objects.

When one's attitude is predominantly theoretical he is endeavoring, so far as in him lies, to conform his thinking to the nature of his object, taking it for granted that it has some specific nature which can be understood in terms that do not reveal any dependence on his variable personal equation. He wants to know, in short, what the object really is, quite apart from anything else that he may propose to do or expect to feel in connection with its presence. This attitude is most clearly exhibited in the theoretical scientist when pursuing

his proper task, and truth is the final value at which anyone aims in situations of this kind.

When one's attitude is predominantly practical, the situation is reversed. Here he is striving to impose on whatever objects are involved some preference of his own; his relation to them takes the form of an aggressive effort to bring them into harmony with a purpose they would not otherwise serve. Instead of conforming his mind to their given nature he is trying to transform them in accordance with some aim which he is eager to realize—they become means to his chosen end, so far as he can render them such. Now it is this kind of situation, in which interested action—very different from the mere endeavor more clearly to reveal the nature of objects—is contemplated, to which the concepts of ethics are relevant. We judge such acts as right or wrong or as good or bad—in the restricted practical sense of "good" and "bad" rather than the broader sense in which "good" is identical with "valuable" in general and "bad" with its opposite.[2] We also judge in the same terms a person's attitude, habit, disposition, or character—holding them to be good if they are such as naturally express themselves in good acts, and bad if they are naturally expressed in bad acts.[3]

There is a third attitude which is most readily indicated by the word "appreciative." The distinctive feature of the situation in which this attitude prevails is that in it there appears a spontaneous, effortless harmony between our chosen value and what we find confronting us. Without earnest endeavor or aggressive action, such as is normally involved in the quest

[2] This ambiguity in these value-concepts is unfortunate, but since it is entrenched in traditional usage one has to accept it and explicitly remove the ambiguity so far as his own usage is concerned. Wherever there is danger of misunderstanding without doing so, I shall modify "good" and "bad" by the adjective "practical."

[3] According to certain theories of ethics, this characterization of the field is too broad. In their view, the field of ethics is restricted to those practical situations in which competing ends appear, or even in which there is felt conflict between the demands of duty and the pull of inclination. For our present purpose it is unnecessary to decide between the broader and the narrower definitions, but I shall assume provisionally that the broader one is legitimate.

for truth and in the active molding of means to make them serve our ends, an object suddenly appears before us—a flower, a landscape, a mountain peak, the heaving sea—in the appreciative perception of which we find a spontaneous satisfaction. And it is not only scenes in Nature that enter into this relation; the phrase "fine art" reminds us that there are human activities specifically devoted to the artificial creation of objects capable of arousing such appreciation. It is situations of this kind to which the concepts of esthetics apply. The most general terms by which we refer to the contrasting values here are "beauty" and "ugliness"—the former, of course, denoting positive esthetic value and the latter negative. But, as in ethics there are terms referring to more specific kinds of goodness in conduct or character than the word "goodness" itself does, so in esthetics there are terms referring to the particular sort of beauty or its opposite that is appropriate to particular contexts of appreciation—terms such as "grandeur," "charm," "loveliness," "harmony," and the like, with their opposites.

Judgments about homogeneous values

Let us now locate the important similarities and differences between judgments about homogeneous values and judgments about heterogeneous values, with the special problem in mind, how to render the judgments as rational as possible in each case. The reader will recall that homogeneous values were defined as those which fall completely within one or another of the three fields of values just described, while heterogeneous values consist of those which overlap two or more of those fields. We shall begin with judgments of homogeneous value, and since judgments claiming the particular homogeneous value of truth, *i.e.*, scientific propositions, have already been examined in detail, there is need here only to deal with the second and third of the value-fields just distinguished. This means that we shall engage in a more thorough analysis of the judgments in which properly ethical or esthetic concepts are employed.

The primary distinction quite necessary in this analysis is that between the particular judgments in which these concepts appear as predicate attributes and the general judg-

ments of ethical or esthetic theory in which they appear as subject terms. It has been noted already that judgments regarding less general ends depend for their meaning on judgments about the most general ends. We are now to see the nature of this dependence more exactly, in the case of homogeneous ends. Typical of the former class are such judgments as the following: "That act is despicable"; "His deed was very courageous"; "John's attitude on that occasion was wrong"; "Mary has a beautiful character"; "Your proposal is a fair compromise"; "Those snow-capped mountains are majestic." The authors of these assertions are in each case attributing ethical or esthetic value, positive or negative, in some particular situation to which, as the above paragraphs indicate, such value-concepts are appropriate. Their purpose is obviously to identify specific instances of these values. Among judgments of the latter class are these: "Justice is that ethical virtue which consists in giving every man his due"; "Grandeur is the beauty which objects reveal when they arouse a sense of vastness"; "Sincerity is of greater value than temperance"; "Practical goodness is self-realization"; "Beauty is unity in variety." And judgments of this class need to be divided further. The first two among those just mentioned are definitions of ethical or esthetic terms of limited scope by employing as genus the broader concepts "virtue" and "beauty." Their aim is thus to fix the meaning of those terms by specifying in just what kind of ethical or esthetic context they are properly applied. The third is a comparative evaluation of two ethical virtues, stating that one of them stands higher than the other in some explicit or implicit scale of moral values; such a judgment might appear as part of a systematic exposition and defense of the hierarchical system of values assumed. The last two judgments are proffered definitions of the most inclusive and ultimate concepts of ethics and esthetics, telling us how and where they are to be identified in our experience. The proposed definition of practical goodness tells us that, in any case of determining an ethical end, the imagined consequences that are relevant to guiding choice are those in which

one's potential selfhood promises to gain full expression; were the definition different—for example, "practical goodness is pleasure"—we should be asked to find our clue in somewhat different experiences, namely, particular sensations of pleasure or pain. The suggested definition of beauty tells us that, in deciding what is or is not an instance of positive esthetic value, our judgment should be guided by the specified character of the object; another definition would have told us that a different guide to evaluative analysis would be the appropriate one.

How particular judgments depend upon those of general theory

Let us concentrate now on the manner in which the particular judgments depend upon the judgments of general theory. Consider again the judgment "That act was despicable." Presumably no difficulty will arise about the subject of this judgment, that is, in identifying clearly and objectively just what act is being referred to. But difficulty may well arise about the predicate. What, exactly, is meant by "despicable," and what kind or range of acts does this adjective properly cover? Obviously, answers to these questions will take the form of judgments of general value-theory, such as were illustrated in the second class of cases just discussed. Similarly, special importance attaches for the same reason to judgments which define the most inclusive concepts employed in any value-field, such as the last two judgments above considered, defining "practical goodness" and "beauty." For the less inclusive concepts will be defined, as the above instances show, by judgments employing as genus these quite inclusive concepts, and it is only when we come to definitions of the latter that the ultimate court is known, to which one employing these concepts intends to appeal for determining in some cooperatively understandable way the meaning of the entire hierarchy of value-concepts. A definition of "despicable" would doubtless include this term under the genus "practically bad," and only when one has defined "practically bad" by indicating how to identify it in experience has he pinned down in intelligible form the meaning of the entire set of value-concepts from the most specific to the most general.

Judgments of heterogeneous value

Now let us engage in a similar analysis of judgments of heterogeneous value, that is, judgments about values which, in virtue of their meaning, overlap two or more of the three fields of value. These include the values with which one is typically concerned in such subjects as law, religion, and metaphysics. It will have been noted that the concepts referred to when heterogeneous values were illustrated in Chapter 25 were drawn from these disciplines. And again it is necessary to distinguish between particular judgments which employ concepts of heterogeneous value as predicates, and general judgments of theory in which they appear as subject terms. But in this case it will be well to clarify a few of the typical concepts involved before pursuing the distinction. By doing so we shall bring into bold relief the reality of heterogeneous values and the special problem which judgments about them face.

The characteristic concepts of legal thinking, for example, are concepts of heterogeneous value, because their definition involves a reference both to ethical ends and to truth concerning facts. By saying that they refer to factual truth, it is meant, of course, that they do so not merely in the general sense in which any end-concept refers to given materials whose nature must be respected if the end is to have any chance of realization, but in the more specific sense that it is part of their purpose to try to determine which among competing judgments conforms to certain specific facts that all recognize to be relevant. This point will become clear if we consider the difference between "justice" in its ethical meaning and the same word in its legal meaning. The ethical concept of justice has factual reference in the general sense that its realization can come about only in and through the social relationships of people, which exist as present actualities. The legal concept of justice not only exhibits such general factual reference as this; it also contemplates the circumstance that constitutions and legislative enactments have authoritatively decided what kind of act is just in many social situations. Accordingly, any judgment employing the legal con-

cept of justice is responsible, among other things, to conform to these established decisions, while an ethical prophet accepts no such responsibility at all; he may, indeed, regard the legal interpretation as radically unjust and demand, on ethical grounds, its replacement by a quite different interpretation.

Is there any doubt that legal terms also refer to ethical values? If so, consider a legal concept that, at first sight, might appear to be merely descriptive. One might think that "murder" is simply a term of factual description. But when the difference between "murder," "manslaughter," and "accidental homicide," as these words are employed in legal thinking, is examined, it is evident that they are terms of evaluation as well as of description. When any of them is used, one person has (supposedly, at least) killed another; here is the requisite factual basis for their application. Also, as far as possible, legal thinkers try to discriminate clearly the experiential differences between the situations to which each applies; murder is premeditated killing, manslaughter is killing on sudden impulse, etc. It might seem as though no difficulty would arise in identifying any case of killing under one or another of these concepts, and indeed there is none in the simpler cases where the evidence is clear. But many cases are not so simple; when the situation is carefully examined, it appears that the evidence is not coercive with respect to any given classification, and then choice is made by giving attention to the comparative values involved. This is quite obvious in a situation in which, let us say, a doctor, contrary to law, has terminated the life of a sufferer from incurable cancer. Should this act be called murder? It was clearly premeditated, and yet it was done from a humane motive and was presumably approved by the one whose life was ended. With these circumstances in mind, one strongly hesitates to apply such a concept with its inevitable associations. "Murder," then, is seen to be a value-concept and not merely a fact-concept.

Legal concepts prove to denote heterogeneous values

Study of such an instance enables us to understand legal situations that might otherwise be very puzzling. Why do

juries so frequently bring in a verdict of manslaughter when the facts as revealed in the trial would just as easily, if not more easily, have justified a verdict of murder? It is because they are convinced that certain mitigating circumstances are present, not specifically recognized in the laws applicable to the case, which make the penalty for murder harsher than would be equitable. Suppose now that we say to the jurors: "See here, it is not your business to revise the law; your sole duty is to determine just what law was actually violated in this case. If the facts are according to the law describable as a case of murder, then it is your business to apply that concept to them, not to refuse to do so because you do not think the guilty person ought to suffer the prescribed penalty." But if the jurors have philosophized somewhat about their responsibility, they will have a ready answer to this plea. "In establishing the laws about murder and manslaughter, legislators were not simply describing facts but were attempting to secure certain social values, and they framed their definitions so that, in the typical cases before their minds, these social values would be secured by closely following the letter of the law. They did not, however, anticipate a case involving these peculiar circumstances, and we see clearly that to apply here the letter of the law would conflict rather than harmonize with the larger values that the authors of the law were most concerned to realize when they established it. The spirit of the law is fulfilled by calling this manslaughter rather than murder." In short, murder and manslaughter are value-concepts as well as concepts of factual truth. Not only does each attempt truthfully to describe a certain social phenomenon, but also its use in a legal judgment implies some estimate of the heinousness of the act and some decision as to the penalty that would be appropriate to safeguard the social values at stake.

A similar conclusion is enforced when one examines another set of legal concepts. The United States Constitution gives to Congress two main responsibilities. On the positive side Congress is authorized to legislate in such ways as will secure the

health, safety, order, and welfare of the people. On the negative side it is forbidden to invade the guaranteed rights of individual citizens, such as the right to life, to various specified liberties, and to the peaceable use of one's property. Now these concepts, "health," "safety," "liberty," "property," "right," and the rest, are obviously value-concepts; they denote desirable ends which were agreed upon by the founders of our Republic as of paramount concern. This is clear from our earlier discussion of the first and third terms in this list. But, as dealt with in law, they do not merely refer to values of social ethics. They have a reference to factual truth as well, in the specific sort of truth reference which applies to these concepts. A legislature is not free to pursue these ends in any way it might think desirable. It is bound by the established sense of the community as to the pragmatic meaning of the concepts referring to them. This sense is expressed in the constitution and in prior judicial decisions which act as restrictive precedents, limiting the avenues of action that are genuinely open to legislative consideration. These precedents exist as present facts, which must be respected by any new application of the concepts in question. It is not that precedent is absolutely coercive, since any precedent may be subject to varying interpretations, but it does impose a degree of consistency and of continuity with decisions made in the past that would otherwise be unnecessary. The legal meaning of words referring to these ends, then, combines reference to the envisioned ethical goods with responsibility to conform to the factual embodiment of the community's notions as to how those goods may properly be sought.

Consider especially the two concepts "property" and "right," in their legal significance. Their factual reference is perhaps more obvious at first sight than their ethical meaning. "Property" refers to definite possessions, tangible or intangible, that are legally under a person's control; "right" refers to certain specific immunities and privileges that are guaranteed in the constitution and have been protected by countless judicial decisions. But control of property and enjoyment of these

rights is never absolute. It is limited in manifold ways from time to time, and the limitations clearly reflect a sense of the essential responsibility of such privileges to the social good. A man controls his property, yes; but he may not use it for illegal purposes, and he must pay on it whatever taxes the government may see fit to assess. A man enjoys the right to life, yes; but he must endanger his life as a conscripted soldier if the government so orders in the interest of national safety. These limitations show that property and right are subordinate to what the social good requires; they must therefore be understood, in part, in terms of the community's pursuit of that good. They indicate certain aspects of the end which the community has chosen to seek, reflecting the circumstance that social well-being demands protection of the individual in certain ways and also his obligation to serve the needs of the community in certain ways. There is, hence, a difference between "property" and "possession," the latter being the term to use when mere description of a person's actual control over something is wanted. And "right," in its legal meaning, is a translation of the Latin *ius*, which, according to Dean Pound, was a "convenient ambiguity, lending itself to identification of what ought to be and what is." [4]

Here, also, particular judgments depend on those of general theory

So legal judgments are clearly judgments of heterogeneous value. But the difference between particular and general judgments in this field plays a role that is analogous to the difference noticed above in the case of judgments of homogeneous value. "This act is manslaughter" is a particular judgment employing a concept of heterogeneous value as predicate. Similarly with "Use of this property as a grocery store is your right," and "This legislative act unreasonably restricts the liberty of such and such citizens." On the other hand, "Manslaughter is a crime," and "Crime is an omission of a duty commanded, or a commission of an act forbidden, by a public law," are general judgments of legal theory. And it is clear here, too, that whenever any dispute arises about the meaning of "manslaughter" and the situations in which the

[4] *An Introduction to the Philosophy of Law*, p. 31.

term may be properly applied, the disagreement can be resolved only by appeal to such general judgments as these. Indeed, in the end we are brought to the definitions of the most general legal terms, which indicate the phase of human experience to which we are supposed to turn for our ultimate foundation. Now we have seen that different definitions of the most general ethical and esthetic concepts are championed by different schools of theory, and the same is true in the case of definitions of these inclusive legal concepts. Hence, the task of distinguishing which among competing value-judgments is most rational, in this field as well as in those, leads inescapably to the problem with which our remaining chapters will be occupied, namely: How can we adjudicate the issues which divide these various schools in a cooperative and impartial way?

Concepts of heterogeneous value illustrated in religion

The contrast between judgments of homogeneous value and those of heterogeneous value will be clarified in another way if some illustrations from the field of religion are examined. When the idea of God was briefly discussed, in Chapter 25, it was pointed out that all three realms of value are involved in the meaning of such a concept. The same circumstance appears when any other distinctively religious concept is analyzed, for (in the Western world) the meaning of all of them depends on their relation to the concept "God." This is what distinguishes them from concepts referring to the same experiential material but referring to it in a nonreligious context. God, however, is a more complex entity even than a heterogeneous value; in order not to anticipate considerations to be introduced in due time, let us pick as our present illustration the concept of "sin."

Sin is a heterogeneous value, of greater complexity than the heterogeneous values in the field of law. When a piece of conduct is characterized as a sin, it is not merely condemned from a moral or legal standpoint; it is judged in its relation to God, as the supposed ultimate ground of all types of value that men experience, and its meaning is affected accordingly. A man, let us say, defames another by abusive

language. If we view the situation merely from the standpoint of the science of psychology, here is an *act,* that is, an item of behavior of a certain sort, capable of description in exact and objective terms, among which "sin" is conspicuous by its absence. From the point of view of ethics, it is an act that is *wrong;* use of this further category implies the passing of an adverse ethical judgment—the deed is morally valued in a certain way. Legally, it is presumably an *injury* which, as such, is capable of adjudication, with attainment of appropriate redress, through the courts of justice. In neither of these two fields, likewise, will the category of "sin" appear, except in communities which have not yet detached law and morals from the religious matrix in which, in all social groups, they were at one time embedded. When the act is called a *sin,* the moral aspect of an adverse evaluation is preserved. But something distinctive is added. The act is now judged in its religious meaning; it is viewed in its relation to God, as the being who gives coherent and final unity to all facts and all desirable ends. Since God is believed to be the source of beauty in the world, to call an act sinful is to attribute a quality of ugliness to it. Since ultimate control over all facts is believed to lie in His hands, an act expressing failure to recognize this truth has an element of falsehood in it. The criterion that is assumed here varies, of course, with the theory of God adopted. If it is taken for granted that God is a personal being who has given us His code of commands as a superhuman legislator and judge, the sin will consist in a violation of one of those commands, *e.g.*, "Thou shalt not bear false witness against thy neighbor." If He is conceived in some other way, the sin will consist in whatever is implied by that conception, *e.g.*, as a violation of the duty to treat other men as our brothers, arising from the dependence of all alike on the fatherhood of God. But however it may be interpreted in detail, "sin" will exhibit a religious meaning, and to assert that any act is a sin is to view it as a heterogeneous value of the complexity characteristic of religious values.

It is hardly necessary to pause at length over the distinction between specific and general judgments of heterogeneous value as it applies in this field, for the analogy with which we are now familiar carries over in every essential respect. "That act is sinful" is, of course, a specific religious judgment. Should any question arise as to the meaning or appropriateness of the predicate term, it can only be answered by some general judgment of religious theory, *i.e.*, of theology—"Sin is any violation of the will of God." And when perplexities appear about the interpretation of such a general judgment as this we shall have to appeal to definitions of the ultimate concepts of religion—to that part of theology which endeavors to explain in socially understandable terms what kind of being God is, why it is justifiable to affirm His reality, and by what procedure one determines what His will for man's conduct requires. But here, again, no one theory on these matters has gained general acceptance. Different religions, different sects within any religion, and different schools of theologians present us with varying theories. The problem how to render our judgments rational, then, in this field too, culminates in the task of applying an impartial method in resolving the issues between these varying ultimate interpretations.

Transition to the values of metaphysics

It will help us pass from religious values to the still more complex values of metaphysics if we pause to consider an implication of the circumstance that, for religion, God is believed not only to embody all three types of desirable value but also to exercise ultimate control over the world of facts. This means that He is assumed to have a certain factual character Himself; or rather, to express the situation more accurately, that He is thought to be a complex reality, uniting in ultimate harmony the world of fact with the three realms of value. The same holds, in essence, of the entity referred to by the religious man as his "soul," and the implication in question can be brought out most easily by examining the meaning of this word. That it does not refer merely to objective facts is shown by the circumstance that the

science of psychology, whose task it is to deal exhaustively with the facts of man's inner life, discovers no soul. In times past, to be sure, it did find such an entity, but those were precisely the times when science had not yet become disentangled from problems of religious evaluation; its method and findings expressed a particular theory of value, and through it were intimately allied with a positive and distinctively Christian faith. When science became emancipated from this commitment, developing a method by which facts could be verified objectively—that is, by which their causal conjunctions could be established to the satisfaction of any normal and competent person whether he is religious or irreligious, whether theist, pantheist, deist, or agnostic—it gradually became clear that psychology not only could no longer take for granted the reality of the soul but must definitely dispense with such a concept. The technical term now employed in science which comes nearest to "soul" is the term "personality," for this is the word a modern investigator is likely to use when he thinks of the circumstance that a human being is not merely a subject of study in terms of his distinguishable ideas, behavior patterns, emotions, and desires, but needs also to be considered as a unified whole. In common sense parlance the nearest factual equivalent is probably "self." Every person, we assume, has a "self"—that is, a distinctive entity which binds together all his acts by its characteristic sort of coherence and is, in some sense, the inner spring revealing its nature in them.

What, then, is a soul? Well, for the religious believer this concept refers to an existent fact; if he did not believe that he has a soul destined either to win or to miss salvation, the very core of his idea of himself would be radically changed. But it is also a value-concept. It is his personality, or self, viewed not merely as an entity that exists for the psychologist to analyze, but also as something participating, in accordance with its own selective choice, in the various ends which lie open to man's commitment and devotion. It is one's personality considered as capable of ethical relations with other per-

sonalities, as capable of creating beauty and responding to it when found, as capable of envisioning God and seeing all things that happen in the pattern assumed when they are unified by that vision. It is no accident that the situations in which "soul" still retains positive and definite meaning in an age increasingly dominated by the scientific habit of mind are situations arousing these capacities into action. A lover turns toward his beloved a "soulful" glance—that is, a glance which expresses all the generous outpouring of admiration, trust, and loyal affection that he has it in him to give. The performance of an otherwise great artist "lacks soul"—that is, it lacks rich and full responsiveness to the values that the author wished to disclose in his creation of the work performed. And even persons who are not themselves religious will often speak of men whose names stand out in the spiritual history of mankind as "great souls," referring thus especially to those whose vision of some unifying meaning in life was free from dogmatism, cant, unrealistic sentimentality, or conceit. The status of the concept, then, is fairly clear in relation and contrast to that of the concepts by which common sense and science denote the bare objective facts of man's inward life. It refers to a more complex reality.

Metaphysics and its category "reality"

But how, more precisely, are we to conceive such complex realities as God and soul? Are there, perchance, more complex realities still, and heterogeneous values of a more comprehensive order than any thus far considered? To answer these questions we must give brief attention to the field of metaphysics, whose central task is to deal systematically with the concept of "reality."

We may begin by surveying the main situations in which this concept has been employed in the preceding pages. In our very first chapter, we began talking the language of metaphysics, although we did not know that it was metaphysics then. It became apparent that reasoning is distinguished from other kinds of thinking by the fact that it accepts the responsibility to square itself with whatever realities the nature of the problem renders relevant. Since then we have discov-

ered that there are three major kinds of reality, each with distinctive features which affect the methods and criteria which reasoning must employ in dealing with them—formal relations, facts, and values. What, then, is the status of judgments about reality, and of metaphysics as the enterprise whose distinctive task is to establish reasonable judgments of this kind?

The answer to this question may be reached most simply by considering the difference and relation between fact and truth. Facts constitute one kind of reality, while truth is one sort of value which is itself a different kind of reality. But truth is the specific sort of value that essentially consists in the agreement of a proposition with the facts to which it refers, and because of this intimate connection between truth and fact people sometimes use these two words interchangeably. It is especially easy to talk about a true proposition as though it were a fact. Whenever there is doubt about what *is* the fact in any given situation, what we are directly dealing with is a set of alternative propositions claiming the value of truth about that fact. We now confront a parallel situation in connection with reality. Reality is the all-inclusive genus under which belong formal patterns, facts, and values (homogeneous and heterogeneous) as subordinate species. And the metaphysician is trying to reach judgments which will have the value of agreement with whatever reality is relevant. Thus reality itself, like fact, transcends the field of values, while agreement with reality, like truth, is a value, and an exceedingly important one. The difference between them is that truth, being agreement simply with fact, is a homogeneous value, while agreement with reality is a heterogeneous value, since reality includes formal relations and the various kinds of values along with facts.

As we noted in Chapter 7, until recently truth was generally defined as agreement with reality rather than as agreement merely with fact. But the contemporary trend strongly favors the narrower definition, and we have thus far been accommodating our terminology to it. Unfortunately, there

is as yet no generally accepted word by which to replace "agreement with reality" in this altered historical context, and therefore to denote the ultimate heterogeneous end which metaphysics seeks. Instead of coining a new technical term to serve this purpose, we shall content ourselves with that phrase itself, even though a briefer equivalent would be preferable.

The somewhat complex scheme of relationships which this analysis has endeavored to clarify will be helpfully exhibited by the following table:

- **Reality**
  - **Formal relations**
  - **Facts**
  - **Values**
    - **Homogeneous**
      - **Truth (whose essence is agreement with facts)**
      - **Practical goodness**
      - **Beauty**
    - **Heterogeneous**
      - **In law**
      - **In metaphysics (the major one being agreement with reality)**
      - **In religion**

Metaphysical categories

Now in the field of metaphysics there is a special word by which we customarily refer to the distinctive concepts that are appropriate to its universe of discourse; they are called "categories." Hence "reality" is the basic metaphysical category. It plays an analogous role in this field to that of "fact" in science, "good" and "right" in ethics, "God" in religion, and "beauty" in esthetics. How are we to understand this category when it is used to perform its appropriate function?

The metaphysician's universe of discourse is just *the* universe—that is, the totality of everything that can be significantly talked about, without limitation, in the most generous possible use of the word "significantly"; it is the sum of all realities. Thus the metaphysician finds confronting him, as material for his responsible consideration, the scientist's order of fact, natural and social, the realm of formal struc-

tures, and the various fields of value, heterogeneous as well as homogeneous. Now since problems arising within each of these fields are met without his distinctive aid, by the formal logician, the empirical investigator, the ethical philosopher, the theologian, etc., working according to their own appropriate methods, his role as a metaphysician consists essentially in dealing with problems that arise from the interaction of these various fields with each other. When theology conflicts with science as it historically often has, what is to be done, and by what ultimate criteria is a reasonable decision on such a matter to be reached? Or when theology conflicts with ethics, that is, assigns attributes to God that appear inconsistent with our highest ideal of what is morally good, by what standard may we wisely decide where the proper solution lies? When art conflicts with morals, demanding the right to pursue "art for art's sake," irrespective of how its creations affect men's conduct for better or worse, to what final norm should we appeal in reaching a reasonable adjustment? In the light of these illustrations it is clear that the metaphysician's task is a peculiarly challenging and difficult one. The work of the ethical theorist culminates in his effort to establish rational criteria by which to settle issues regarding the meaning of ultimate moral concepts, that of the esthetic philosopher in his quest for appropriate criteria by which to do the same for the distinctive concepts in his field; that of the theologian in his search for principles by which he can clarify, in the presence of rival theological schools, his basic concept of God and the other ideas that depend upon it. So the metaphysician, centering his attention on the foci of conflict and of needed adjustment between these various enterprises, is thus confronted with the unique task of trying to establish reasonable *criteria of criteria.* Where we need to know whether it is wisest to follow the guidance of a criterion drawn from ethics, one arising in esthetics, one taken from law, or one derived from theology, or whether some synthesis of these is possible and desirable, it is his business to give us such rational guidance as can be rendered avail-

able. Clearly, in order to do this he must seek criteria that are more ultimate than the criteria claiming to be ultimate in each of these fields, since what he hopes to find is to be used in reconciling conflicts between them. And in order to accomplish this task reasonably, his criteria must be stated in terms of concepts that apply impartially and in the same sense to anything whatever—form or fact, goal or means, homogeneous end or heterogeneous end—that can be sensibly discussed in any field. Such is the distinctive nature and role of the concepts that are called metaphysical categories.

Their role illustrated by a conflict between esthetics and ethics

In order to render it quite evident that the metaphysician's problems are genuine ones, and cannot be solved without the aid of special criteria provided by no other discipline, consider a situation that may at any time confront the creative artist. His primary aim is to produce an object of beauty, supplementing and enhancing the beauties that nature spontaneously offers him. Is he then occupied merely with esthetic values; are the criteria by which his achievement is to be judged merely esthetic criteria? Well, it might be thought that this is the case, and, indeed, when his product is finished and submitted to the judgment of connoisseurs in the branch of art concerned, such are the criteria that will be applied.

But his performance cannot avoid being viewed in another dimension,[5] that of ethics. His active creation of any work of art is a form of conduct, involving at every step a choice between alternative goods of conduct. Such choices are ethical. It is an ethical choice when he decides to create things of beauty rather than to engage in some other career; it is also an ethical choice when, as an artist, he decides to attempt this particular creation rather than some other that he might have tried to produce. For different achievements will have different consequences in his own subsequent experience and in that of other people; they will foster different attitudes, different feelings, and different courses of action. These are matters of ethical concern. He may neglect this aspect of his work, and justify doing so by pleading the slogan "Art for

[5] And doubtless still others, *e.g.*, religion.

art's sake," but this will not change the fact that he has chosen a course which is bringing about one set of social consequences rather than another.

Consider in this light, for instance, two different artistic portrayals of the same theme. Look at the accompanying photographs of two well-known statues of Abraham Lincoln. In their work on these statues the artists were obviously guided by radically different aims, and in each case the aim is subject to ethical as well as to esthetic criticism. It is subject to esthetic criticism because they expect the completed product to arouse appreciative feeling. It is subject to ethical criticism because that product elicits its natural response in the way of action as well as in the way of immediate enjoyment. So far as the artist himself or anyone who contemplates his work admires the figure thus portrayed, he is moved to imitate, in his own conduct, Lincoln's character as the artist interprets it. But that character varies markedly in the two cases, and therefore somewhat different kinds of conduct are encouraged. The idealization of the Great Emancipator indulged in by St. Gaudens emphasizes such qualities as strength, dignity, poise, and humane feeling. The more realistic representation by Barnard emphasizes such traits as homely simplicity, patience, and an anxious sense of heavy responsibility for the woes of the world. Each artist was, therefore, engaged in the promotion of certain distinctive ethical values as well as a certain distinctive esthetic result.

Naturally, the artist would like to forget about this ethical dimension of his performance. To take it into account is obstructive and confusing in his quest for a realization of high esthetic quality in the result at which he aims; he wants to be free to appeal single-mindedly to esthetic canons, unhampered by any moral responsibility for what he is doing. Hence his eagerness to adopt the principle, "Art for art's sake." The ethical prophet, on the other hand, naturally wants to make the ethical dimension completely determinative wherever he sees it properly to apply. So he demands that the artist accept a rigorous moral censorship over his work, accepting the ob-

Lincoln—By St. Gaudens

Lincoln—By Barnard

ligation to render his art an instrument for the realization of high ethical ends. His slogan is "Art for the sake of human goodness in character and conduct." This guiding principle, when followed without qualification, would issue in such a social situation as we are historically familiar with in the periods when Puritanism was in control—a situation in which such arts as dancing, the theater, and the opera were proscribed as essentially immoral.

The principles formulated in these contrasting slogans, then, when carefully examined, prove to be neither esthetic nor ethical canons merely. They claim to tell us what to do when both esthetic and ethical criteria seem to be relevant, and where obedience to the former leads in a different direction than obedience to the latter. What are they, then? The answer is clear: they are metaphysical criteria, though not explicitly expressed as such.

What, then, is the nature of metaphysical criteria?

Now we are in position to see why "reality" is the basic metaphysical category—why this word, together with its opposite, "appearance," as men employ them in daily life, provide the essential clue to what the metaphysician is about. Consider a few instances of their use. Do the railroad tracks really converge in the distance or do they only appear to do so—that is, would the judgment that they converge be verified if all the relevant considerations are taken into account, or does it merely reflect the limited perspective of the particular position from which I am now viewing those tracks? Is democracy really more inefficient than fascism, or does it only appear to be—that is, are its slowness, blundering, and waste intrinsic defects which would continue to appear as such when one takes account of its long-run purposes, its ideal of voluntary cooperation, and its commitment to the method of rational persuasion, or do they trouble us merely because we view it in too narrow a perspective, the perspective of an unqualified demand for immediate results? Well, in the same way the metaphysician is asking: Does such and such a solution appear to be the reasonable one only because we have confined ourselves to the limited and one-sided per-

spective of science, or ethics, or religion, or art when the question concerns more than that limited field; or is it really reasonable, *i.e.*, would its reasonableness be clearly maintained when we assume the broader perspective of the genus including all these fields? From the standpoint of one committed to the cooperative method, this would mean the perspective of impartial eagerness that each of these major enterprises of intelligent humanity should do its job in the best way that it might, with the widest scope for growth that is possible through harmonious cooperation with the others. And so his task finally reduces to that of searching for the criteria by which we can reasonably distinguish between reality and appearance in this quite unlimited realm of discourse. This is what is meant when he says that he wants to know what the total universe, in which all these great quests play their role for the guidance and enrichment of human life, is "really" like—what it becomes when we detach ourselves from the partial and unbalanced perspectives that give distorted "appearance" to the objects of our vision and see them from the vantage point of the whole.

The metaphysical category of "reality" is thus more inclusive than the scientific concept of "fact." The two are often confused because in many situations where the word "real" naturally occurs, one feels no need to appeal beyond the perspective assumed by factual science in order to solve his problem, and in such cases reality and fact are the same. Is the stick really bent in water, or does it only appear to be so? This question, like the one about the railroad tracks just mentioned, can be answered without going beyond the realm of considerations with which empirical science deals; here, "real" is equivalent to "factual." But the word "real" covers a much broader realm than this, and hence its meaning must be clearly distinguished from the meaning of "factual."

As might be expected, however, the problem of what the ultimate criteria of reality are is a problem to which metaphysicians reach varying answers. Hence they divide into different schools just as the ethical theorists, the esthetic

philosophers, and the theologians do. The "naturalistic" schools, for example, are those which, in the formulation of their criteria, emphasize relevant considerations arising in the field of science as decisive, reinterpreting in terms of scientific truth, so far as they can, all considerations by which the theologian, the ethical theorist, the art critic, etc., find themselves confronted. The "idealistic" schools are those whose criteria of reality reflect acceptance of some ultimate doctrine about values, fitting the realm of scientific truth by some technique of harmonization into the system thus constructed. And of course, as the metaphysical issue just referred to indicates, there are other schools reflecting somewhat different guiding principles than could be stated in terms of the issues separating these two groups. How is one to tell which of the criteria proposed as ultimate by each of these schools of metaphysicians is preferable to the rest? By what supercriterion of reality can he reasonably and impartially evaluate them all?

Our answer will be, once more, that some way must and can be found to apply the method of democratic cooperation to the solution of this problem.

In the light, now, of this analysis of homogeneous and heterogeneous values, what are the distinctive logical properties of judgments dealing with ends—the properties that mark their difference from propositions claiming formal validity or factual truth?

Logical properties of end-judgments

First, the circumstance that the referents of the concepts employed in these judgments are values, not facts or formal relations as such—and values whose pragmatic meaning suffers from the vagueness and syntactic meaning from the lack of objectivity described in Chapter 25. Second, the explicit or implicit appeal of these judgments to the criteria which the preceding chapter has explained. From the viewpoint of determining their rationality, *i.e.*, of seeing just what needs to be done if they are to be adjudicated on a cooperative basis, the fundamental criterion is the rule of democratic impartiality. The third property consists in the irreducible factor of personal decision. Despite the fact that he cannot demon-

strate to others the validity of his chosen criteria, at least in the scientific sense of "demonstration," the asserter of the judgment nonetheless commits himself to them. He does not wait, he cannot wait, in sceptical neutrality; for he has to act, and any mode of action implies that certain ends are preferred to others, and any choice of ends reflects the appeal to certain criteria of value rather than to others by which he might have been guided instead. If he attempts to wait, external circumstances will sooner or later decide for him, and the results of that decision will have their own value, positive or negative—most likely very different from any of the values that he would voluntarily choose. So one cannot avoid decisions between values, even though any decision is a venture whose validity he cannot render convincing to all other intelligent persons.

These three properties are exhibited by judgments of homogeneous ends and judgments of heterogeneous ends alike. But the latter will, in all cases, exhibit a further property too, which arises from the nature of heterogeneous value—namely, fourth, complexity. In the illustrations employed from the field of law, the end sought combines both factual truth and ethical goodness; in the one aspect a judgment about the end must correspond with the relevant given facts, while in the other it must guide realization of a social value which no present facts fully embody. It is thus part of the nature of the judgment to express the way in which these two factors are combined, and the relative weight that is to be given to each in the concrete situation judged. This complexity is greater in the case of religious and metaphysical judgments than it is with legal ones, since the reference of the former includes all three fields of value rather than two fields only. And it is important to notice that when truth is one of the values entering the combination, as it is in these disciplines, the property of factual reference is present in a special form. It consists not merely in the general responsibility to respect the potentialities of the relevant factual material; it includes also a specific responsibility to conform to certain given facts, or to facts as interpreted in a certain way.

We may note in passing that this analysis explains why, to minds habituated to scientific reasoning, concepts of heterogeneous value are apt to seem grotesquely vague and incurably ambiguous, and why religious and metaphysical concepts seem to offend most seriously in this regard. They intentionally combine factors which, from the viewpoint of analytic understanding, need always to be carefully discriminated. But one cannot help recognizing the legitimacy of such a combination, in the proper context, if there are problems in human experience whose distinctive features demand concepts of heterogeneous value for their adequate formulation. Should anyone be in serious doubt as to whether such problems exist, the field of law is at hand to resolve his doubt.

Terminology to be employed

In view, then, of these distinctive properties that characterize end-judgments, in what terms shall we express the distinction between a reasonably justified end-judgment and one that, by comparison, is not reasonably justified? In the field of formal relations the appropriate terms are validity and invalidity, as applied to inferences; in the field of facts they are truth and falsity, as applied to propositions. Here we need a different terminology. But just as no technical term has as yet come into general use for meeting this need in the subfield of metaphysics, so none has been generally accepted for meeting it in the field of end-judgments as a whole. In this case, too, it is probably wiser not to attempt to coin a new term. We have found it natural, thus far in Part IV, to talk about the distinction as one between more rational and less rational judgments about ends. Let us continue to use this terminology. We shall, then, simply regard assertions about ends, whether homogeneous or heterogeneous, as more or less rational, and the criteria which are applicable for deciding between any pair of competing end-judgments as criteria of superior rationality.

EXERCISES

1. Just how do particular value-judgments depend upon those of general value-theory?

2. Explain clearly each of the logical properties of judgments of homogeneous value.
3. Explain the distinctive characteristics of judgments of heterogeneous value.
4. Illustrate these characteristics in detail by some judgment employing the legal concept of "property." By one employing the religious concept of "sin."
5. Study each of the following pairs of arguments. Formulate as clearly as you can the differences in general ethical theory reflected by each pair, *i.e.*, the definitions of "good" or "right" that seem to be implied.

   A. (1) I have a distasteful task to do. I shall therefore postpone it as long as possible, for if I do so something may happen to make it unnecessary for me to do it at all.

      (2) I have a distasteful task to do. I shall therefore do it as soon as possible, so as to have it off my mind and free my attention for things in which I am more interested.

   B. (1) This is an unjust law. I therefore ought to do everything I can to make it ineffective.

      (2) This is an unjust law. But I ought to obey it till it is removed from the statute books, for it is more important to preserve law and order than to have every right of individuals respected.

   C. (1) "I should worry, I should care,
   I shall marry a millionaire;
   If he should die, I should cry;
   I'd go marry another guy."

      (2) "Love is for aye.
   Though the unrolled whole of life
   Be void of thy warmth,
   I shall not turn to another's arms.
   Thou shalt be mine unmixed."

CHAPTER 28

CONFLICTING END-JUDGMENTS IN LAW AND SOCIAL ETHICS

Conflicts between value-theories will be explored in two fields

Let us now explore a fundamental conflict between value-theories. In this exploration we shall deal with two of the fields of value which the distinction between homogeneous and heterogeneous values requires us to recognize. And to reveal the difference in complexity which is bound up with this distinction, we shall pick a conflict which involves a field of homogeneous value, namely, ethics (in certain of its political bearings), and a field of heterogeneous value, namely, law. It will be necessary to probe the conflict deeply enough to locate the crucial issue at stake and to reveal the contrasting value-theories which it reflects; also to exhibit in detail how such an issue would be dealt with by the method of democratic cooperation which was introduced in Chapter 26. It will appear that this method consists in a definite sequence of procedures by which the contrasting criteria of competing value-theories are systematically criticized in such a way that a superior and clearly more rational criterion can emerge. The illustration is intended to be typical, *i.e.*, to bring out the way in which any conflict of ends is resolved when the ends are championed by different people and the method of impartial reasonableness is employed to resolve it. The first half of this program will be carried out in the present chapter; the second half, occupied chiefly with the systematic statement and detailed exposition of the cooperative method, will be the theme of the final chapter.

Since we propose to deal in this way with the problem of end-judgments in two different fields, it will be necessary to

state our illustrative conflict in two different forms. However, as the discussion proceeds, it will be evident that the differences arise simply from the fact that in the one case we are in the realm of discourse of a homogeneous value, while in the other we are in that of a heterogeneous value. Except for these differences, we are facing a single issue, viewed now in the context of the statesman's choice between alternative social ideals, now in that of the judge's decision as to the constitutionality of laws endeavoring to realize those ideals.

The conflict in the field of law

The natural order to follow in discussing these problems would be to take the issue in political ethics first, since judgments of homogeneous value are logically simpler than judgments of heterogeneous value. But in this particular case the difficulties in the way of rationally judging competing theories are best brought out by following the other order. So we shall begin with the conflict in the field of law. And in the light of the preliminary remarks on the nature of legal concepts in the preceding chapter, let us plunge quickly into the heart of this problem. Legal concepts, it was seen, are concepts of value rather than merely concepts of fact; and the value involved is heterogeneous, not homogeneous. The meaning of any such concept includes a reference to ethical ends and also to the end of truth, the latter reference being revealed in the responsibility of any judgment employing it to conform to certain given statutes or prevailing customs which explicitly limit the ways in which the ethical ends involved can be pursued. Legal reasoning must thus aim both at truth and at an ideal of social value whose bearing on any given situation is not yet determined.

And now that we are to deal, not merely with the general nature of legal concepts but with conflicts arising from the reasoning of judges, it will be necessary to square ourselves with a prevalent misconception, according to which value-concepts play no essential role in a judge's thinking; it is just, so we are told, a chain of precise syllogistic reasoning. The major premises for any piece of judicial inference are at hand in the authoritative rules previously established by legisla-

tures or constitutional assemblies, or in some cases by the common law; the minor premises are supplied by the description in legal terms of the human behavior about which the question of legality is raised. Since the meaning of the words employed in the premises has been clarified by previous decisions now at hand as precedents to guide their interpretation, what should prevent a judge from drawing his conclusion with placid confidence that it is the only outcome that the premises could validly allow? "It was assumed that the function of the judge consisted simply in interpreting an authoritatively given rule of wholly extra-judicial origin by an exact process of deducing its logically implied content and in mechanically applying the rule so given and interpreted." [1]

But this could be the case only if one were dealing with formal or factual concepts—indeed, concepts whose syntactic or semantic meaning has been so rigorously defined that there is no disagreement about it, and whose bearing on the problem to be solved leaves no margin of ambiguous uncertainty. Since, however, legal concepts refer to ethical ideals as well as to given social facts, one who deals with them inevitably meets all the perplexing difficulties that are involved in the determination of ends. The consequence is that their syntactic meaning depends on some theory not accepted by all competent legal thinkers, and that their pragmatic meaning must be formulated anew each time they are used in the reasoning of a lawyer or a judge. And this means that his personal value-commitment can hardly be eliminated when he decides what "health," or "order," or "freedom" implies in their bearing on some critical social issue. "Freedom" is not merely a given fact; it is also an end, and as such its meaning is not fixed in any fashion that prejudges all its possible applications. When all relevant precedents clarifying its interpretation have been conscientiously respected, still the judge finds an undetermined margin between what recognized authority clearly prescribes and the social values at stake in the concrete case before him, and this gap must be closed by his own decision.

[1] Roscoe Pound, *An Introduction to the Philosophy of Law*, p. 100 f.

But how can he close it, and thus determine the meaning of the word in this context, without being influenced by his own sense of what constitutes the relevant social values, and how they promise to be most fully realized in the situation at hand? Does not the evident lack of objectivity in legal judgments primarily reflect exactly this circumstance?

To be sure, every conscientious judge will endeavor to reduce the play of this personal factor to the minimum. Knowing that it is his business not to make law, but only to interpret a law already formulated, he will allow, so far as he can, the settled moral sense of his community to guide the evaluations which he makes; he will estimate the comparative values at stake in the way he believes any reasonable, prudent, and responsible man would estimate them. But the crucial point is that there is never complete agreement among reasonable, prudent, and responsible men as to what ought to be done in the more novel and complex problems that come up for decision. Hence the judge's own theory of law, which rests upon his general ethical, economic, and social philosophy, cannot fail to be reflected in the conclusions at which he arrives. President Theodore Roosevelt probably overstated the matter, while yet bringing out the important truth clearly, when he said: "The chief lawmakers in our country may be, and often are, the judges, because they are the final seat of authority. Every time they interpret contract, property, vested rights, due process of law, liberty, they necessarily enact into law parts of a system of social philosophy; and as such interpretation is fundamental, they give direction to all lawmaking. The decisions of the courts on economic and social questions depend upon their economic and social philosophy; and for the peaceful progress of our people during the twentieth century we shall owe most to those judges who hold to a twentieth-century economic and social philosophy and not to a long outgrown philosophy, which was itself the product of primitive economic conditions." [2]

[2] In his message to Congress on December 8, 1908.
A more judicious statement of the same point is found in B. N. Cardozo,

And what, more exactly, does a judge's own social philosophy express, so far as its fundamental evaluations are concerned? The answer seems to be: the whole shifting complex of his ideals, hopes, and interests, which are in part aspirations common to him and to most other men of his time, in part urgent demands of the social group upon whose prosperity his well-being most closely depends, in part mere personal prejudices, biases, or ambitions. And he is unaware how large a part each of these three factors plays; were he aware, any conscientious judge would, as a matter of course, allow none but the first to determine the outcome. Especially is it difficult, as the Marxian philosophy has made abundantly clear, for him to distinguish the first from the second; all thinkers easily assume, unconsciously, that the welfare of society as a whole is identical with the welfare of their limited social group, because to them it seems obviously only capable of being attained through the assured prosperity of their group. As Mr. Justice Cardozo says: "The spirit of the age, as it is revealed to each of us, is too often only the spirit of the group in which the accidents of birth or education or occupation or fellowship have given us a place. No effort or revolution of the mind will overthrow utterly and at all times the empire of these subconscious loyalties."[3]

Impracticable solutions of conflicts in legal theory

When one first confronts the serious nature of the problem of achieving rationality in legal decisions which results from the inevitability of this personal equation, he is apt to think that a solution could be found in one or the other of two directions.

Why not give up, he might ask, all attempt to secure essential historical or logical continuity between judicial decisions—all pretense at applying consistently a single set of legal prin-

The Nature of the Judicial Process, p. 167 f.: "I have spoken of the forces of which judges avowedly avail themselves to shape the form and content of their judgments. . . . Deep below consciousness are other forces, the likes and the dislikes, the predilections and the prejudices, the complex of instincts and emotions and habits and convictions, which make the man, whether he be litigant or judge. . . . The great tides and currents which engulf the rest of men do not turn aside in their course and pass the judges by."

[3] *The Nature of the Judicial Process,* p. 174 f.

ciples? Why not abandon the responsibility of law to conform to any rules set up in the past? What would be lost if we frankly recognize that judges' minds are bound to vary, and that our real safeguard lies rather in having conscientious, competent arbiters than in making them follow any constitutional dictates or other precedents? The conclusion to draw from this reasoning would be that the best course is to do away with our present legal foundations, allowing judges and juries to decide on their own discretion, guided by the ethical sense of justice built up in the course of lengthy experience and reflection, what ought to be done in each case. In the totalitarian countries many areas of legal dispute are dealt with in this fashion, and even in democratic countries the recent trend has been in the direction of allowing a much greater margin of discretion than had previously been customary. Without denying that some evils can be corrected in this way, it can nonetheless be said that there is a serious danger in carrying this principle to the limit—the danger, namely, that the dependability of a regular system of law would be lost; nobody would know where he stood in reference to the social judgment on his conduct; moreover, officials assigned the responsibility of passing judgment on asserted violations of right would lack the guidance that a definite conceptual expression of the developed sense of the community on such matters would furnish. This method would clearly increase rather than decrease the uncertainty and variability of our reasoning in these situations. It would mean an enormous multiplication of the number of decisions which leave behind them a sense of injustice and rankling grievance, for the same issue might be decided by one tribunal today in one way, and by another tribunal tomorrow in another. It seems as though the very idea of impartial justice demands at least some minimum of continuity and consistency; and apparently we cannot advance toward the realization of a greater measure of rationality in this field by abandoning such halting approximation toward a stable standard as the history of law has already secured.

Why not, then—for this would seem to be the other plausible alternative—provide a definite legal concept to take care of every possible shade or mixture of the social values that might be involved in any situation, and define what we mean by each of these terms so fully and rigorously that nobody will ever have any doubt which of them ought to be applied to any given set of facts? This, however, is exactly our difficulty in dealing with evaluation generally. Everything would be vastly easier in these baffling matters if we could anticipate all events that might possibly happen; determine, therefore, in the light of them our comparative evaluations; thrash out a social agreement on those sufficiently fundamental to require agreement; and then define to the last detail our descriptive vocabulary. Then it would doubtless be possible to develop a system of law so precisely and completely formulated that assimilation of future cases would reduce to the simplest type of syllogistic thinking. But in a world incessantly producing novelties that we do not know how to predict, and in which the experience and consequently the evaluations of every individual are constantly changing but changing differently, how is such a situation to be brought about?

Conflicts are most crucial when ultimate social ends are at stake

Neither of these solutions, then, is practicable; we must search more deeply. And the next step needed is to sharpen our analysis. At just what point does this subjective factor, with its critical influence on judicial decisions, appear most clearly? The answer is: in those issues whose settlement turns upon the judge's interpretation of the most general concepts of legal theory—that is, his conception of the ultimate social ends which it is the basic function of the law to secure. A judge's convictions about these ends are, indeed, reflected in his interpretation of the meaning of all legal concepts, just as in the field of esthetics the meaning of subordinate concepts is controlled by one's theory as to the nature of the most general esthetic values. But in cases that are analyzed in terms of these subordinate concepts, holders of different views about ultimate issues often agree pretty closely on their pragmatic implications with respect to those cases. When contrasting

notions as to the general social ends of law are directly involved, however, so that acceptance of one notion by the judge will mean his support of the plaintiff's claim in the controversy while acceptance of the other will turn the scale in favor of the defendant, it is evident that the social philosophy to which he is committed becomes the crucial factor. And this is the case with special frequency in an epoch of history during which control of events threatens to pass from the hands of one social class to those of another, so that the theory of value in which the dominant interests of the former group find justification is thrust into sharp competition, in the minds of judges as in those of other thinkers, with theories reflecting the interests of the groups struggling to displace it.

The Western world is in such a period today. Control by the business leaders, who have dominated social life for a century and a half or longer, is increasingly limited and more and more seriously threatened by the influence of other groups, notably labor, organized farmers, and the technical experts who actually manage the economic machine. Now the theory justifying, in the main, the interests of those whose power is on the wane, is the theory of individualism which later in the chapter we shall carefully analyze; from its viewpoint the ultimate purpose of law is to secure the maximum of free self-assertion on the part of each individual member of society. Every person is assumed to possess a natural (and hence absolute) right to engage in whatever activities his enterprise and ambition might lead him to project, subject only to the obligation to respect a similar right on the part of others and to support a government whose function is simply to protect these rights. The interests of those who are competing with increasing success for a share in power are expressed in no single social philosophy, but they all agree on the proposition that the wants of people who have been insufficiently satisfied under the previous regime must, in the interest of equal justice, be given much greater weight, and that this can only be effectively accomplished through broader governmental control. This common demand reflects, as far as it goes, a legal theory

which might be summarized in the following principle: The end of law is the satisfaction of the largest number of human needs and wants, pursued with equal consideration for all groups of persons whose interests are vitally involved. From the standpoint of this theory no right, no form of individual liberty, is a natural right as such; it is a social function rather than a private possession, and is therefore properly limited in any person's case not only by the similar private rights of others, but by the demand for satisfaction on the part of the various groups whose concerns it affects. Further regulation by the state is of course held to be necessary, to establish the limitations thus required.

This is often the case in questions of constitutional law

The clash between these contrasting conceptions of the social values to be secured by law has been displayed in many judicial decisions in recent years; in a large number of the cases where both majority and minority opinions are filed, one opinion reflects the former of these competing theories and the other the latter. In many cases the former has been the majority view, while in many others, especially the more recent ones, it has been relegated to the status of minority opinion. One finds it especially instructive to follow the struggle between the competing theories as fought out in the nationally prominent arena of the United States Supreme Court, in its decisions on the constitutionality of Congressional or state legislation. Prior to 1937, in general, the Court's settlement of major controversies reflected support of the traditional American individualism, interpreted in such wise as to protect freedom of enterprise, in its customary form, for members of the business class. The decisions were sometimes rather close, the nine judges on the Court not infrequently dividing five to four, so that the ultimate meaning of American fundamental law depended on the social evaluations of a single judge. The shift came in 1937, and it is of special interest to note that it came without any immediate change in the composition of the Court. Later, through the death or retirement of the conservative members of the Court and their replacement by men who sympathize with a less individual-

istic social philosophy, the shift was presumably made permanent, so far as the present generation of Americans is concerned.

We shall now examine, as a detailed illustration of the problem arising from these fundamental divergences of conviction in legal thinking, a case in constitutional law where the Court, within a period of ten months, reversed itself, first giving a majority decision expressing the philosophy of individualism, and then meeting the same issue by a majority decision reflecting a quite different perspective on the social ends at stake.

The issue whose constitutionality was in question in these two decisions was whether a State Legislature has the power to enforce within the state a minimum wage for women. In 1933 the New York State Legislature had passed a law declaring it to be against public policy for an employer to employ any woman at an "oppressive or unreasonable wage," and establishing a procedure for determining and enforcing minimum wages in the various industries within the state. The criterion to be used in this procedure was twofold; the wage must be at least equal to the fair and reasonable value of the services rendered, and it must also be sufficient to meet the minimum cost of living necessary for health. This law was declared unconstitutional by the New York Court of Appeals, affirming the decision of the Supreme Court of the State of New York. The case was carried to the United States Supreme Court, which also, on June 1, 1936, declared the law unconstitutional.[4] This decision prevailed by a majority of one, five members of the Court supporting it while four (Justices Hughes, Brandeis, Cardozo, and Stone) disagreed with their colleagues. The majority opinion was delivered by Associate Justice Butler; the sentences essential to his main argument will now be quoted.

The majority decision of the Supreme Court in 1936

> Upon the face of the act the question arises whether the State may impose upon the employers state-made minimum wage rates for all

[4] Morehead *v.* New York *ex rel.* Tipaldo, 298 U. S. 587 ff.

competent experienced women workers whom they may have in their service. . . . So far at least as concerns the validity of the enactment under consideration, the restraint imposed by the due process clause of the Fourteenth Amendment upon legislative power of the State is the same as that imposed by the corresponding provision of the Fifth Amendment upon the legislative power of the United States.[5]

Referring to the Court's decision in the earlier Adkins *v.* Children's Hospital case (1923),[6] he continued:

This court's opinion shows: The right to make contracts about one's affairs is a part of the liberty protected by the due process clause. Within this liberty are provisions of contracts between employer and employee fixing the wages to be paid. In making contracts of employment, generally speaking, the parties have equal right to obtain from each other the best terms they can by private bargaining. Legislative abridgement of that freedom can only be justified by the existence of exceptional circumstances. Freedom of contract is the general rule and restraint the exception. This court has found not repugnant to the due process clause statutes fixing rates and charges to be exacted by businesses impressed with a public interest, relating to contracts for the performance of public work, prescribing the character, methods and time of payment of wages, fixing hours of labor. Physical differences between men and women must be recognized in proper cases and legislation fixing hours or conditions of work may properly take them into account, but "we cannot accept the doctrine that women of mature age, *sui juris,* require or may be subjected to restrictions upon their liberty of contract which could not lawfully be imposed in the case of men under similar circumstances. To do so would be to ignore all the implications to be drawn from the present day trend of legislation, as well as that of common thought and usage, by which woman is accorded emancipation from the old doctrine that she must be given special protection or be subjected to special restraint in her contractual and civil relationships. . . . Enough has been said to show that the authority to fix hours of labor cannot be exercised except in respect of those occupations where work of long continued duration is detrimental to health. This court has been careful in every case where the question has been raised, to place its decision upon this limited authority of the legislature to regulate hours of labor and to disclaim any purpose to uphold the

[5] A stronger argument in some respects is presented by the minority opinion in the West Coast Hotel case which was decided the following year. See below, pp. 682 ff.

[6] This decision had pronounced unconstitutional a minimum wage law passed by Congress for the District of Columbia.

legislation as fixing wages, thus recognizing an essential difference between the two. It seems plain that these decisions afford no real support for any form of law establishing minimum wages."

The decision and the reasoning upon which it rests clearly show that the State is without power by any form of legislation to prohibit, change, or nullify contracts between employers and adult women workers as to the amount of wages to be paid.

Again referring to the Adkins decision:

The Act takes account of the necessities of only the employee; to the extent that the sum fixed exceeds fair value of service rendered, it amounts to a compulsory exaction for the support of a partially indigent person for whose condition there rests upon the employer no peculiar responsibility; the statute exacts from the employer an arbitrary payment for a purpose and upon a basis having no causal connection with his business or the contract or the work the employee engages to do; the declared basis is not the value of the service rendered but the extraneous circumstance that the employee needs to get a prescribed sum of money to insure her subsistence, health, and morals. The Court said: "The ethical right of every worker, man or woman, to a living wage may be conceded. . . . The fallacy of the proposed method of attaining it is that it assumes that every employer is bound at all events to furnish it. The moral requirement, implicit in every contract of employment, viz. that the amount to be paid and the service to be rendered shall bear to each other some relation of just equivalence, is completely ignored. The necessities of the employee are alone considered and these arise outside of the employment and are as great in one occupation as in another. . . ."

A statute requiring an employer to pay in money, to pay at prescribed and regular intervals, to pay the value of the services rendered, even to pay with fair relation to the extent of the benefit obtained from the service, would be understandable. But a statute which prescribes payment without regard to any of these things, and solely with relation to circumstances apart from the contract of employment, the business affected by it and the work done under it, is so clearly the product of a naked, arbitrary exercise of power that it cannot be allowed to stand under the Constitution of the United States. . . .

To distinguish this from the *Adkins* case, petitioner refers to changes in conditions that have come since that decision, cites great increase during recent years in the number of women wage earners, and invokes the first section of the Act, called "Factual background."

The Act is not to meet an emergency; it discloses a permanent policy; the increasing number of women workers suggests that more and more

they are getting and holding jobs that otherwise would belong to men. . . .

These legislative declarations, in form of findings and recitals of fact, serve well to illustrate why any measure that deprives employers and adult women of freedom to agree upon wages, leaving employers and men employees free so to do, is necessarily arbitrary. Much, if not all, that in them is said in justification of the regulations that the Act imposes in respect of women's wages applies with equal force in support of the same regulation of men's wages. While men are left free to fix their wages by agreement with employers, it would be fanciful to suppose that the regulation of women's wages would be useful to prevent or lessen the evils listed in the first section of the Act. Men in need of work are as likely as women to accept the low wages offered by unscrupulous employers. Men in greater number than women support themselves and dependents and because of need will work for whatever wages they can get, and that without regard to the value of the service and even though the pay is less than minima prescribed in accordance with this Act. It is plain that, under circumstances such as those portrayed in the "Factual background," prescribing of minimum wages for women alone would unreasonably restrain them in competition with men and tend arbitrarily to deprive them of employment and a fair chance to find work. . . .

The New York court's decision conforms to ours in the *Adkins* case, and the later rulings that we have made on the authority of that case. . . . And in each case, being clearly of opinion that no discussion was required to show that, having regard to principles applied in the *Adkins* case, the state legislation fixing wages for women was repugnant to the due process clause of the Fourteenth Amendment, we so held and upon the authority of that case affirmed per curiam the decree enjoining its enforcement. It is equally plain that the judgment in the case now before us must also be

Affirmed.

Let us summarize this argument.

By the due process clause of the Constitution individuals are guaranteed certain freedoms, including freedom of contract, which legislatures can abridge only in exceptional circumstances. A universal requirement to justify any case of such abridgment is that it be nondiscriminatory. Now certain conditions have been recognized by the courts as justifying restrictions on freedom of contract, *e.g.*, in the case of laws regulating hours of labor. But laws establishing minimum

wages for adult women are in a different situation. They are discriminatory on two counts: (1) they require an employer to pay in accordance with a wage scale that has no relation to his business or to the work performed, being determined by the extraneous circumstance that the employee needs a certain amount of money to maintain health; and (2) they discriminate against men, who need the protection of minimum wages as much as women do. Indeed, because of this latter discrimination these laws are not calculated to attain the ends which the New York Legislature had in mind. Since men compete with women for many jobs, they will take over the positions where employers are unable to conform to the minimum wage scale prescribed. Thus women needing employment will be unable to compete successfully against men.

The majority decision in 1937

On March 29, 1937, the United States Supreme Court, by an equally close decision, upheld the constitutionality of a law which had been passed by the legislature of the State of Washington in 1913.[7] This law also prescribed a minimum wage scale for women, the criterion being that the wage must be "adequate for the maintenance of the adult woman worker." It had been upheld as constitutional by the Supreme Court of Washington. The membership of the United States Supreme Court which decided the case was the same as in the previous year, but in the meantime Associate Justice Roberts had changed his conviction; hence the former minority now became the majority of the Court.[8] The majority opinion was delivered by Chief Justice Hughes.

> The principle which must control our decision is not in doubt. The constitutional provision invoked is the due process clause of the Fourteenth Amendment governing the States, as the due process clause invoked in the *Adkins* case governed Congress. In each case the violation alleged by those attacking minimum wage regulation for women is deprivation of freedom of contract. What is this freedom? The Constitu-

[7] West Coast Hotel Company v. Parrish *et al.*, 300 U. S. 379 ff.

[8] It was during the preceding winter that President Roosevelt's proposal to increase the size of the Court, so as to secure a majority favorable to liberal social legislation, had stirred the country.

tion does not speak of freedom of contract. It speaks of liberty and prohibits the deprivation of liberty without due process of law. In prohibiting that deprivation the Constitution does not recognize an absolute and uncontrollable liberty. Liberty in each of its phases has its history and connotation. But the liberty safeguarded is liberty in a social organization which requires the protection of law against the evils which menace the health, safety, morals, and welfare of the people. Liberty under the Constitution is thus necessarily subject to the restraints of due process, and regulation which is reasonable in relation to its subject and is adopted in the interest of the community is due process.

This essential limitation of liberty in general governs freedom of contract in particular. More than twenty-five years ago we set forth the applicable principle in these words. . . .

". . . Freedom of contract is a qualified and not an absolute right. There is no absolute freedom to do as one wills or to contract as one chooses. . . . Liberty implies the absence of arbitrary restraint, not immunity from reasonable regulations and prohibitions imposed in the interest of the community." *Chicago, B. & Q. R. R. Co. v. McGuire*, 218 U.S. 549, 567.

This power under the Constitution to restrict freedom of contract has had many illustrations. That it may be exercised in the public interest with respect to contracts between employer and employee is undeniable. [Precedents are cited] . . . In dealing with the relation of employer and employed, the legislature has necessarily a wide field of discretion in order that there may be suitable protection of health and safety, and that peace and good order may be promoted through regulations designed to insure wholesome conditions of work and freedom from oppression. *Chicago, B. and Q. etc.*, p. 570.

The point that has been strongly stressed that adult employees should be deemed competent to make their own contracts was decisively met nearly forty years ago in *Holden v. Hardy, supra*, where we pointed out the inequality in the footing of the parties. We said:

"The legislature has also recognized the fact, which the experience of legislators in many States has corroborated, that the proprietors of these establishments and their operatives do not stand upon an equality, and that their interests are, to a certain extent, conflicting. The former naturally desire to obtain as much labor as possible from their employes, while the latter are often induced by the fear of discharge to conform to regulations which their judgment, fairly exercised, would pronounce to be detrimental to their health and strength. . . ."

And we added that the fact "that both parties are of full age and competent to contract does not necessarily deprive the State of the

power to interfere where the parties do not stand upon an equality, or where the public health demands that one party to the contract shall be protected against himself. The State still retains an interest in his welfare, however reckless he may be. The whole is no greater than the sum of all the parts, and when the individual health, safety, and welfare are sacrificed or neglected, the State must suffer."

It is manifest that this established principle is peculiarly applicable in relation to the employment of women in whose protection the State has a special interest. That phase of the subject received elaborate consideration in *Muller v. Oregon* (1908), 208 U.S. 412, where the constitutional authority of the State to limit the working hours of women was sustained. [Further precedents are cited]. . . .

This array of precedents and the principles they applied were thought by the dissenting justices in the *Adkins* case to demand that the minimum wage statute be sustained. The validity of the distinction made by the Court between a minimum wage and a maximum of hours in limiting liberty of contract was especially challenged. 261 U.S., p. 564. That challenge persists and is without any satisfactory answer. As Chief Justice Taft observed: "In absolute freedom of contract the one term is as important as the other, for both enter equally into the consideration given and received; a restriction as to the one is not greater in essence than the other and is of the same kind. One is the multiplier and the other is the multiplicand. . . ."

The minimum wage to be paid under the Washington statute is fixed after full consideration by representatives of employers, employees, and the public. It may be assumed that the minimum wage is fixed in consideration of the services that are performed in the particular occupations under normal conditions. Provision is made for special licenses at less wages in the case of women who are incapable of full service. The statement of Mr. Justice Holmes in the *Adkins* case is pertinent: "This statute does not compel anybody to pay anything. It simply forbids employment at rates below those fixed as the minimum requirement of health and right living. It is safe to assume that women will not be employed at even the lowest wages allowed unless they earn them, or unless the employer's business can sustain the burden. In short, the law in its character and operation is like hundreds of so-called police laws that have been upheld." 261 U.S., p. 570. . . .

We think that the views thus expressed are sound, and the decision in the *Adkins* case was a departure from the true application of the principles governing the regulation by the State of the relation of employer and employed. Those principles have been reinforced by our subsequent decisions. [Illustrative cases are cited]. . . .

What can be closer to the public interest than the health of women and their protection from unscrupulous and overreaching employers? And if the protection of women is a legitimate end of the exercise of

state power, how can it be said that the requirement of the payment of a minimum wage fairly fixed in order to meet the very necessities of existence is not an admissible means to that end? The legislature of the State was clearly entitled to consider the situation of women in employment, the fact that they are in the class receiving the least pay, that their bargaining power is relatively weak, and that they are the ready victims of those who would take advantage of their necessitous circumstances. . . . The legislature had the right to consider that its minimum wage requirements would be an important aid in carrying out its policy of protection. The adoption of similar requirements by many States evidences a deepseated conviction both as to the presence of the evil and as to the means adapted to check it. Legislative response to that conviction cannot be regarded as arbitrary or capricious, and that is all we have to decide. Even if the wisdom of the policy be regarded as debatable and its effects uncertain, still the legislature is entitled to its judgment.

There is an additional and compelling consideration which recent economic experience has brought into a strong light. The exploitation of a class of workers who are in an unequal position with respect to bargaining power and are thus relatively defenceless against the denial of a living wage is not only detrimental to their health and wellbeing, but casts a direct burden for their support upon the community. What these workers lose in wages the taxpayers are called upon to pay. The bare cost of living must be met. . . . The community is not bound to provide what is in effect a subsidy for unconscionable employers. The community may direct its lawmaking power to correct the abuse which springs from their selfish disregard of the public interest. The argument that the legislation in question constitutes an arbitrary discrimination, because it does not extend to men, is unavailing. This Court has frequently held that the legislative authority, acting within its proper field, is not bound to extend its regulation to all cases which it might possibly reach. The legislature is "free to recognize degrees of harm and it may confine its restrictions to those classes of cases where the need is deemed to be clearest." If "the law presumably hits the evil where it is most felt, it is not to be overthrown because there are other instances to which it might have been applied. . . ." [Precedents are cited.] This familiar principle has repeatedly been applied to legislation which singles out women, and particular classes of women, in the exercise of the State's protective power. [Precedents again.] Their relative need in the presence of the evil, no less than the existence of the evil itself, is a matter for the legislative judgment.

Our conclusion is that the case of *Adkins v. Children's Hospital, supra,* should be, and it is, overruled. The judgment of the Supreme Court of the State of Washington is

Affirmed.

Again, let us summarize.

The Constitution does not guarantee freedom of contract. It does prohibit the deprivation of liberty without due process of law, but the latter clause recognizes that liberty is not an absolute right. The liberty safeguarded is liberty in a social organization, which means that it must be limited by the exercise of power on the part of the legislatures in fulfilling their responsibility to protect the health, safety, morals, and welfare of the people. Now what is true of freedom in general is true of freedom of contract in particular, as is shown by many previous decisions of this Court. Some of these precedents specifically recognize that in contracts for employment the inequality of bargaining power between the parties often leads employees to accept conditions which they know are detrimental to health, and that if their health is to be protected it must be done by law. Furthermore, previous decisions have recognized that this principle is especially applicable to the employment of women; so far as concerns the regulation of their hours of labor there is no controversy regarding its applicability. But it is impossible to point out any relevant difference between limiting working hours and establishing minimum wages. To be sure, if a minimum wage law were discriminatory in the sense of being arbitrary or unreasonable, it would be an invalid abridgment of freedom of contract. But in the present instance that is not the case. The wages are fixed after full consideration by representatives of employers, employees, and the public. There is no discrimination against the employer; women will not be employed at even the lowest wages allowed unless they earn them, or unless the employer's business can sustain the burden. There is no discrimination against men, because the legislature is quite within its rights in recognizing that women need protection more than men; the laws may be extended to include men, too, if the need for such extension becomes apparent. It was, therefore, a proper exercise of legislative judgment to attempt by this means to safeguard the health of women, and through it the health and welfare of the community at large.

Analysis of the logical problem involved

Here are two contradictory conclusions, reached by the same Court within a period of a few months, with respect to the same problem of constitutional law. By what procedure would it become evident which line of reasoning is the more rational? Can we find a standard in the light of which they can be adequately judged?

An examination of the contrasting arguments will quickly reveal where the crucial issue lies. The Constitution guarantees to individuals and minority groups certain freedoms, which have been universally recognized to include freedom of contract. It also gives the legislatures power to safeguard by appropriate laws the health, safety, morals, and welfare of the people. Now, when a law which has been adopted to secure this end abridges the freedom guaranteed certain individuals, which should give way? The Constitution is silent on this point, and precedents can readily be found on both sides. In the 1936 decision the majority held that the legislative power should give way; the controlling consideration is to preserve freedom of contract, as this phrase had traditionally been interpreted. In the 1937 decision the majority held that freedom of contract should give way; the controlling consideration is the legislative right to protect the health and well-being of the people. In the face of such a contradiction, to what shall we appeal? Well, theoretically the judges are supposed to rest in the end on the intention of the framers of the Constitution. But these framers are all dead, and their intentions doubtless differed as widely as those of the members of any constitutional convention. Furthermore, how can we tell what any group among those constitution-makers would want if they could foresee the day when their problems would be things of the past and quite different ones challenge settlement? Would those who wished to uphold individual liberty then still wish to do so today? Would those who championed large powers then for the legislatures still do so today? Of course, the thing that we may be reasonably sure each of them would always want is the stable and increasing well-being of the entire community, but how is this to be secured? By pro-

tecting, come what may, the freedoms of the individual? Or by emphasizing the power of the legislature to limit those freedoms?

Superficially viewed, the above arguments disclose a criterion that is explicitly accepted by both sides. Any abridgment of freedom, to be constitutional, must be reasonable, *i.e.*, not arbitrary or discriminatory; and any exercise of legislative power, to be valid, must be adapted to the end announced. But when these concepts of reasonableness and wise adaptation of means to ends are examined in the context of the two decisions, it is evident that the former, at least, points toward two different criteria rather than toward a single standard. The decision of 1936 implies that any restriction on freedom of contract is arbitrary if it seems to bear unfairly on employers and if it does not treat men and women alike. The decision of 1937 implies, rather, that a restriction is arbitrary if it does not give intelligent consideration to all the social values at stake; different groups may be treated differently if the difference can be justified on this ground. So it is only the words that are shared in common; the real criterion underlying them is different in the two cases. And is it not clear that in the last analysis the difference is due to the fact that the theories of social value, assumed as appropriately controlling the decisions, are different? The majority in 1936 believed that the supreme end to be promoted is economic individualism—the right of all participants in the economic process to make what arrangements they may agree upon, unrestrained by law except where restraints are clearly needed to insure that the process may continue successfully. The majority in 1937 believed that the supreme end is the health and security of the community at large. These differing final values determined the conclusions reached in each case, and they did so by determining the meaning of the concepts employed, so that the implications of those concepts led in quite different directions as the two chains of inferences were developed.

Where can we find the key to a rational choice between

these rival values? How can a genuinely common criterion that is applicable to such conflicts be established?

The general issue of individualism vs. collectivism

In this situation reasonable appraisal is complicated by the fact that it is dealing with heterogeneous values. Since the problem is a legal problem, an adequate solution must be able to show not only that it takes proper account of the alternative moral ends at stake, but also that it conforms to the limitations on pursuit of those ends that are at hand in the relevant provisions of the Constitution as previously interpreted. It must show in this way a definite continuity with solutions already accepted. Let us examine what is in essence the same issue, when freed from this second requirement. It then becomes the general issue in social ethics between the theory of individualism and what may be called, for want of a less prejudicial term, the theory of collectivism. Every statesman today must take his stand on this issue. Which of these two alternative goods—individual liberty or community security and order—is superior to the other when they appear to lead to different policies and contrasting institutional practices?

Before it is systematically attacked, this question needs preliminary clarification. The clarification may begin with two points of warning. First, it is evident from the above analysis of the corresponding legal problem that neither of these values can be adopted absolutely, to the exclusion of the other. A community in which individual liberty were complete would, as matters now stand, be a social chaos; no person would be safe in his life and property, and no organized protection of the community itself against external threat would be possible. On the other hand, complete suppression of individual liberty is likewise impossible; supervision of people's behavior by the officers of the law, in the interest of maintaining some chosen pattern of social regulation, can obviously be carried only to a certain point. Hence the question is one of degree and emphasis; granted that some measure of liberty is inevitable and some minimum of order necessary, in which direction, on the whole, does true progress lie; in the

direction of increasing the area of liberty, or in that of widening the area under social control? Second, it is likewise evident to all who are not dogmatically convinced that certain liberties, or a certain form of social order, are dictated by the nature of things, that the question is incapable of the same answer at all times and under all circumstances. What answer is appropriate depends on the various interests that are present in a given community at a given time, and on the specific problems which in that situation most insistently press for solution. There is now general agreement, for example, that in Western Europe two hundred years ago it was very desirable to expand the scope of individual freedom in the economic field; it seems fairly evident that the enormously successful application of scientific knowledge to the processes of industrial production and distribution, with the vast advance in the standard of living which has thus been secured, would have been impossible otherwise, or at least would have taken place much more slowly. There is also general agreement in the democratic countries that by the opening of the present century economic liberty had, at too many points, degenerated into license, threatening the security of the majority of citizens, violating their sense of justice, and thus weakening the foundations of social security. Some expansion of the area of socialized control was therefore demanded, lest liberty become destructive of conditions vital to man's continued existence and well-being. Were this not so, law would never have been faced with the issue above discussed. With these illustrations in mind, it is patently necessary to think of the question as definitely limited in space and time; no answer to it in general terms can, perhaps, even be profitably considered. We shall limit it then as follows: Today, in the democratic part of the world, and especially in the United States, which policy needs to be emphasized—that of securing a larger field for freedom or that of broadening the scope of social regulation?

It is true that even in a limited community at any given time neither policy can be reasonably carried out in uniform

fashion. Particular social stresses may call for regulation of previously unfettered activities, while other problems may best be met by relaxing controls now in effect. In 1933, in the United States, for example, among the earliest acts of the Roosevelt administration were a radical extension of Federal control over the nation's banks and repeal of the prohibition amendment to the Constitution—a restriction of liberty in one direction, accompanied by an increase of it in another. Nonetheless, so far as a community's major political, economic, and educational institutions are concerned, one usually finds that at any particular time it is seeking to solve its problems either by increased socialization or by a more complete individualism. Indeed, in countries with two strong political parties the vital difference between them often lies in the adherence of one to a program of securing greater liberty wherever there is no obvious need for restrictions, and the commitment of its rival to the extension of social control wherever such control promises to be effective, provided that the liberties recognized on all hands as essential are not destroyed. At the present time, for example, the Republican Party, supported by many Southern Democrats, champions the former of these two policies, while the majority wing of the Democratic Party pursues the latter. Which of the contrasting emphases is in the right, or, if neither is wholly right, where can a third value, clearly superior to the ideal sought by each of them, be found?

The case for individualism—general considerations

Well, what case can be made, in broad outline, for a larger measure of individual liberty? In answer, let us concentrate first on the general arguments in favor of a maximum degree of human freedom, passing then to the considerations that carry special weight at the present moment, or to those who share the distinctive background of American institutions.

Among the general arguments, a contention likely to be vigorously stressed is that people naturally want freedom from restraint and are happy only when they can exercise it. All of us want to be able to do whatever we find ourselves eager to do, and this is precisely what constitutes freedom in its social meaning—the absence of barriers imposed by other

people to our chosen activities. In lack of such freedom we are haunted by the unhappy sense that we cannot be ourselves; for happiness depends, in the last analysis, on one's ability to exercise his powers in his own way, to display whatever talents he has, and to develop into the unique personality that he has it in him to become. To be thwarted is not only painful in itself, but it is also to be stunted in the growth that would otherwise be possible toward one's individual maturity and desired destiny. The social groups to which we belong have, of course, a right to bring us under control whenever we use our freedom in ways that harm others, but if they go beyond this measure of repression they unjustifiably limit our selfhood and violate our right to happiness. If one is strong-willed enough he may refuse to obey unreasonable restrictions; unless, however, the refusal is recognized as justifiable by the community, this only brings upon him the penalties of the law and of public opinion, with the sufferings and more drastic restrictions that they involve.

A community needs varied human resources

In the next place, it seems clear that individuals are of much greater use to society when they are allowed to develop and express their distinctive capacities than when they are forced into some stereotyped mold, made to conform to a common pattern. A modern community is confronted from time to time by the most varied and complex problems; unless it can draw upon equally varied resources in its individual members, it is unable to meet satisfactorily many of the difficulties into which it is plunged. An epidemic of some new disease suddenly strikes; attack is threatened by an enemy employing new weapons; a dust bowl drives millions from their homes and settled callings—if individual liberty has not been permitted in the fields of medical research, of military invention, and of the sociology of mass migrations, no promising solution of these problems may be at hand, and the community may be dealt a crippling blow before, under the stress of the emergency itself, the right way of meeting the challenge can be found. Moreover, activities done under compulsion are, in general, badly done. People are effective workers only when

they are doing what they freely choose to do and therefore what they enjoy doing. Of course, there are always dull and disagreeable tasks that must be performed for the sake of social health and well-being. But in an age of science, methods can more and more be devised whereby such routine tasks are taken care of by machines; it is the community's responsibility to advance in this direction as rapidly as possible, thus freeing its human constituents for activities that give play to initiative, resourcefulness, and creative discovery, allowing them to realize and express constructively their individual selfhood.

One particular aspect of this need on the part of society itself for members enjoying a large measure of individual freedom deserves separate mention. Without such freedom, a community finds it exceedingly difficult to progress toward a more perfect adjustment to the conditions under which it lives, and especially toward fuller mastery of the natural resources which lie at its disposal. New inventions which promise significant advance in these matters are rarely the creation of political administrators; unless, therefore, other individuals are allowed the maximum opportunity to originate new ideas and to experiment with them, even when the ideas appear futile or perverse to persons possessing social authority, the community will stagnate in whatever happens to be its present rut. This is especially obvious in the field of scientific inquiry. One of the greatest lessons of modern empirical science is that the truths most clearly verified at any given time are approximations rather than finalities, and must be held tentatively in the expectation that they will be revised when new data, more exact measurements, or improved instruments of research are at hand. But this means that science can only progress toward fuller understanding and control of nature as individual thinkers are given freedom (and even, if possible, encouragement) to investigate facts in their own way and to experiment as they like with varied methods and tools of research. The belief characteristic of an authoritarian age, that every important truth is already known and merely

needs firm lodgment in the minds of each new generation, yields to the realization that man has as yet only scratched the surface of nature's mysteries, and that if they are to be laid bare with desirable rapidity each generation must be roused to exercise to the full whatever capacity for original discovery it possesses. It must be allowed to correct freely old ideas by new, holding no established truths as sacred or immune to criticism. This principle, moreover, applies just as obviously to the religious doctrines and moral standards of a community as to its scientific beliefs. In these matters, the tendency is especially strong for men to cling to traditional notions and the hoary institutions through which they find expression, resenting any intimation that these are not absolutely right; their deepest emotions of hope, fear, trust, comfort, and security are so intimately bound up with that prized heritage that any critical question in its regard seems like a threat to the very foundations of community life. But the most casual glance at moral and religious history shows that ideas now the object of such reverent attachment were once new, that only gradually did they gain power to replace older notions to which they proved superior, and that the fanatical worshipper of tradition is thus simply the rigid follower of a dead radical. It shows also that so far as this attitude of conservative loyalty governs social policy in these matters, no distinction can be drawn between the prophetic pioneer of a better way of life than that now generally accepted and the criminal rebel against rules necessary for social order; the former and his followers, although they are the greatest benefactors that society has ever produced, must win their way, if at all, only through the fire of persecution and martyrdom. If the Christs of our day are not to be crucified or the Socrates forced to drink the hemlock, freedom to propose and exemplify changes that touch even our tenderest sentiments must be allowed, and the community must learn through such freedom to distinguish changes that may be constructive from those that would destroy essential social values.

Social organization is for the sake of individual well-being

A guiding criterion of broad scope emerges from these ar-

guments which, when formulated, binds them together in a coherent whole. It is this: The proper relation between the various social institutions of a community and its individual members is that the role of the former is always to protect and serve the well-being of the latter. This principle then becomes the cornerstone of the individualistic theory. Well-being in the last analysis, it holds, is the well-being of individuals, realized in the exercise of the fullest measure of freedom that is consistent with community order and with the cooperation required by the complex activities of a modern social group. Institutions exist for the sake of their individual members. Especially is this true of the national state which, because it holds the reins of police and military power, is in a peculiarly favorable position to wield unjustified authority over individual citizens if its administrators so desire. Leaders who aspire to positions of governmental responsibility always need to be watched, because the very qualities which make them leaders in politics usually include firm conviction in the rightfulness of their policies and impatience with the restraints on executing them which the freedom of other people imposes. Accordingly, they are constantly tempted to seek greater power, wider discretion, and more detailed control of the activities of their fellows, in the interest of promoting more quickly the causes they have espoused. In the case of political leaders it is especially imperative to resist such demands for increased authority since, as individual citizens do less and less for themselves and entrust others with more and more power to do for them, the very habit of exercising initiative and assuming personal responsibility is likely gradually to become lost. Sooner or later, some would-be dictator will seize such a situation to extend his executive control without effective restraint from the legislative and judicial branches, and the community will become enslaved. To prevent this catastrophe it is necessary for individual citizens and minority groups sedulously to preserve their rights and, except in serious emergencies, steadfastly to refuse any increased authority to their executives beyond the powers necessary for the

maintenance of order and defence against external attack.

Further arguments for a maximum of liberty

Three special arguments may be added to these general ones, reflecting the atmosphere of discussion characteristic of the United States at the present time.[9]

For one, it is contended that the rather extreme individualism which has prevailed in the United States in the past is the regime under which we have become a virile and prosperous nation and have won the highest standard of living ever appearing anywhere in the world. Those who are impressed by this argument will recognize, of course, that the conditions under which this degree of individualism was possible have now changed—still, the question is one of emphasis and direction; shall we not continue to realize, wherever it is not clearly inappropriate, the pattern of life which has already proven highly successful?

For another, the contrary emphasis would constitute, it is insisted, an imitation in our own fashion of the totalitarian institutions which are fundamentally opposed to democracy, which intrinsically threaten the peace of the world, and whose baneful consequences in the lives of people in the dictator-ridden countries have now become clearly apparent. Let us be willing to learn from them the secret of greater decisiveness and greater coherence in action, but let us always do so in ways that will make these virtues instrumental to the welfare of individual citizens and to the peace of the world rather than a mark of their exploitation by tyrants who seek power without limit both at home and abroad.

For a third, it is urged that the extensive governmental control already realized in the United States during recent years has shown that the serious evils of bureaucracy are quite unavoidable when a nation moves in this direction. It makes no difference what the ideology guiding such an enlargement of centralized regulation may be—whether it is that of Fascism, Communism, or New Dealism—the net effect is to saddle a community with a horde of public servants who are more concerned to preserve the red tape of established routine than to

[9] This was written in 1944.

meet the pressing human problems by which they are faced, and who become psychologically incapable of seeing any possibility of further progress in the methods by which these problems are solved. Now it is only justifiable for a social process to be placed under the care of government rather than left to private initiative, if the realistic understanding that private enterprise encourages, together with its readiness for continued improvement, will be preserved while the larger measure of social responsibility made possible by political control is secured in addition. But if the gain in the latter direction does not clearly balance a serious loss in the other, the change is unwise. When one looks at many of the politicians who win positions of authority in the state, one sees that there is not even any guarantee of a deeper sense of human responsibility among them than can be taken for granted among unofficial wielders of social power. Enough of these evils have already crept into American administration to indicate the necessity of calling a halt on this program of socialization, and of returning at many points to the older regime of freedom on the part of individuals and minority groups to order their actions as they please.

The case for collectivism—general considerations

Such, then, is the case for liberty. Let us see now the contrasting argument for increased regulation of American life under present conditions. And here again we shall start with the more general considerations, passing later to those specifically enforced by the present problems facing the United States in a rapidly changing world.

First, in the long run, community order and cooperation are more basic than individual freedom. As is evident everywhere on the surface of the planet except among the democratic countries, societies can continue to exist and to make themselves effective with very little liberty for their members, but no group of people could continue to exist without realizing the conditions necessary for social order. Order, hence, does not depend upon the possession of any special degree of liberty by the mass of the citizens, while liberty, in any form which makes it desirable to anyone, does depend on the pres-

ervation of social order. There is no freedom worth anybody's having in a madhouse or a gangster hide-out, save the very precarious freedom of those who happen to be on top at the moment. It is absolutely necessary, in short, that people form the habit of respecting the main rules of justice, and of collaborating in providing the food, shelter, clothing, and other essentials that everyone requires; it is not absolutely necessary that any but the administrative leaders enjoy a considerable measure of freedom. Such liberty as people exercise at any given time thus rests on a foundation of order and security; any amount of the former may be surrendered under pressure of events, but the latter can never under any circumstances be surrendered without dissolving the society which has lost it.

This decisive circumstance indicates, second, that the basic task of a community which wishes to enjoy liberty is not to pursue that end directly wherever it seems possible to do so without serious danger. The task is, rather, to develop individuals who will find the liberty they wish in activities that contribute to the security and progress of the social whole. Freedom may always be exercised in two ways—in acts whose effect is openly or subtly to weaken the bonds of social unity, and in acts whose tendency in the short or long run is to strengthen them. It is clearly very important that the former kind of freedom be discouraged and the latter encouraged in every way possible. Now the policy of allowing a large measure of individual liberty, and only checking it by penalties when it breaks loose in antisocial forms or threatens to do so, is far from equal to this task. A society must take the responsibility, through education and all other forms of discipline that promise to be effective, to train its members in the way they should go, making sure that their first concern is not to seek personal happiness but to play their role acceptably in the life of the community. They must learn to find happiness in making their contribution to the enfolding whole apart from whose security no worth-while existence would be open to them at all. For individuals do not know, at first, what it is

that they really want, and often, when free from social restraint, pursue quite illusory goods. Disappointed when failing to win the objects they desire, they are still more disappointed when success in winning them has been achieved. For many things sought by impulsive desire turn to dust and ashes in our hands when found; they fail miserably to bring the satisfaction hoped for. We are really members one of another, but we are not likely to see this truth vividly and to transform our desires in accordance with it, except as a carefully planned course of education, and detailed social control in other forms, mold our habits and character so that they fit a wisely chosen social pattern. But this means that the dominant emphasis in the relation between any community and its individual members must be supervision and control rather than permission for everyone to do what he happens to wish.

True liberty is much more than absence of social control

Third, when we realize clearly that individuals can find liberty in various ways, and need wisdom in distinguishing true liberty from false, it becomes evident that the traditional notion of freedom as essentially a negative thing—the mere absence of social restraint—is radically inadequate. On the one hand, life cannot be divided into two distinct areas, one characterized by the exercise of freedom and the other by obedience to social demands. The more mature a person is, the more he finds that what he really wants to do is just what society expects him to do; he seeks no uncooperative freedom, for his highest aim is to serve the whole of which he is a part. On the other hand, freedom, if it is to be a practically significant affair, involves the possession of those facilities without which no one can pursue any intelligent ambition even when no restraint bars his way. Freedom requires security in life and limb, for without it a man is not free to do anything but protect himself against attempts on the part of others to enslave him. It demands cultural and technical training, for otherwise he is not free to find out what he really wants to do or how to carry out his purposes when they are formed. And it presupposes the possession of property—a place to call his home, some capital, at least a few modern tools—without

which he is unable to express his knowledge in action and make an assured start in whatever constructive tasks he is eager to perform. Apart from these things freedom is empty rather than real.

The American slaves were supposedly freed by the Emancipation Proclamation of 1863. But because, in the South especially, they have remained a downtrodden class, deprived of the positive opportunities and facilities that are necessary to effective freedom, their liberty has remained largely illusory. It is instructive to contrast this situation with that revealed in a statement made to Mr. Wendell Willkie on his visit to the Soviet Union in 1942. Mr. Willkie had suggested to the superintendent of production in a large factory that the latter had, after all, no freedom, because if he should happen to hold political or economic ideas that are unorthodox according to Marxian standards he could not express them. The superintendent replied, "Mr. Willkie, you don't understand. I've had more freedom than my father and grandfather ever had. They were peasants. They were never allowed to learn to read or write. They were slaves to the soil. When they sickened, there were no doctors or hospitals for them. I am the first man in the long chain of my ancestors who has had the opportunity to educate himself, to advance himself—to amount to anything. And that for me is freedom." [10]

To be sure, this program of training individuals to be truly free, and of providing them with the conditions essential to positive freedom, can only be carried out by granting large powers to our political leaders. And there is a danger in this; vast power and authority tend insidiously to corrupt those who wield them. But if the performance of their high tasks is kept responsible to the people through fair and frequent elections (and the other democratic safeguards tested by experience), this tendency can always be checked before it brings serious harm. The proper motto is: Give large powers to your chosen leaders, and then be vigilant to see that they are exercised in the interest of true social welfare. The average

[10] *One World*, p. 68.

citizen, if left alone, does not know how to realize the virtues of responsible citizenship, but carefully chosen leaders can be trusted to train those over whom they have authority to fit into an orderly social pattern.

Now this effective subordination of the desires and interests of individuals to those habits of cooperation that dependably promote the welfare of the whole, which is always necessary in the long run, is still far from realized in the democratic countries, and especially in America. The general conclusion to be drawn, then, is that extension of social control over individuals and minority groups, at present, marks the path of sound progress, and should be our policy wherever it promises to be effective.

A guiding criterion emerges from these arguments also, which gives them cohesiveness when it is clearly stated. It is that the true role of the individual members of a community is to serve the whole of which they are a part, and therefore to view the institutions which symbolize that whole, especially the national state, as having rightful authority over them. This principle then becomes the cornerstone of the collectivist theory. In its light, the state does not exist merely to serve its individual members; its greatest task is so to order their lives that they will become as capable as possible of fulfilling their social responsibilities in a modern community.

Special arguments

In addition to these general arguments, the champion of this position finds two special reasons which seem to him peculiarly compelling when one contemplates American history and considers the postwar problems which confront the United States. The latter, all are agreed, will be of terrific complexity, and will require continued collaboration with those nations in whose company the war has been won. These problems include demobilization of enormous armies of soldiers and providing stable employment for them, the restoration of normal economic processes throughout the world, the progressive elevation of standards of living everywhere, relief and tactful supervision of the conquered countries to the end that they may become willing partners in the pursuit of

peace and cooperation between nations, the organization of some world federation with capacity for flexible readjustment and sufficient power so that no nation will be tempted to gain its ends by violence, and the preservation at home of those fundamental securities that people everywhere demand as just and right.[11] Now these problems are not problems that can be solved in a few years; they will require decades, if not generations, before any comprehensive solution can be really attained. How is it possible to meet problems of this sort by a return, in any degree, to the individualism of an era of national isolation and factional irresponsibility? Rather, they obviously demand a large further extension of the controls exercised by government, especially the national government, for only thus can the large-scale planning that is necessary be implemented in action. The policies to be carried out require that American business men, labor organizations, farmers, and other groups, all gear their activities into a single coherent pattern of world-wide collaboration.

The other reason lies in the ghastly evils resulting from the turning of liberty into license, which American history of the last fifty years prominently exhibits. The robber barons of the epoch of railroad expansion, the jugglers of utility systems during the 1920's, are but outstanding illustrations of the numerous ways in which freedom of economic initiative in the United States has been exploited to build enormous private fortunes and autocratic empires in defiance of the public interest. The frequent success of such antisocial promotion schemes has even had its insidious effect upon the American character; the avowed hero of ambitious boys has all too often been the clever manipulator who knows how to put across big deals and cares little whether they are accomplished by shady means or not. When we note that many of those who today clamor for a return to the libertarian tradition of the past appear to want a renewed opportunity for exploitation of this kind, we see that to accede to their wishes would be a tragic mistake,

[11] Including security against antisocial use of knowledge of how to split the atom.

plunging us once more into hideous evils from which we had at last begun to escape. Whatever the dangers of socialization may be, they can hardly threaten, as seriously as this retreat would do, the security with justice at home and wise cooperation abroad that are basic to any further progress of mankind toward sane ends.

Such, then, are the contrasting arguments. How can this conflict between two theories of what constitutes the ultimate essence of social value be subjected to rational adjudication? Where can an adequate criterion applicable to both be discovered? Let us see in our final chapter how this problem would be attacked by the method of democratic cooperation.

EXERCISES

1. Explain clearly just how the relational theories of value differ from the psychological theories. From the realistic theories.
2. State each of the ultimate criteria which seem to be implied by the argument for individualism. Those implied by the argument for collectivism. Is there any common criterion that seems to be recognized by both arguments?
3. At just what points in the arguments do the two Supreme Court decisions summarized in the chapter reflect divergent value-theories?
4. Study the majority and minority opinions in the following case, where the issue is whether a certain petition states a cause of legal action or not. On what vital point do the opinions conflict? What fundamental differences of evaluation seem to be involved, and how do they affect the interpretation of important concepts, such as that of business "competition"?

The Passaic Print Works, a manufacturer of calicoes established in Passaic, New Jersey, appealed to court for damages against the Ely and Walker Dry Goods Co. of St. Louis. The petition alleged that the defendants, who were jobbers in St. Louis, having on hand a limited quantity of calicoes of the brands which the plaintiff had been accustomed to sell in largest quantities to St. Louis jobbers, issued circulars to retail dealers, in which they offered to sell these brands, as long as their stock should last, at prices below those asked by the plaintiff of jobbers. The petition further stated that plaintiff was informed and believed that this action was taken by the defendants, not for any

legitimate trade purpose of their own, but for the purpose of injuring the business of the plaintiff; and that the action did in fact injure and destroy the plaintiff's trade in St. Louis and the adjacent territory, by causing other jobbers to cancel their orders to the plaintiff, and by compelling the plaintiff to reduce its prices to meet those charged by the defendant. The central questions before the court concerned the legal status of business competition and the legal significance of the motive behind an act.

The majority opinion was as follows:

" . . . The owner of property, real or personal, has an undoubted right to sell it and to offer it for sale at whatever price he deems proper, although the effect of such offer may be to depreciate the market value of the commodity which he thus offers, and incidentally to occasion loss to third parties who have the same kind or species of property for sale. The right to offer property for sale, and to fix the price at which it may be bought, is incident to the ownership of property, and the loss which a third party sustains in consequence of the exercise of that right, is *damnum absque injuria*. We are thus confronted with the enquiry whether the motive which influenced the defendant company to offer for sale such calicoes of the plaintiff's manufacture as they had in stock at the price named in the circular, conceding such motive to have been as alleged in the complaint, changed the complexion of the act and rendered the same unlawful, when, but for the motive of the actor, it would have been clearly lawful."

After summarizing in the light of precedents the conditions under which a bad motive changes the legal complexion of an act, the opinion reaches the following conclusion:

"It is wiser, therefore, to exclude any inquiry into the motives of men when their actions are lawful, except in those cases where it is well established that malice is an essential ingredient of the cause of action, or in those cases where, the act done being wrongful, proof of a bad motive will serve to exaggerate the damages.

"The case at bar falls within neither of the exceptions to the general rule above stated—that, if an act is done in the exercise of an undoubted right, and is lawful, the motive of the actor is immaterial. No one can dispute the right of the defendant company to offer for sale goods that it owned and were in its possession, whether the quantity was great or small, for such a price as it deemed proper. This was the outward visible act of which complaint was made, and, being lawful, the court will not hold it to be otherwise because of a secret purpose entertained by the de-

fendant company to inflict loss on the plaintiff by compelling it to reduce the cost of a certain kind of its prints or calicoes."

The plea that in consequence of the act of the defendant certain jobbers were induced to break their contracts with the plaintiff is denied legal value on the ground that no names are mentioned of persons thus influenced, nor was it shown that jobbers were not privileged to cancel orders at pleasure. In the light of all relevant considerations, therefore, the decision is that the complaint did not state a cause of action.

One of the judges, however, presented a dissenting opinion. " . . . It is conceded that, if the defendant had advertised these prints for any legitimate trade purpose, for the purpose of selling them for gain for themselves, for the purpose of converting them into money because they preferred their advertised price to the goods, or for the purpose of competing in trade with the plaintiff, they would have had a justifiable cause for inflicting upon it the damages of which it complains, and these damages would then have been *damnum absque injuria.* But, if they had advertised them for any of these purposes, this case would have constituted an exception to the general rule of law. The general rule is, that whenever one injures a man's business, profession, or occupation he is liable for the damages he inflicts. The exception is that, where the injury is caused by competition in trade or the lawful exercise of a right which the inflictor has, then the injury is justifiable and no damages can be recovered. But, where such an injury is inflicted, the presumption always is that the rule, and not the exception, applies, and if the inflictor would justify, he must show that he falls within the exception. The question in this case, therefore, is not whether or not the motive or intent of the defendants will make acts unlawful which were otherwise lawful, but whether or not the intent and purpose of the defendants will justify an otherwise unlawful act, and excuse them from the payment of damages for which, under the general rule of law, they are liable to the plaintiff. It is whether or not the petition shows that they advertised the goods for legitimate trade purposes, so that their acts fell within the exception, which justifies the infliction of damages, and not under the general rule, which requires them to compensate the plaintiff for the injury they have caused. The opinion of the majority assumes that the defendants advertised the prints for a legitimate trade purpose, so that their acts fell within the exception to the general rule. It overlooks the legal presumption that injury to one's business entitles him to compensatory damages, and the plain averment of the petition that the acts of the de-

fendants were not done for any justifiable cause, but were committed for the sole purpose of inflicting upon the plaintiff the injury they caused. . . .

"Now, no one will dispute the rules of law that the plaintiff in this action had the right to conduct its business of manufacturing and selling prints without the injurious interference of strangers, and that the defendants were subject to the universal rule that they must so use their own property and rights as to inflict no unnecessary injury upon their neighbors. The averments of this petition are that they were not using any of their property or exercising any of their rights for any legitimate trade purpose, but that they were using them for the express purpose of inflicting injury upon the plaintiff, and that they succeeded in imposing the infliction. These allegations seem to me to bring this case under the general rule of law, and to clearly negative the claim that it falls within the exception. . . ."

The judge then cites a number of precedents, whose tenor is well summed up in the following statement from the Massachusetts Supreme Court, which he quotes:

"'Everyone has the right to enjoy the fruits and advantages of his own enterprise, industry, skill, and credit. He has no right to be protected against competition, but he has a right to be free from malicious and wanton interference, disturbance, or annoyance. If disturbance or loss come as a result of competition, or the exercise of like rights by others, it is *damnum absque injuria*, unless some superior right by contract or otherwise is interfered with. But, if it come from the merely wanton or malicious acts of others, without the justification of competition or the service of any interest or lawful purpose, it then stands upon a different footing. . . .'

"Under the legal principles to which reference has been made, and under the authorities which have been cited, the petition in this case states a good cause of action for interference with and injury to the business of the plaintiff by preventing it from obtaining custom it would otherwise have obtained, without any justifiable cause or excuse, and for this reason the demurrer should have been overruled, and the case sent to trial." [12]

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CHAPTER 29

COOPERATIVE EVALUATION

The problem of method in dealing with conflicts of value-theory

What procedures promise to point the way toward a reasonable resolution of such conflicts as the preceding chapter has illustrated? We are ready to consider this question in detail. It has been intimated that these conflicts can be adjudicated by the use of a definite method for securing maximum rationality in judgments about ends; let us show what this method is and how it can actually perform such a role. Thus far only its general character—as a form of democratic cooperation—has been indicated.

But before we plunge into details it is important to state again the significance of this method in the perspective of our whole analysis of reasoning as evaluation. Even before that analysis commenced, it became evident that the basic logical difficulty about evaluation appeared in those situations where the values that are involved cannot be allowed for by different thinkers in the same way; where they can be definitely allowed for there is no special problem. Then the analysis showed that these situations were those in which the values are not means to some clearly understood end—here is the field, in general, of applied science—but are ends in process of being determined; since they have not yet been given definite shape there is no way of accurately taking them into account. Worse still, different people often choose ends which require contrary courses of action for their successful pursuit. As we proceeded, the form which such conflicts take when they are pushed to their ultimate grounds emerged clearly. Each competing end implies, directly or indirectly, some definition of the most general concepts employed in

whatever field of value is involved. Now different philosophies of value, championed by different schools of thinkers, adopt different persuasions as to how those ultimate concepts should be defined; thus each value-theory is committed to a distinctive set of criteria for resolving any problem about ends. Unless, therefore, we can see some way of rationally adjudicating between these theories, recognition of their competing claims leads inevitably to an unrelieved scepticism in the field of end-judgments, and to dependence on force rather than reason when the conflicts must be resolved for purposes of social action.

In the past, this problem has appeared almost insoluble. Each school has been blithely confident in its commitment to its own criteria; it has, therefore, taken for granted that reconciliation with its rivals can only be satisfactorily secured by conversion of the latter to its own viewpoint. To be sure, this assumption has not always been conscious. Two opposing groups of thinkers will often, so far as their overt attitude is concerned, seek to understand each other's position and to find some satisfactory principle of mutual adjustment. But they have usually failed to realize that unless they succeed in detaching themselves from their adopted theories sufficiently to view them tentatively rather than absolutely, this quest is hopeless. In the absence of such detachment, no genuinely common understanding between them can result, let alone any impartial basis for reconciliation. The reason for this is that any concept employed in the discussion will be interpreted by one group in terms of its ultimate criteria, and by the other group in terms of its rival criteria. We have seen that the meaning of subordinate concepts in any field of value is determined by the meaning of its most general concepts, and that if the assumed definitions of the latter are different, then the significance of all the concepts employed will be different. There might be considerable verbal similarity because of the tendency of different theories to use the same words, but this only increases the confusion when there are no common referents for those words. In such a situation, no progress

toward any real reconciliation can be made. After the verbal sparring is over, the crucial issue arising from their ultimate divergence of conviction remains just where it stood before.

Consider how this point is exemplified by the issue in social philosophy outlined in the last chapter, between the individualist and the champion of more extensive social regulation. The former would solve the conflict by asking what form and degree of social control best promotes the richest and freest life for every individual member of society. But this is precisely his ultimate criterion, as a champion of freedom, for determining how the rights of individuals and the powers of society should be balanced. The latter would solve the issue by asking what kinds of freedom for members of the community best assure security, effective cooperation, and vigor in the social whole. But this is his ultimate criterion as a collectivist. As long as each sticks confidently to his own procedure for resolving whatever disputes about social policy arise, it is evident that a fundamental impasse is faced; each is really demanding that the other be converted to his own viewpoint if the conflict is to have any chance of terminating in agreement.[1]

Some practicable procedure is demanded

But one who has followed the analyses in the preceding chapters appreciatively will find this impasse both intolerable and irrational. Having considered with sympathetic fairness contrasting viewpoints in each of the fields of value discussed, he will be conscious that each has something substantial to say for itself that ought not to be lost from sight in any reasonable adjudication of the conflict; he will realize that he would be poorer in evaluational wisdom rather than richer if he committed himself absolutely to any one of the theories to the complete exclusion of its rivals. Every bit of the insight that sincere, earnest, and intelligent men have gained in reflecting on these matters will not seem to him

[1] In a sense, the constructive method soon to be proposed demands this too. But it is not a sense which allows any imposition of one thinker's beliefs upon others. Each places his own previous criterion in jeopardy, on an equal basis, so that a genuinely impartial criterion may emerge. What all must be converted to is willingness to participate in this process.

more than he needs in trying to find out how to choose his ends aright. Since at present, however, such insight finds expression in theories of value that stand in apparent contradiction to each other, he cannot help seeking a dependable method for their critical assessment and reconciliation. And such a method, if it can perform this role successfully, will clearly constitute for him the embodiment of reason when beings capable of reason face these crucial issues of value-theory.

Happily, in our analysis of the criteria that are actually employed in forming end-judgments, we found the central idea of the method needed. In any situation where the concerns of other people are affected, a thinker who wishes to be reasonable must follow the rule of impartiality—the rule, that is, which so guides any such process of social adjustment that the different ends of all people concerned will be allowed for on an equalitarian basis. This principle gave us our essential clue, for it seemed clearly to apply to cases of conflict between different theories of value. It implied that there is a procedure of democratic cooperation by which such conflicts can be reasonably resolved, but that procedure has as yet been described only in very general terms. We must now explain it more fully and precisely.

An adequate explanation of what democratic cooperation involves when it is employed in meeting this problem will answer two main questions. First, what are the essential prerequisites to any application of the proposed method; and second, what specific sequence of steps would one pass through who proposes to follow it conscientiously?

The prerequisites of democratic cooperation —1. Frank and modest recognition of differences in evaluation

We begin by stating the prerequisites to its application.

The first prerequisite is frank recognition of the fact that people honestly differ in their ultimate convictions regarding values, and that there is no greater initial likelihood that a given thinker is right in any conclusion on which he differs from others than that they are right against him. When one recognizes this clearly, he will forswear all dictatorial dogmatism, all pretense of absolute assurance, and will hold his

convictions tentatively, subject to revision and correction. He will say, in effect, "I see clearly that the mere fact that I have been led to a particular judgment about some end does not give it any greater likelihood of being the reasonable one than the fact that someone else has been led to a different judgment gives it the right to claim that it is justified. And when I view in this way the criteria which at present seem to me appropriate I am eager to understand the contrasting criteria of others, so as to find out what factors they have taken into account that I might have neglected, in the light of which my own might be rendered more adequate. What I want is the very opposite of suppression, so far as concerns other people's right to determine and announce their ends; I wish for them the freest opportunity to champion the values suggested by their experience of good or bad, so that I may have the widest range of alternatives to consider in correcting the ends to which my vision has previously been limited."

Notice how completely this attitude contrasts with that of the would-be dictator. The latter is so certain of the rightness of his position that he wants to leave no opportunity for his judgment about ends to be corrected; he is willing to suppress, for this reason, the contrary contentions of others, so that his present convictions will be left in full possession of the field. The cooperative thinker is so certain that no individual, at present, is wholly right, that he seeks every opportunity and encouragement for his evaluations to grow wiser through critical comparison with those of others who differ. There is something much more than tolerance here. Tolerance gives others an equal chance, because one does not want to be deprived of an equal chance oneself. The attitude of democratic cooperation reaches far beyond this, for it finds a positive good in the differing evaluations of others—the good of embodying a mine of suggestions whereby one's own choice of ends can be made progressively keener, more realistic, and more inclusive. Just as the scientist is glad rather than sorry to have available a variety of hypotheses regarding any moot problem in his field, so that the chance of finding a satisfac-

tory answer is increased, for the same reason the open-minded inquirer in the field of ends is glad to have a variety of evaluations available for his consideration.

Important consequences emerge from such a statement of this first prerequisite, which may be clearly illustrated by reference to the issue in legal and social philosophy discussed in the preceding chapter. It is evident that this attitude of democratic cooperation requires of the individualist that he abandon his traditional contention that some freedoms for the individual have the status of "natural rights." For as long as one views them in this fashion they seem to him to lie beyond the possibility of legitimate criticism—he thinks of them as absolutely demanded in virtue of the native endowment of man. He feels that he must therefore cling to them without budging, come what may. Similarly, it requires of the champion of governmental regulation that he abandon all fanatical adherence to any particular form of collectivism as intrinsically required by the social nature of man. Each of these rivals must sincerely and fully recognize that claims to individual right, and likewise pretensions to institutional authority, are in no case final. Reason must judge them by some criterion of comparative value that will gradually emerge from the impartial analysis and systematic synthesis of these competing viewpoints, and any sort of dogmatic commitment, explicit or implicit, is entirely inappropriate because it blocks this procedure at the very beginning. The same principle applies in any conflict of ultimate ends, and of the theories by which they are justified.

2. Accepting and protecting an open arena for thought and discussion

With this basic point stated, the second prerequisite becomes clear. What is the essential social condition that is necessary if this continued growth in evaluational wisdom on the part of each member of the community is to be encouraged, or even freed from serious obstructions? The answer is: an open arena for thought and discussion of all questions involving alternative values when they arise. Each participant will want his rivals to be free to present their ideas on matters of value as persuasively as they can, so that whatever is

good in them will have the fullest chance to exert the influence that it deserves to exert upon those who have previously neglected or unduly depreciated it. The vital requirement for all participants—never to be subordinated to any other consideration—is the determined maintenance of a free field for discussion. The cooperative thinker, then, will accept the responsibility to play his full part in this social task. And acceptance of such a responsibility implies that all who do so, whatever form their present theories of value may take, have really agreed upon a certain supervalue as of transcendent worth—namely, the value of continued growth in insight into desirable ends through wider mutual understanding and the constructive criticism which it makes possible. All have tacitly accepted the principle that the ends which their several theories, taken out of this context, would imply as supreme, are subordinate to this still greater and more inclusive end; they are to be continually clarified and corrected in whatever manner loyalty to this higher end may require. So far as representatives of different schools are willing to join wholeheartedly in this process of searching discussion, and combine their forces to maintain it, the Kantian's final value is no longer respect for the Golden Rule, in his sectarian interpretation of it; the Utilitarian's is not universal happiness; the individualist's is not the maximum of freedom; nor is the collectivist's social harmony as he has previously defined it—all have accepted as still more important the value of progressive enlargement in their vision of what is good and right through the contribution that each can make to that process. The supreme end for all is continued cooperation in the clarification and realization of the varied goods that appeal to the desire and aspiration of men.

3. Readiness to participate in formation of new theories and the systematic reconciliation of those now available

Third, when one begins to participate in this democratic process of unhampered growth, he finds himself ready to advance in two complementary directions. On the one hand he will recognize that even the various theories of value now available do not take into full account all the desirable ends that man is capable of glimpsing; he will therefore be eager

to gain pioneering insight into novel values that have thus far escaped all contending theories. When these are clearly formulated, they will add to the stock of philosophies of value now competing in the arena. On the other hand he will want to aid in the process of reconciling present theories, by finding how to detect the dependably positive values of each and to combine them in a more inclusive synthesis. So far as this can be done by the sequence of steps soon to be sketched, a larger measure of uncoerced agreement will be secured among those who had previously been opposed to each other.

Now the former of these two modes of progress, though exceedingly important, apparently cannot be given any detailed methodology.[2] All that can be said at present about such a quest for novel perspectives of human good seems to be that one who seeks to contribute here must be sensitive to previously unnoted values in the experiences that come to him, and must be ready to see what plausible theory of desirable ends would be suggested if any value previously unemphasized were given a central place. His is a pioneering task which knows few rules. But the latter enterprise can be given a systematic technique, under guidance of the idea of democratic cooperation in the determination of ends, and this technique will now be explained in its essential features.

The steps of cooperative method: a. Provisional acceptance of all competing theories

It includes five distinguishable steps, in a definite sequence.

a. Each participant must adopt impartially all the theories which he proposes to try to reconcile, not merely the one theory that his own thinking has previously inclined to accept. The reason for this requirement is that unless he does so he will be unable to consider seriously and sympathetically the constructive values that each of them may embody, nor will he be ready to give them the weight they deserve in the synthesis he is eager to work out. He will assume, unconsciously, that the theory toward which he naturally gravitates is just for that reason more likely to be right than the others. Hence such generous and inclusive adoption is necessary, however difficult psychologically it may be. Of course, it is

[2] See below, p. 728.

important to note clearly in what sense of "adoption" this is to be done. It would be quite impossible for a thinker to adopt more than one theory as unqualifiedly adequate, since some assertions in each contradict some assertions in the others. But they are to be adopted tentatively, subject to continued criticism and revision. On this account it is possible to welcome simultaneously any number of differing theories, just as it is possible for a scientist to entertain at once several different hypotheses between which he is trying to decide by empirical verification. We are not forgetting that when a thinker engages in action he will need to adopt some theory—the one which at the time seems to him most adequate—in another sense, namely, as an immediate practical guide. But we are now considering what is appropriate when he is assessing various criteria of value and trying to find the criterion that is best.

Except those which are essentially uncooperative

This hospitality, however, must be qualified in one fundamental respect. Any theories of value that by their very nature are inconsistent with the attitude of cooperation on an equalitarian basis cannot be included in this generous fashion, but must be rejected outright. That is, convictions about value which are undemocratic in their essence must be regarded as intrinsically irrational and necessarily inadequate. And it is important to know how to identify them, both for this reason and because the danger from their presence must be socially guarded against.

To take the latter point first, the protagonists of any theory must be assumed to be undemocratic if they betray, when they enter the arena of discussion, intolerance of other convictions. The democratic thinker will allow these persons freedom of speech, but no opportunity to express their intolerance in action, for there would be a practical contradiction in such generosity. And the reason is clear. Whenever given a chance, as was observed in another connection earlier,[3] the champions of intolerant philosophies are likely forcibly to suppress upholders of competing theories and to compel ac-

[3] See above, chap. 26, pp. 620 ff.

quiescence in their own. The would-be dictator naturally resorts to compulsion whenever he sees any prospect of success, for since he is convinced that his values are absolutely right and all others are absolutely wrong, why should he unnecessarily tolerate the presence of falsehoods that lead people astray? Thus an unwise generosity leads to the victory of ungenerosity; the social condition essential to the further exercise of hospitality and friendly cooperation will have been destroyed. Obviously, no intelligent man can afford to give any of his rivals a free field to achieve such a solution as this. He does not wish his cooperation to become the effective cause whereby the very opposite attitude to that of cooperation assumes control of the scene.

But even if intolerance is not openly displayed, convictions are shown to be undemocratic by their very content if they include doctrines that specifically claim opportunities and basic rights for certain persons or groups which are not allowed to others. Examples of such are the doctrine that the white race is superior to all other races and has a right, therefore, to reduce them to the status of colonial dependents, and the doctrine that some particular ecclesiastical body has authority to order the conduct of other persons. These notions are clearly irreconcilable with the attitude of democratic hospitality and therefore are incapable of free cooperative synthesis with other theories of value. They are incompatible with openness to continued growth through mutual sharing. Of course, any opponent of democracy may hold some doctrines whose content is not affected by his undemocratic views; these will not need to be rejected unless the rejection proves to be required on other grounds after critical evaluation.

b. An impartial language in which to compare them

b. Next, cooperative evaluation requires a procedure by which sharable meanings can be given to all value-concepts employed, that is, an impartial common language in which to discuss whatever issues are at stake. The significance and necessity of this have been seen already; we have noted that each competing philosophy has its own distinctive way of

solving conflicts between itself and its rivals, the way implied by its own chosen criteria which rest upon its definitions of ultimate value-terms. The very meaning of all words denoting ends, as it uses them, is determined by these criteria for telling what is essentially right and good, and they are therefore logically incomparable with the same words as employed by any other philosophy. Each theory of value, in brief, determines its own distinctive universe of discourse for the discussion of any problem concerning value. For the Kantian, the concept "ethical goodness" means unconditional respect for the Golden Rule; it implies nothing with regard to human happiness. For the Utilitarian the same concept means devoted pursuit of the greatest general happiness; it implies nothing with regard to any binding rule in the Kantian sense. So long as this is the case, the conflict between the two is necessarily irreconcilable, for they are not talking about the same things by the words that provide the counters of discussion.

Consider as a further illustration of this point a central concept in legal and social philosophy, such as the concept of "justice." If one defines "justice" in terms of the individualist's criteria, and again in terms of the collectivist's criteria, it is clear that he is dealing with two virtues and two sorts of obligation rather than one. For the decision as to what is the just thing to do in any given situation is reached by two different procedures, and it is never certain that the search for justice will, in both cases, lead to the same solution of a puzzling social problem. In many instances it would, but in many it would not. Suppose, for example, that the question is raised: How should an inheritance tax be graduated according to the value of the estate? From the individualist's viewpoint a just solution would be one which protects the right of all individuals to amass property and to determine its disposal on an equal basis. The same percentage of tax on large and small estates might easily appear just to him, in the light of this concept. Whereas, to a collectivist, a just solution would be one which promotes the largest measure of social

unity. From this standpoint, a tax which takes a much larger percentage from the estates of the rich than from the estates of the poor might easily appear just. Thus the two social ideals, though referred to by the same name, are not identical, and do not lead in all cases to the same results. If, however, one can define each of them in neutral terms, as denoting such and such a species of "social value," for instance, and then see what each definition would imply in the situations which our unsophisticated common sense picks out as those to which the notion of justice is relevant, the two concepts become impartially comparable.

Clearly, then, progress toward greater agreement on these issues cannot be made until a basis of mutual understanding is secured with respect to what it is that they are talking about. So the step essentially needed at this juncture consists in establishing a common realm of discourse—in finding a way of cooperatively determining some single, mutually acceptable meaning for each of the value-concepts that participants in the discussion propose to use. Otherwise, when we debate these matters we never really get together; our several assertions pass each other on parallel lines that never meet. Without a set of definitions which transcend the cleavage between rival schools of thought, no medium is available for talking about the issues upon which we want to focus our joint attention, except with persons who belong to the same school of value-theory as ourselves.

There is a further, more specific reason for setting up these cooperative definitions. In their absence one must perforce use, in discussion with the holder of some competing value-theory, either the latter's terminology, determined by his chosen criteria, or one's own. Now one might generously be willing, in the interest of fruitful argument, to adopt the first of these alternatives and try to translate his own position into that unaccustomed terminology. Indeed, a cooperative thinker would always be willing to make such an attempt. But he would find that this is not always possible. Some issues that seem relevant to him, and perhaps quite important, are not

capable of adequate statement in the conceptual medium provided by his rival. Let us take again an illustration from the conflict between Utilitarianism and Kantianism in the field of ethics. If a Utilitarian should attempt to discuss his differences from Kantianism in the latter's language, he would meet no serious difficulty in stating, in that medium, his insistence that the happiness of every member of the moral community is to count equally with the happiness of every other; for this is one of the implications of the Kantian Golden Rule. But he would meet a sheer impossibility when he tries to state in that language one of his major problems as a Utilitarian—the problem of comparative evaluation of different sorts of pleasure. For to the Kantian this is not an ethical problem at all. Before the latter could deal with it this would have to be split into two questions: a psychological one—the question how people actually decide between alternative objectives; and an ethical one—how they should decide which objectives, if any, are right. In this division, the problem as conceived by the Utilitarian has disappeared, and has become replaced by a somewhat different perplexity. Hence, until a cooperative terminology is established, discussion of these matters is almost futile. Either the concepts employed remain quite incomparable, or else one or the other disputant must abandon problems that are genuine and important to him, because of inability to state them in his rival's language.

Such a consideration implies, of course, that even after a system of cooperative definitions has been established, it will need to be progressively revised as new schools of value-theory appear, not all of whose important problems are capable of formulation in the language currently used in the arena of discussion. This necessity complicates the problem of democratic cooperation, but it is unavoidable.

The essential principle here brought out may be illustrated from the history of science. As new scientific problems emerge, it is not always possible to state them in concepts already available without redefinition of those concepts; implications of the current definitions may be foreign to the

essential nature of the problems faced, and quite irrelevant to fruitful investigation of them. Thus early modern scientists found it necessary on this account to redefine all the fundamental terms of physical science, such as "matter," "form," "space," "force." In value-theory the same need is obvious.

How this can be accomplished

When a thinker states his position in terms of such neutral (and therefore more general) concepts, by that very fact he indicates his willingness to view it in impartial relation to its live alternatives instead of insisting on its finality and demanding that the views of others be fitted into the realm of discourse which it determines. And it is evident that the selection of such concepts cannot be guided merely by the formal consideration of securing more general ones; any concept, however inclusive, can be interpreted differently by different philosophies if they insist on so interpreting them. The selection must be guided by the cooperative purpose of finding an inclusive concept which can be shared on a nonpartisan basis, in terms of which the position of each can be stated without prejudice. The search for such a category testifies to one's desire to recognize in advance positive values in the theories of others, and to encourage them to take the opportunity and accept the responsibility to show just what those values are.

It is requisite, then, to develop cooperatively a system of concepts in terms of which the conflicts of value-theory can be impartially stated, and which are sufficiently inclusive in scope so that all problems felt to be real by any competent participant in the discussion can be formulated in the medium thus provided. In the case of the ethical and social conflicts above used for illustrative purposes—between Kantianism and Utilitarianism, and between individualism and collectivism—philosophy has already haltingly endeavored to do this, and we have been profiting by the result. Its technique has been to replace "good" and "right" as ultimate concepts by the still more general concept of "value." To make "good" basic in ethical theory at the present time is to commit oneself un-

avoidably to associations with Utilitarianism, while to make "right" basic is to accept the context of a Kantian approach; "value," by contrast, is neutral as between these schools. It could serve in this role because it was taken over from the field of economics, where it had already acquired a quasi-scientific impartiality and precision. Having gained this character, it could be used as a medium for unprejudiced statement of each of those positions in ethical theory without loss of anything really significant; "good" is "value as determined by a process of comparing ends," while "right" is "value whose authority is determined by its relation to a universal rule or law." Similarly with the issue in social philosophy. "Freedom" is "social value as realized in the fullest feasible development of the individual members of a community," while "regulated order" is "social value as realized in the constructive interaction between individuals, to promote the unity of the whole." What is further needed to complete the clarifying process initiated by the general acceptance of this neutral term, is systematic elaboration of the concept "value" under the guidance of such definitions as these, to provide impartial meanings for all other important concepts in the field. Thus a vocabulary will become available for the cooperative statement of all relevant problems that really perplex thinkers, and for comparing the various solutions proposed.

What, more explicitly, is meant by the "systematic elaboration" of a chosen ultimate concept in this manner? Well, first, each contestant will do what has just been briefly illustrated, *i.e.*, state his claims in terms of this neutral concept instead of in the partisan language which he has formerly used. That is, the individualist will accept the responsibility to describe impartially the distinctive kind of social value that individual freedom realizes, while the collectivist will assume the obligation to specify the distinctive kind that is secured by community control of the individual; the two theories are then rendered genuinely comparable. Second, all other concepts required in their discussion will be redefined in the same way, so that the use of none of them will prejudge the issue

in favor of one side or the other. Thus the word "cooperation" will probably be necessary; it must be so defined as a species of social value that it will imply, when used by both participants, neither the voluntary collaboration of completely free individuals that the champion of liberty would originally mean by it, nor the regimented interaction imposed by some source of authority, naturally preferred by the champion of social control. Or, if "cooperation" is to mean one of these things, some more neutral term must be chosen by which to refer to what is common to them both. It is imperative, in brief, to emancipate debate from the situation in which activities called "collaboration" by one are called by the other "regimentation"—a situation in which discussion can accomplish nothing because the arguers insist on using words differently.

c. Comparative analysis in terms of this language

c. As such a terminological apparatus is rendered available, it becomes possible to analyze each distinctive theory of value in terms of that neutral and impartial language.[4] This places thinkers in a position to compare any pair of competing theories in systematic detail. Until a common language in which to state them has been established, this is impossible; the theories are not comparable point by point but are only pragmatically comparable as a whole. That is, one can compare over the years the type of character exhibited by a conscientious Kantian, and the conduct in which it finds expression in various situations, with the corresponding character and conduct of a good Utilitarian; he reaches, thus, some basis for judging which of the two best meets in the long run the ethical needs of man and society. But as long as this is all that can be done, there is no rational basis for choice except between those two alternatives as a whole; no way has been

[4] It should be borne in mind that these two processes must, to a large extent, go forward concurrently. The language cannot be invented first, and later employed in impartial analysis, except in the sense that its most general concepts must be fixed upon before the analysis can begin. Otherwise, the invention of the language and the neutral analysis of conflicting theories are two sides of the same process; the appropriate subordinate terms of discourse are discovered in and through the dissection of those theories in such a way as to make them impartially comparable.

discovered for selecting the dependably valuable features of each and synthesizing them in a philosophy superior to either. If this is to be possible, we must be in position to break up each theory impartially into its significant details, seeing thus not only how it guides people in general but also what specific factor in it is responsible for the distinctive results reached in this, that, or the other value-situation. Then we will be able to see if and how Utilitarianism deals more wisely with some situations and Kantianism more wisely with others; and we can take rational account of this circumstance.

Issuing in tables of agreement, contradiction, and possible synthesis

Since such an analysis is controlled by the definite purpose of guiding a systematic comparison of competing theories, it will be conducted in such a way that when the comparison is completed its net result can be embodied in three tables. Those tables will reveal: (1) points of essential agreement between any two theories undergoing comparison; (2) points on which they flatly contradict each other; and (3) points on which there is neither agreement nor contradiction. Let us note illustrative items falling under each of these three kinds that would undoubtedly appear when the theories of individualism and collectivism are analyzed in this way.

Both these theories agree that it is well for people to be free when their freedom does not obstruct the security and unity of society; that the talents and resourcefulness of individuals constitute an asset to any community; and that the freedom of an individual must be regulated when it threatens definite harm to others. So far as such points of agreement are concerned, we will naturally take it for granted that they will become a part of whatever theory is adopted as more adequate than either of these rivals. Only when they prove to be in conflict with some other plausible theory in the field will they be placed in doubt.

On certain points the two theories are in flat contradiction. For example, the individualist holds that absence of constraint, as such, is a good thing; the collectivist denies this. The individualist contends that the best government is that which governs least; the collectivist denies this. What the lat-

ter wants is not the minimum of government, but the kind of government which most fully develops the capacity of men to cooperate toward social ends. The individualist is convinced that the state's proper task is to serve the individual; the collectivist denies this, believing that, in certain respects at least, the individual's proper task is to serve the state.

Finally, statements are made on either side that are not explicitly accepted by the other, but are not contradicted either. The individualist insists, for example, that some opportunity to choose one's career is a good thing, that self-restraint is always preferable to external compulsion, and that any form of compulsion employed must be capable of clear justification. The collectivist has not denied these assertions, nor has he made them his own. In turn, the latter insists that true individuality requires positive social conditions, and that effective coordination of many activities of individuals is necessary for social security and efficiency; these contentions have been neither accepted nor denied by the individualist.

d. Selection between alternatives where the theories are in flat contradiction

d. The next step needed is obvious from these tables. As for the first table, no issue arises regarding these points of agreement. In the case of the second table, however, it is clear that so far as these points of flat contradiction are concerned, one alternative will have to be rejected and one alone accepted. How is the selection between them to be reasonably performed? There seems only one answer, and it is implied by the whole conception of an open-minded, cooperative method. We shall need to trace the specific consequences to which each of the contradictory alternatives leads in actual human practice, and assess them in the light of the clearest and most inclusive vision of social value to which we can attain, under guidance of the democratic ideal. In other words, at these specific points the procedure must be applied which, in the absence of detailed analysis in terms of a neutral language, is applied to competing theories as a whole by a thinker who sees possible value in each.

And what does it mean, more concretely, to find our criterion of selection here in the idea of democratic cooperation itself? It means that the issue between the competing alternatives will always be held open until, in the process of unfolding their implications and observing what they actually involve in social experience, it becomes evident that some implication of one of the alternatives is inconsistent with the full realization of democratic cooperation. Then that alternative can be rejected, and its rival tentatively accepted as more rational.

In many cases the result will be that under certain conditions one of two contradictory alternatives works out better, and under others the other. Then the problem is to identify the contrasting sets of conditions as accurately as possible, so as to take account of them in the general criterion that will emerge. In many cases one of the two will have to be entirely abandoned.

For example—with reference now to the last of the contradictions above noted—we would examine, on the widest historical scale, the results reached when governments have taken it as their essential function to serve the individual citizen, and the results reached when they have demanded an unconditional loyalty from their citizens, suppressing them when they refuse to give it. In this particular case it will surely be a decisive consideration to note that the most intelligent, conscientious, and morally prophetic citizens will always refuse to give unconditional loyalty to any government; the object of their loyalty can never be anything less than the highest attainable good for all mankind, without distinction of race, creed, or nation. Thus, any country which chooses the second alternative must deprive itself of the contribution which such persons are prepared to give. This is a decisive point because such deprivation is inconsistent with the very idea of democratic cooperation in the determination of ends—it not only irrationally shuts out some citizens from participation in that process, but the ones excluded are precisely those whose contribution is likely to be of the highest value.

e. Synthesis of the features that are not contradictory

e. The final step is a harmonious reformulation of the contentions of the two theories, with respect to the points brought out in the third table, and a synthesis of the result thus reached with the points of initial agreement and the surviving alternatives of the second table. In performing this step with the theories of individualism and collectivism the disputants, in effect, will be sorting the various specific situations contemplated by these surviving contentions under two heads—those in which security or unity is the primary, immediate need, and those in which progress or fuller opportunity for individual activity is the obvious requirement. This will be done, of course, in the neutral language which the cooperative procedure will develop. In the former situations the crucial question will be: What kind of social control will best meet this need, and how much freedom can be allowed without prejudicing its success? In the latter cases it will be: Just what freedoms will promise to assure the appropriate values here, and in what ways must they be limited to make sure that antisocial license is not encouraged? These values, thus clarified, do not stand in opposition to each other, but are mutually supplementary; they can be progressively realized together. Thus a harmonious reconciliation of previously contrasting judgments will be achieved. As a result of this process, what has been distilled from the two viewpoints and retained as possessing constructive promise will be synthesized in a more inclusive theory of social values—a theory which will recognize liberty as the desirable good wherever circumstances so indicate, and unified control as the desirable good wherever a persistently needed form of cooperation is likely to fail without it. It will embody systematic wisdom in meeting all situations of both these kinds.

Exactly what form such a harmonious unification of two competing theories of value would take in detail cannot, of course, be described without working it out successfully, and presenting the synthesizing concepts in which it would be stated. This is a gradual process, and can only be cooperatively achieved.

No such synthesis, when achieved, will make any claim to

finality. There will appear, from time to time, situations in which the most rational harmonization of theories previously available will not seem entirely satisfactory. The problems that thus challenge thinkers will then be met by further detailed analysis, projection of new theories, critical comparison, and systematic reconciliation of the sort that has just been described. The process never ends, just as the process of seeking scientific explanation never ends. But each gain is a genuine gain as compared with the views left behind.

Democratic cooperation is not a new method; and it is self-corrective

This method of democratic cooperation is not at all a new way of dealing with the issues arising from fundamental differences of value-theory. When it is consciously practiced by thinkers, their work under its guidance will constitute simply a foreshortening of the process which haltingly and with much waste and cantankerousness goes forward now. Wherever progress is made at present in reaching a genuine resolution of any serious conflict of values—at least a resolution that can plausibly claim to be reasonable—it is made by the use of this method. What makes it an exceedingly slow and cumbersome process now is the fanatical dogmatism by which many men commit themselves passionately to one particular theory as to what is good and right, and the consequent heated hostility with which they view other theories that to an impartial eye are just as plausible as their own. The process does move irregularly onward even now, but it is terribly obstructed by emotional resistance, partisan self-justifications, hostile criticism, and irrational animosities in general. Conscious adoption of the method of democratic cooperation will reduce these obstructions to the minimum, and allow progress toward the discovery and formulation of the most adequate philosophy of value that is possible at any given time, to proceed at as rapid a pace as human resources permit.

This circumstance not only shows that the method thus illustrated seeks to impose no foreign criterion upon the ongoing process of adjusting value-theories to each other, but it also indicates that this cooperative procedure is self-corrective in the same fundamental sense in which the in-

ductive method of factual science is self-corrective. The self-correction proceeds differently in detail, of course, because of the distinctive nature of end-judgments and their role in reasoning. But the same logical need is met in both cases. Induction in factual science is self-corrective because if it should fail to reach an adequate result in any given instance, all that is needed is to secure a larger sample under guidance of the same canons and criteria. Similarly, if the results reached by employment of the method of cooperative evaluation should prove insufficient in any given case, all that is needed is a more intensive application of the same procedures which led to those results. A more obviously neutral general concept may be required; more thorough comparative analysis of the competing judgments in its terms may be demanded; or a more complete synthesis of the items brought out by such analysis may be possible. In any case, it is in one or more of these directions that the correction will be appropriately sought.

When we have in mind, however, the larger social situation which this method hopes to meet, the greatest immediate need is a larger number of thinkers who are prepared, in attitude and conviction, to practice it. And in the presence of this need, a further comment about dogmatism is in order.

Dogmatists in the field of value still flourish, in large part, because they fail to accommodate themselves satisfactorily to a difficult challenge that life thrusts upon all men. Both reflection and decisive action are needed in any successful life, but one must realize that what is essential as a guide to decisive action is inappropriate as a guide to constructive reflection about ends. To act with energy and confidence when action is needed, one must take whatever value appears at the time to be supreme as a "practical absolute." [5] That is, if one acts at all under its guidance he must act with the decisiveness and vigor that are natural when he is convinced beyond peradventure of doubt that the value chosen is right; nothing is gained, ever, by hesitant, weak, uncertain action. But at

[5] A phrase proposed by Professor E. S. Ames.

periods when action is not required, and one is endeavoring, in part through discussion with others, to find out more clearly the value of his ends in comparison with theirs, this attitude of aggressive assurance is entirely inappropriate. It blocks the way to free and equalitarian examination of these competing goods, creating an atmosphere of conflict instead of cooperation, and prevents the one who exhibits it from fair and sympathetic consideration of the contribution the other disputants might make toward the helpful revision of his system of values. Hence, it is very unfortunate to carry over the attitude appropriate when action is needed into the situation where the merits of the end that would have guided action are to be critically examined. It is exceedingly easy to do this, but the proper ideal is clearly to gain such alertness of readjustment that one can pass at will, according to the needs of the moment, from unqualified hospitality to criticism of his ends, in discussion, to determined action in quest of whatever ends occupy the supreme place when action is called for. Indeed, one of the chief virtues which come from giving the ultimate place in one's life to the value of democratic cooperation is that when one does so he is assured that no inconsistency will ever appear between the end controlling impartial discussion, whenever he engages in it, and the supreme end which all his other actions are designed to serve. He can be reasonable at all times, whatever he happens to be doing, in thought or in action. He has found the secret of a fully integrated self.

And it appears to be universally applicable

If democratic cooperation is the reasonable way to make decisions about human values, it appears to be so universally; there is no need to recognize any boundary, this side of which it is appropriately applicable but beyond which it should give way to some other method. Its alternative is some form or degree of dictatorial dogmatism, and the latter, if bad anywhere, is bad everywhere; it is the prostitution of reason to serve an end which is intrinsically inconsistent with complete openness to all the values that might prove relevant to its quest. Dogmatism is intellectual exploitation of others, prac-

ticed instead of treating them as equal members of the community of all who seek to realize the goods that life makes possible. And the most unreasonable form of such exploitation is that in which one does not even attempt to convert his opponents by argument, but skilfully uses their minds, with whatever value-commitments characterize them, as tools for serving his own ends or those of some limited group to which he belongs. Reason as shrewd calculation is appropriate when one is dealing with physical things; man may legitimately reduce them to mere instruments for carrying out his purposes. But to apply nothing but shrewd calculation in dealing with other beings that are capable of reason is essentially unreasonable. The ghastly penalty incurred by the clever propagandist, in playing without scruple upon the minds of his subjects, is not that he fosters erroneous belief on their part instead of what he knows to be true, but that he can no longer believe in truth himself—in the dignity and impartiality that mark it as the supreme end pursued by every sincere and honest intelligence. And impartiality is a social value. It means not merely freedom from prejudice in weighing values as they present their several claims within the context of one's own experience, but also freedom from prejudice in weighing one's own set of values alongside the values of others within the context of human experience as a whole—the experience not of one's local community, or economic class, or nation, or race, but of all mankind. The way of rational evaluation is thus the way of generous humanism; it involves commitment to a universal democratic goal and acceptance of the responsibility to render one's own mental powers its loyal servant. Nothing is more difficult than this, but nothing less is required.

And so far as one can successfully carry out that commitment, he has vastly expanded the area of the universe which he can view impartially. The thinker who takes other theories of value than his own merely as objects of hostile criticism and refutation, shows thereby that he cannot reach out in an unprejudiced way beyond the fields of common sense and nat-

ural science. When it comes to evaluational perspectives he is controlled by emotional commitment to his own perspective and by irrational bias against other views. Tied to his own limited ends, he is unable to see them in detachment; they have mastered him instead of becoming stages in his growth. The cooperative thinker, by contrast, can view impartially the total realm of all possible perspectives on the varied values that appeal to men, determined by all seriously proposed criteria for distinguishing between good and bad, better and worse. He thus carries to completion, in the realm of values, the same quest that the scientist has pursued successfully in his objective approach to the realm of fact.

To choose this way of dealing with values is itself, as we have admitted, to make a practical decision between alternative ultimate values; no coercive proof that this decision is right can be given. But since reason is universally social in its origin—the effect of causes that transcend any limited community in their historical operation—it is but an expression of one's sense of gratitude to render it, so far as in him lies, sincerely universal in its constructive present function.

Our minds with their ways of reasoning are a gift to us from a social heritage that knows no geographical limits. Deeply embedded now in their tried and tested habits are the homely respect for empirical fact of the English mind, the passion for clarity and lucidity of the French, the scholarly thoroughness of the Germans, the sense of universal order of the Romans, the speculative enthusiasm of the Greeks along with their eagerness for mathematical consistency, the moral passion of the Hebrews, the urbane and socially responsible pragmatism of the Chinese, the feeling for the infinite vastness and unity of the cosmos that sank so deep into Hindu mentality.[6] We cannot repay these past donors for what they have given us, but by unreserved commitment to the way of democratic cooperation we can share these boons with their descendants everywhere, and make still more effective in our

[6] The sentence would become too long if other important nations, such as the Russians, Persians, Arabs, and Japanese, were included in this list.

day the freedom and equality implied in the very nature of reason.

Fallacies of evaluational reasoning

In dealing with inferences from given premises and with reasoning about facts, we endeavored to classify and describe the main fallacies into which thinkers fall through failure to perform these processes correctly. It is time to make the same attempt with reasoning that aims at the establishment of a rational end-judgment, supplying illustrations of the blunders that have not already been illustrated. It will be quite a pioneering task to organize the errors which may be committed in reasoning about ends, and we cannot hope to cover all the errors that a complete classification would include. Let us, however, do the best that we can. Our main division will be between *fallacies which may appear in any reasoning concerning ends,* and *fallacies which consist in a failure of democratic cooperation* at some point. Most of the fallacies which will be listed have their analogues among the inductive fallacies discussed in Chapter 22.

Fallacies of reasoning concerning ends in general

Among the fallacies which belong in the former of these two classes is that of *insensitiveness* to some of the relevant ends in an evaluational situation. Just as an inductive thinker may fail to notice some of the facts which should be noticed if his conclusion is to apply to the whole field which it claims to cover, so in forming an end-judgment one may be blind to some of the values which need to be taken into account. If the judgment is to give wise guidance to action throughout the whole field of experience which the action affects, all the relevant values must be considered. In excluding neutral nations from the San Francisco Conference, for example, the four sponsors of the conference were blind to the value of understanding how the defeated powers would be likely to react to their proposed policies—an understanding which could hardly be expected of those embittered by the long years of war but which representatives of the neutral nations could contribute. How serious this blindness will prove to be cannot be told in advance, but that it will be serious no reasonable person can doubt.

Another fallacy which belongs in the same class is similar to the fallacy of false analogy in inductive reasoning. It is the error of *applying the wrong criteria* to an evaluative situation. Expressed in proverbial terms, it is the fallacy of "barking up the wrong tree." In forming the policies governing their relations with Germany after Hitler came to power, for instance, Britain and France followed criteria which would have been relevant had they still been dealing with the Weimar Republic, but which were entirely irrelevant in view of the radical change which had taken place in the German government.

The remaining fallacies in this class which will be mentioned consist in adopting one or the other of two irrational extremes instead of the middle course which wise application of the proper criteria would prescribe. We have said that reasoning in any field is responsible to avoid self-contradiction. In the case of reasoning about ends, the two most popular ways of violating this requirement are *vacillation between two incompatible ends,* and *attempting to realize both those ends at once*—in proverbial parlance, "trying to have your cake and eat it too." In 1929 and 1930, the Hoover administration vacillated between policies which implied that it recognized the presence of a serious depression, and policies which implied a denial that any such depression existed—the outcome being that no program drastic enough to meet the human needs created by the catastrophe was initiated. The other extreme is exhibited by any political party when it writes its preelection platform. Eager to appeal to as many groups of citizens as possible, it makes varied promises, all of which cannot possibly be realized together, since they would require incompatible courses of action. We have also said that wise reasoning about ends respects the limitations of the factual materials in which, if at all, the end sought must be realized. This requirement may likewise be violated in either of two extreme ways. One is the wishful thinking which consists in *assuming that any end is attainable merely because it is desirable.* Specific illustrations of this fallacy are hardly needed; any failure to distinguish between an ap-

pealing fantasy and an ideal having some chance of realization would be an instance. The other extreme may be appropriately called the "*sour grapes*" fallacy. Its essence lies in the assumption that an unattainable good is shown by that fact to be no good at all. Thus, many who disbelieve in the immortality of the soul comfort themselves by insisting that immortality is, after all, not desirable. But while the wise procedure here is surely to turn one's attention toward ends that do seem to have some chance of attainment, this does not require us to deny that many other ends would bring a distinctive satisfaction if they were not ruled out by the nature of things.

Two other fallacies we shall mention in this context. One of them violates the requirement that good reasoning about ends will forecast the consequences of alternative courses of action as fully as the problem involved requires. Such a violation may take the form of *imprudence*—that is, failure to anticipate as many of the likely consequences as an intelligent man could fairly be expected to anticipate. At the other extreme is *inability to reach any sensible decision*, through continuing to compare the alternatives beyond the time when a decision has to be made if it is to give needed guidance, or through considering forecasts so distant in time that the chances are exceedingly slight of events happening as they are predicted. The second consists in demanding an unattainable certainty in our end-judgments, thus failing to recognize that such judgments, like the truths of factual science, are tentative rather than certain—at best possessing high likelihood of proving adequate. Here, at one extreme, stands the believer in *absolutism*, insisting that his judgments lie beyond any possibility of rational correction, who is hence rigidly closed to any new experience that might indicate a desirable revision. At the other end is the *scepticism* which leads to paralysis in action because its holder, distressed by his realization that certainty is unattainable, forgets that resolute action can accomplish many aims even though there is no guarantee of success in any particular quest.

Fallacies of failure in democratic cooperation

Among the fallacies which consist in a failure of democratic cooperation in some form is, of course, the *assumption of special privilege* for the end-judgments or value-theories of certain groups as against those championed by others. An outstanding example is the provincialism of most Western philosophers and theologians in confining their attention to the Occidental tradition in their fields. They thus show that they take it for granted that Oriental theories are intrinsically less likely to be adequate than those originating in the West.

Another is the fallacy of *assuming that one's own theory of values is established merely because a coherent defense can be worked out,* which takes account of the theories presented by others. The thinker who falls into this fallacy forgets that since he is interpreting the theories of others in terms of his own criteria, there is nothing astonishing in this; his rivals, too, given sufficient ingenuity, can elaborate an equally persuasive defense of their convictions, allowing for his position in the same way that he has allowed for theirs. This might be appropriately called the "little Jack Horner" fallacy; pulling out a plum does not prove "What a good boy am I!" when the pie was so baked that anyone could pull it out merely by sticking in a thumb.

At least a pair of important fallacies in this class, like several in the class discussed above, may be committed in either of two extreme forms, the rational procedure consisting in taking a middle course. One is a violation of the principle that the reasonable thinker will assume in advance that an adequate resolution of any conflict of value-theories would find a place for some values precious to each. This principle may be violated either by the *dogmatism* which seeks aggressively to impose one theory upon all of its rivals[7] (an attitude sufficiently illustrated in the preceding chapters), or by the *undiscriminating appeasement* which admits positive value in any theory merely because it is championed by somebody, without taking the responsibility to discover by analysis what that positive value is. There is nothing more provoking in a serious

[7] And hence naturally affiliated with absolutism.

argument than to have one's opponent remark, "Oh! I suppose there is something in what you say," when he obviously has no interest in finding out just what there is in it that his position has not properly recognized. It must not be forgotten that when two opposing views are subjected to careful analysis there is always the possibility that one of them may show nothing that the other does not sufficiently provide for. To assume in advance that this is the case is dictatorial presumption, but to assume that it never will turn out this way is likewise unjustifiable. The other fallacy is committed by those who recognize that analysis of this sort provides a necessary foundation for any rational resolution of a conflict of ends, but who rush into a *premature synthesis*. This irrational eagerness is likely to take one or the other of two forms. A thinker may "throw out the baby with the bath," *i.e.*, discard some important values that might be given a place in the synthesis if he were less impatient to get the problem settled. Or he may follow the principle "Love me, love my dog," *i.e.*, retain all the subordinate values associated with a value that is to enter a projected synthesis, without showing a reasonable probability that they are compatible with the other ends which that synthesis is to include. The former of these irrationalities is exhibited in those who are so eager to establish socialistic governments that they are willing to sacrifice hard-won civil rights along with the abuses of individualism which obviously need correction, instead of seeing whether the hope is not justified that such rights can be preserved in an adequately socialized order. The latter is exhibited by those defenders of "free enterprise" who are willing to concede that some governmental control of the economic order is now inevitable, but who insist that all the main privileges of business leaders that have been associated with free industrial initiative in the past can somehow be reconciled with political control. Adequate analysis would almost surely show that not all of those privileges are compatible with an effectively regulated economy.

Challenging problems about rational evaluation

When reasoning as evaluation was contrasted in Chapter

25 with reasoning in the formal and factual sciences, it was suggested that a frank and honest recognition of the differences would show itself, in part, in the admission that after any solution of the major difficulties about end-judgments has been proposed, serious problems still remain to which a more adequate answer is wanted. Six such problems are here listed, in the hope that some readers will cooperatively participate in the task of working out a more satisfactory answer to them than the preceding pages have provided.

1. In classifying values, we were forced to recognize, on at least one occasion, that the distinction drawn is relative rather than absolute. In distinguishing between values as means and values as ends, it was noted that the difference between them is relative, not absolute; in many situations, for example, we commence our evaluation by merely comparing several means to an end that is taken for granted, and finish by redetermining the end itself. Now to leave an analysis in these terms is unsatisfactory. To render it more adequate, we should either find some way to replace this distinction by one which can more successfully be viewed as absolute, or else, if we have to be content with a relative distinction, we should find some way to describe accurately the factors to which it is relative, and how it is relative to them.

2. In listing the criteria that a reasonable person employs when choosing between alternative ends, we discovered five criteria, and then indicated how they are logically interconnected under the dominance of the fifth—namely, the rule of impartiality as between our own preferred ends and those of others. But it was necessary to recognize that, on the one hand, the other four criteria function to some extent independently of this rule, while on the other hand problems which arise about their mode of functioning can only be adequately solved by a method which is essentially derived from this rule. It is not entirely satisfactory to leave an analysis in this form. Further clarification of the interrelations of these criteria is needed, aiming to show, if possible, in just what respects these criteria are independent, and in just what respects they constitute an interdependent whole.

3. The distinction between homogeneous and heterogeneous values is not only terminologically awkward, but is unsatisfactory on other accounts. The difference toward which it points is real and important, but to recognize that difference in just this way leads to difficulties that might be avoidable. Among the difficulties are these: It suggests that in some sense homogeneous values are more natural and legitimate than heterogeneous ones, which would be a hard doctrine to justify; and it seems to leave judgments of heterogeneous ends without any criterion by which to tell just how the various fields of value covered are combined in them and what weight is intended to be given to each.

4. The method of democratic cooperation, as here presented, needs to be continually worked over, to the end of making its successive steps as simple, clear, and self-justifying as possible. Precisely because its fundamental idea is essentially reasonable, it is very important that it be subjected to the most searching criticism in the light of that idea, and made to undergo whatever revisions systematic criticism may dictate. The rapidity of progress in adjusting conflicts between ends will depend not only on the increasing application of that method, but also on the continual rendering of the method itself more adequate to its logical task.

5. A more complete and more adequately organized list of fallacies of evaluational reasoning is obviously desirable. Formal fallacies and fallacies of inductive reasoning can be listed and classified with considerable confidence that no serious omission or misinterpretation has occurred. A comparable treatment of the fallacies that appear in the process of reaching end-judgments is one of our major needs.

6. Finally, the course of analysis in the volume as a whole might be given a more close-knit unity if the interrelations of form, fact, and value in human experience could be more fully clarified. We have assumed that the main key to an understanding of their interrelations lies in the temporal analysis of typical acts of reasoning, and in the pragmatic, syntactic, and semantic dimensions of meaning which have been explained in terms of that analysis. Any larger measure of unity

that might be realized would presumably have to be secured, either by more intensive examination of the distinctions and relations disclosed by this analysis, or by the discovery of another mode of dissecting the material of logic which, while fair to all the relevant considerations here emphasized, succeeds in binding them together in a more systematic whole. Since it is of the very nature of mind to seek the largest measure of unity that proves possible, progress in both these directions is on all accounts desirable.

Summary of Part IV

Let us conclude by a summary of the present Part. After examining the distinctive nature of value as contrasted with fact, we faced the problem of divergences of value-judgment, and listed the criteria to which judgments of ends seem to appeal. The logical properties of end-judgments were then analyzed, in the fields of homogeneous values, and the fields of heterogeneous values. It became clear that the problem of rendering end-judgments rational culminates in the conflicts between ultimate theories of value and the question how they are to be adjudicated. Such conflicts were illustrated in some detail in the areas of law and social ethics. Finally, the method of democratic cooperation was proposed and given detailed explanation as expressing the way of reason in dealing with questions of this kind.

EXERCISES

1. Explain the meaning of the word "prerequisite" in the phrase "prerequisites of democratic cooperation."
2. Why, and in what sense, is it necessary for a cooperative thinker to adopt all competing value-theories?
3. State clearly each step of the cooperative method, indicating exactly what role it performs. Use some appropriate illustration of your own.
4. Show in detail what results you believe would be reached if the cooperative method were applied to the concepts "arbitrary" and "reasonable" as used in the two Supreme Court decisions summarized in the preceding chapter.

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APPENDIX

CONTAINING THE LONGER EXERCISES
REFERRED TO IN THE TEXT

A

Exercise on Chapter 4

(See p. 59.)

Study the following confessions of superstition by contemporary college students. Add, if you can, a case from your own experience or knowledge. Give a concise written answer to these two groups of questions:

1. How do these cases illustrate ways in which superstitious beliefs arise, factors which support their continued influence, and forces which tend to undermine them?

2. Do you think such beliefs entirely erroneous, or is there something in them? If the former, how could we emancipate ourselves more fully from the influence of such notions? If the latter, which beliefs do you think may have some basis in fact, and why? How can the truth be clearly separated from the error?

> a. "I was not superstitious. In fact, I loved to laugh at people who were. Whenever I had a chance I walked under a ladder just to tease those who were so inclined. But now I have one (and only one) superstition. It all happened at a flying station. A friend of mine was up in a plane, flying rather conservatively round the station. Suddenly his plane nosed down and to everybody's horror went into a tail-spin at an altitude of two hundred feet. As he was so low he had no time to come out of the spin, and as a result crashed. The next day I went over to the hospital to see him. He was just regaining consciousness and he said to me: 'Why did I ever light three cigarettes with one match? Somebody told me to blow out the match, and use a new one for the third fellow's cigarette, but I just laughed. But never again.' From that time on I never use one match for three lights and I think I never will as long as I live. Whenever I have occasion to give three men

lights at once I always see my friend's plane the minute after it crashed and I use two matches." (Tozzer, *Social Origins and Social Continuities.*)

b. ". . . The first year my brother was in college he made the track team. On the day of his first collegiate race, in taking off his clothes he stuck his scarf-pin in the jersey of his running-suit. When he got out on the track, he found that he had neglected to leave the pin in the locker building. So fastening it more securely in his jersey, he wore it through the race. He won the race. Ever after he wore the pin and was never beaten in the quarter-mile race, which was his event, during his four years in college till the intercollegiates of his senior year. That day he forgot to bring the scarf-pin with him and failed to score.

"My brother was an intimate friend of W——, of Princeton football fame. Last year just before Princeton played Yale, my brother sent the pin to W——, telling him of its history, and asking W—— to wear it in the Yale game. The Princeton man did and you are familiar with the remarkable showing that W—— made. In the Dartmouth game, however, Sam forgot to fasten the pin to his jersey and failed to distinguish himself on that day. When Princeton played Harvard, W—— took pains to see that the pin was on his person. The game made him the biggest football man of the year. The pin was a piece of Mexican jewelry which my brother had purchased in Atlantic City. It was said to have been made by a Pueblo Indian nearly a hundred years ago." (Tozzer, *Social Origins and Social Continuities.*)

c. "About six or seven years ago, I was engaged in a game of cards with one of my friends. We had already played three games before and I decided to cut matters short. My friend, however, remonstrated with me and urged me to play 'just one more'—a fourth game—and I yielded. The next day my grandfather died and ever since I have been possessed of a sickening, foolish dread of the number four. This superstition, if such I may term it, has taken the shape of several forms. Never since that time have I gone to bed without first making certain that the minute-hand of my watch registered some other minute than four or any of its multiples. If, as luck would have it, it should hover covertly around the four or eight or twelve or what-not mark, I would wait, sometimes shivering in the cold, until the delinquent hand had moved along to the five or nine mark as the case might be.

"Similarly, in all my lecture and reading notes, the number four has played its part, and so persistent has its effect been upon me

that at times it has almost driven me to despair. Thus, in taking down a sentence, if four (or any of its multiples) words chanced to occur between any two successive marks of punctuation, I have inevitably added the word 'dear' to make certain that the number of words was five and not the hated four. Numerous instances of this can be found throughout my notes, and, if the reader is interested, a perusal of my notes in this course will at once indicate this." (Tozzer, *Social Origins and Social Continuities.*)

d. "It was during the middle of my Freshman year at high school that I noticed that my grades in the weekly 'exams' took a sudden drop. Study as hard as I would, I could not raise my low average. I could not account for this drop, for I studied just as hard as ever. Now I am not what you might call a superstitious fool; but I could not help going backward in my mind and searching for some recent event or change of habit that could possibly coincide with the beginning of my change from the high to a low grade. It was not long before I discovered the desired coincidence. I had up to the date of my 'drop' been riding to school by the Subway train. But on the very Friday I had received my first low grade I had met a friend with whom I had ridden to school on a surface car. As I found this route shorter and more convenient, I continued to use it, and was still using it at the time of this strange discovery. On the very next day I returned to the Subway route and the very next Friday my grade rose from a '50' of the previous week to a '90.'

"A foolish coincidence, you will say; but it has influenced my actions in spite of myself. Every day, I go out determined to try the surface cars just for the fun of it, but some unseen instinctive force impels me to seek the Subway route. Even now, while attending ——, I walk every day a distance of almost half a mile to get a Subway train, rather than take the surface car which runs right by my house. But every now and then I meet some friends, who also attend the same college and live on the same street, and I am compelled for companionship's sake to ride with them when they take the surface car. Invariably on such a day I have poor luck in all my work. For instance, I often upset apparatus in my chemistry laboratory when I ride on the Subway, but the damage is never so great as when I use the surface route. What would you call such a coincidence?" (Tozzer, *Social Origins and Social Continuities.*)

e. "Most people think of knocking on wood as a sort of joke; they do it just for fun. I used to think I did, too; it was a sort of humorous notion, and I was rather proud of it. But one night I

was disillusioned. I was lying in bed thinking about something that I was very anxious to have happen. I said to myself, 'It can't help happening.' Unconsciously I felt around for some wood. The bed was made entirely of steel. I rolled over to the edge and felt for the floor; it was carpet-covered. When I found myself getting up and stumbling around in the dark until I found a chair on which to knock, I realized that it was no mere joke. Ever since then I've had an uncomfortable feeling when I predict something and don't knock on wood. I know, when I stop to reason it out, that it's all nonsense, but I have a feeling inside somewhat as if I ought to do it. I know perfectly well that it will have no effect on the ultimate result; but then, of course, it might, and what's the use of taking chances? Now instead of being proud of it, I'm ashamed of it and knock surreptitiously under tables, instead of boldly on the wall as I used to do." (Tozzer, *Social Origins and Social Continuities.*)

f. ". . . You may talk now in the light of calm reason and the throes of philosophic reflection about the foolishness and paganism of superstition. But out on the firing-line of life—not kid-glove life, that benevolent yeast called 'the social whirl'; not even the life of the great middle class; but down at the bottom, the simple, the ignorant human life of the masses—the root from which those higher forms first sprouted, the rude trunk fundamental to the existence of the more beautiful branches—there is where you find your 'slaughter' and there you find no thought of the downright foolishness of superstition, for there they are content to go 'ignorant' if 'knowledge' means a weakening of their blind hold on life, and it does. Where the fighting is thickest, there are the strongest superstitions—and there, too, strange to say, are the men with the strongest personalities. When a man begins to throw away his little superstitions, people say he is building character. Yes, but I am afraid that at the same time he is throwing away little by little his personality, that which made him feel his own peculiarities—himself as distinguished from the rest of the world. He begins to become more and more the literal 'thinking machine'; he is headed straight towards the life of the stoic scientist—a man doubtless very valuable to the world, but whose society, somehow or other, I do not crave. No, sir. I am proud of my superstitions. Every time I spit on my bait I chuckle to myself and thrill with the consciousness that *I* am *human*." (Tozzer, *Social Origins and Social Continuities.*)

g. "I cannot say that I actually believe in superstitions and charms; yet, when I become aware of them, when I am uncon-

sciously doing something, I can hardly refrain from making use of them. Often, I never think of making use of the 'knocking wood' charm, yet at times when I make a remark appropriate for it, I cannot for the life of me refrain from knocking wood, although at the time I know it is all foolishness. I argue against my common sense, that performing these charms can do no harm, but it is best to take every safeguard possible. I have the same feeling when I am walking with a friend and we pass on different sides of a post, or when I go beneath a ladder. But the number 13 has rather the opposite effect upon me. I consider it lucky. I have one superstition, rather original as far as I am concerned, of not risking entering the water if I have done anything peculiar just beforehand. I remember one day I returned a long way back to the bath-house because I had not taken the usual path to the water." (Tozzer, *Social Origins and Social Continuities.*)

h. "I sincerely believe that any full-grown man who has enough brains to be a college student, but who is earnestly superstitious in any way whatsoever, should be an object of pity rather than of derision. Although I have taxed my brain for an hour, I can recollect but one instance in which I ever carried a fetish. Note, however, that I did not make myself an object of pity, according to the above definition, because I was but ten years old. At that time I half convinced myself that an old key, which I had accidentally carried with me to a party, had been the substantial cause of my having a very good time. When I went to a Valentine party some time later, I took the key with me in the hope that it would have a magic influence on my happiness. But for a partner in some of the games that we played that evening I was allotted a certain homely red-haired girl, to whom I have a particular aversion. The next day I threw the key away in disgust." (Tozzer, *Social Origins and Social Continuities.*)

i. "When I was about ten years old I was walking along a road one day, my thoughts full of a great ambition which I hoped I should be able to realize. As I dwelt upon the prospects and difficulties, my anticipations varied all the way from complete confidence to total despair. While my hopes were fluctuating in this way, I noticed a large tree some distance ahead and at the same time a stone of the right size for throwing at the side of the road near me. If I hit the tree with the stone, I said to myself, I shall achieve my ambition; if I fail to hit it, that means that I shall be disappointed and might as well give up my hopes. I picked up the stone, and after taking good aim, threw it. I was exultant indeed

when I saw it hit the edge of the tree. For some time afterward I was completely confident that my big ambition would be realized."

j. "During my first year in high school I fell into calf love with a girl in the same class. Late that year I made the baseball team, and the day before our first game I confided to my sweetheart the nervousness I felt and how I feared that I would not play a good game. She took a ribbon from her hair and told me to tie it around my throwing arm. I did so before dressing for the game next day. In the fifth inning, with our team two runs in the lead, I made a difficult double play which saved our advantage and which proved the turning point of the game. I was acclaimed the hero of the team, and although I naturally said nothing to anybody else about the ribbon, I attributed my good play to its presence on my arm.

"Next year we moved to a distant state and I succeeded in making the baseball team in my new school. I soon lost interest in my former sweetheart, but continued faithfully to wear the ribbon before every important game. Whenever I made a good play I was sure the ribbon had something to do with it. In my senior year, however, I was undressing after a game, and failed to hide the ribbon carefully enough. One of my teammates saw it, and crying, 'A girl's hair-ribbon!' held it up before the rest of the bunch. Such a storm of laughter and ridicule as burst upon me then! You may be sure I didn't wear the ribbon any more. I felt rather uneasy without it the next game, but I seemed to play just as well as before, and after that its absence did not trouble me."

B

(See pp. 341, 361.)

I. Study the following pieces of scientific inquiry. Indicate briefly and clearly in each case:

1. What was the significance of each stage in the inquiry? (*I.e.*, why was it needed, what did it accomplish, and what remained to be proved?)
2. What methodological principle was used at each stage?

A. THE DISCOVERY OF THE CAUSE OF YELLOW FEVER

Yellow fever was early thought to be carried from person to person by contact or by fomites. The latter are any objects upon

which infectious material may be accumulated, such as clothes, bedding, dishes, and the like.

In 1848, the theory was suggested that yellow fever was transmitted by mosquitoes. In 1881, Carlos Juan Finlay, of Cuba, picked out the common mosquito in Cuba, now known as the *ædes calopus*, as the specific carrier. Finlay based his conclusion on two main lines of observation, first that yellow-fever zones corresponded in general with the zones of distribution of this mosquito, and second that these mosquitoes seemed to be especially prevalent in areas of yellow fever epidemics.

In 1900, the danger of yellow fever to the American troops quartered in Cuba was so great that a commission was appointed to investigate its causes. The first experiments on mosquitoes were rather disappointing for the mosquito hypothesis. Mosquitoes were allowed to suck the blood from yellow fever patients and were then placed on the skin of susceptible individuals. They were allowed to remain until they had given several bites. Of the first ten individuals thus bitten, only one came down with the fever.

The hypothesis thus seemed not very promising, but in the course of further work two members of the commission succumbed to yellow fever and died of the disease under conditions which again suggested that the only plausible cause was their having been bitten some days earlier by an infected mosquito.

Accordingly, systematic experiments were devised to find out whether the fever was definitely transmitted by the mosquito or not. The subjects were courageous volunteers from the army of occupation who were willing to run the great risk involved. Two rooms were constructed, both being made absolutely mosquito proof. In one of these fifteen mosquitoes were freed, all of them having fed on yellow fever patients. The other room was kept free from mosquitoes. In the course of the experiments fourteen of the nonimmune volunteers who were bitten in the mosquito-inhabited room came down with the fever, while no one who slept in the mosquito-free room came down with it.

To make the experiment still more conclusive, in another mosquito-free room three nonimmunes slept for twenty nights on bedding brought without washing from beds of patients who had died of yellow fever. None of them contracted yellow fever. Further experiments yielded the same result.

The conclusion thus seemed pretty clear. But why then had the earlier experiments failed? The answer to this question appeared when in the course of continued observations and experiments it was noticed how early in the course of the disease the mosquito whose bite proved dangerous had bitten the patient, and how much

time had elapsed since his infection before he bit the subjects of the experiments. It developed that for the mosquito to become infectious it must bite a yellow fever patient during the first five days of the illness, and that an interval of at least twelve days was necessary after the mosquito had obtained infected blood before its bite will infect man. This last fact accounted for the negative result of the first experiments. But the commission was able to prove that outside of injection of infected blood, the bite of the *ædes calopus* is the only way by which yellow fever can be acquired from one individual by another.

As a result of these experiments, yellow fever has been eliminated from the United States and practically so from the West Indies. In course of time it will doubtless be eliminated from all parts of the world where it still exists.

B. THE DISCOVERY OF HALLEY'S COMET

Before the days of modern astronomy the appearance of a comet was regarded either as an omen of divine displeasure or as betokening an accidental visitor to the solar system, coming no one knew whence and going no one knew whither.

After Kepler, however, had discovered that the path of a planet is an ellipse rather than a circle, and Newton had laid down the laws of gravitative attraction as applying to bodies moving in elliptical orbits, the suggestion seemed plausible that the motion of the comets might be accounted for on the same principles, if they could be regarded as having a very elongated orbit. Newton noted that such an orbit could be charted if careful observations of its position could be obtained at three different dates.

Edmund Halley, a younger friend of Newton, took over the problem at this point. His task was to see how much regularity he could discover in the movements of these strange wanderers. He could gather reliable observations of a number of the brighter comets, and after much toil charted the paths pursued by twenty-four of them, which had appeared between the years 1337 and 1698. In the course of this study he found that three of these twenty-four followed paths so strikingly similar that the suggestion strongly came to him that they were really different appearances of the same body. Moreover, the times between these appearances were roughly the same, the first having been noted in 1531, the second by Kepler in 1607, and the third by Halley himself in 1682. Halley examined older records, and found that comets had been observed in 1456, in 1380, and in 1305. The conclusion seemed highly plausible that these were all appearances of the same comet,

which passed its perihelion every seventy-five or seventy-six years.

The further test needed was successful prediction. It was easy, of course, to predict that if the hypothesis were correct the comet should return again sometime in 1757 or 1758. But could the prediction be made more precise? Halley calculated the perturbations which the planets, especially the large ones Jupiter and Saturn, might be expected to cause in the motion of the comet, and concluded that in this case it would pass near enough Jupiter to be considerably retarded by the planet. Accordingly he assigned an unusually late date for its reappearance, either the end of 1758 or the beginning of 1759. Halley died in 1742, but his prediction was almost exactly verified. The comet was seen first on Christmas Day, 1758, and passed through its nearest point to the sun on March 13, 1759. It has subsequently appeared in 1835 and in 1910.

C. THE DISCOVERY OF THE CIRCULATION OF THE BLOOD

Previous to the work of William Harvey it had been supposed by physiologists that the flow of blood in the arteries and veins was from the heart to the rest of the body and simply carried to the various parts of the body the nutriment supplied by the digestion of food.

Harvey's careful measurements of the quantity of blood in the ventricles of the heart and of the degree and rate of its contractions indicated the impossibility of this hypothesis and strongly suggested that there must be a circulation of the blood through the heart and the body rather than a constant creation in the heart of a new supply. He said: "Let us assume either arbitrarily or from experiment, the quantity of blood which the left ventricle of the heart will contain when distended, to be, say, two ounces, three ounces, or one ounce and a half—in the dead body I have found it to hold upwards of two ounces. . . . Let us suppose as approaching the truth that the fourth, or fifth, or sixth, or even that the eighth part of its charge is thrown into the artery at each contraction; this would give either half an ounce, or three drachms, or one drachm of blood as propelled by the heart at each pulse into the aorta; which quantity, by reason of the valves at the root of the vessel, can by no means return into the ventricle. Now, in the course of half an hour, the heart will have made more than one thousand beats, in some as many as two, three, and even four thousand. Multiplying the number of drachms propelled by the number of pulses, we shall have either one thousand half ounces, or one thousand times three drachms, or a like proportional quantity of blood, according to the amount which we assume as pro-

pelled with each stroke of the heart, sent from this organ into the artery; a larger quantity in every case than is contained in the whole body! . . . (Thus), supposing even the smallest quantity of blood to be passed through the heart and the lungs with each pulsation, a vastly greater amount would still be thrown into the arteries . . . than could by any possibility be supplied by the food consumed. It could be furnished in no other way than by making a circuit and returning." (*De motu cordis*, Chap. 9.)

He succeeded in making further observations confirmatory of this hypothesis. Examination of the valves in the veins showed that they were arranged so as to facilitate the return of the blood to the heart rather than its dispersion from the heart. Study of obstructions in the arteries and the veins indicated that the flow of the blood in the arteries was away from the heart while that in the veins was toward it. Other facts about the structure of the heart furnished additional confirmation.

Four years after Harvey died, in 1661, Malpighi examined with the microscope the tissues in which the smallest arteries and veins were imbedded. He discovered the minute capillaries connecting the arteries with the veins and was able to prove by direct observation that the blood flowed through them from the arteries into the veins and back to the heart.

D. PASTEUR'S WORK ON HYDROPHOBIA

This dread disease raged almost uncontrolled until Pasteur attacked it in a scientific way. Observation of cases seemed to indicate that the rabic virus must be contained in the saliva of the mad dog, that it was communicated by his bites, and that the period of incubation varied unpredictably from a few days to several months.

Pasteur began his experiments by taking some mucus from the mouth of a child who had died of hydrophobia and inoculating it into some rabbits, causing their death in about thirty-six hours. This outcome was confusing, because the normal time of incubation of hydrophobia was considerably longer than this. Pasteur concluded that there must have been many other poisonous germs besides those of hydrophobia in the saliva and that these could have caused the death of the animals.

Accordingly, he inoculated some animals with saliva taken directly from the mouth of a mad bulldog who had been kept in a cage. These experiments were not very conclusive, and Pasteur began to wonder whether the real seat of the rabic virus was not somewhere else than in the saliva. On account of the frenzied con-

dition of a mad dog it seemed likely that the nervous system, and especially the medulla oblongata, might be the seat of the virus. The frenzy seemed somewhat similar to the state of animals whose medulla centers had been seriously injured.

He thereupon extracted medulla tissue from a dog that had died of hydrophobia, being careful during the experiment to exclude all germs except those present in the medulla. When animals were inoculated with this tissue they came down with hydrophobia with considerable uniformity, though the period of incubation varied greatly.

Pasteur's next step was to inject the rabid medulla tissue directly into the surface of the brain. When this was done hydrophobia appeared in every case, and the period of incubation became regular also, namely, fourteen days. Thus a definite and conclusive explanation seemed to have been reached as to the essential cause and mode of development of the disease.

But Pasteur continued his experiments. By increasing the number of inoculations of an animal he proportionately reduced the time of incubation from fourteen days to seven, the latter appearing to be the minimum period necessary for the disease to develop. He varied these experiments sufficiently so that he was able to predict with confidence just when the hydrophobia would appear in the case of an inoculated animal.

These were great achievements, but Pasteur took a further step leading to protection against hydrophobia by developing a serum capable of establishing resistance to it. By drying rabic medulla tissue under sterilized conditions he discovered that in fourteen days it lost all its virulence. He inoculated animals with this prepared tissue, and found that when he followed it with inoculations of tissue that had been drying for thirteen days, and then some that had dried for twelve, etc., he was able to end by injecting tissue from an animal that had died the same day without causing the disease. The serum had established effective resistance to it. When the animals thus prepared with these serums were bitten by mad dogs they proved themselves immune to hydrophobia.

The final test was the use of the serum with a human being. Pasteur ventured to try it in the case of a nine-year-old boy who had been bitten two days earlier by a mad dog and showed fourteen wounds. Twelve inoculations were given, and the treatment lasted ten days. The boy's wounds healed and he remained in perfect health.

E. THE GROWTH OF INDIAN MYTHOLOGIES

As we depart from the area where the peculiar culture of the North Pacific coast has reached its highest development, a gradual change in arts and customs takes place, and, together with it, we find a gradual diminution in the number of myths which the distant tribe has in common with the people of the North Pacific coast. At the same time, a gradual change in the incidents and general character of the legends takes place.

We can in this manner trace what we might call a dwindling down of an elaborate cyclus of myths to mere adventures, or even to incidents of adventures, and we can follow the process step by step. Wherever this distribution can be traced, we have a clear and undoubted example of the gradual dissemination of a myth over neighboring tribes. The phenomena of distribution can be explained only by the theory that the tales have been carried from one tribe to its neighbors, and by the tribe which has newly acquired them in turn to its own neighbors. It is not necessary that this dissemination should always follow one direction; it may have proceeded either way. In this manner a complex tale may dwindle down by gradual dissemination, but also new elements may be embodied in it.

It may be well to give an example of this phenomenon. The most popular tradition of the North Pacific coast is that of the raven. Its most characteristic form is found among the Tlingit, Tsimshian, and Haida. As we go southward, the connection between the adventures becomes looser and their number less. It appears that the traditions are preserved quite fully as far south as the north end of Vancouver Island. Farther south the number of tales which are known to the Indians diminishes very much. At Newettee, near the north point of Vancouver Island, thirteen tales out of a whole of eighteen exist. The Comox have only eight, the Nootka six, and the Coast Salish only three. Furthermore, the traditions are found at Newettee in the same connection as farther north, while farther south they are very much modified. The tale of the origin of daylight, which was liberated by the raven, may serve as an instance. He had taken the shape of a spike of a cedar, was swallowed by the daughter of the owner of the daylight, and then born again; afterwards he broke the box in which the daylight was kept. Among the Nootka, only the transformation into the spike of a cedar, which is swallowed by a girl and then born again, remains. Among the Coast Salish the more important passages survive, telling how the raven by a ruse compelled the

owner of the daylight to let it out of the box in which he kept it. The same story is found as far south as Grey's Harbor in Washington. The adventure of the pitch, which the raven kills by exposing it to the sunshine, intending to use it for caulking his canoe, is found far south, but in an entirely new connection, embodied in the tradition of the origin of sun and moon. (F. Boas, *Journal of American Folklore,* Vol. IX, p. 1 f.

F. THE CAUSES OF ECONOMIC VALUE

If we examine the various objects which command value in exchange, such as the manufactured products in our stores, the foods raised on the farms, the supply of electricity and gas in our houses, the professional services of the lawyer, doctor, or clergyman, and other instances of the same fact, we find that two essential factors are common to all these cases. One is that they are useful for satisfying human wants; the other is that they are sufficiently scarce so as not to be available in the quantities wanted without effort and specialized labor.

But let us see if this outcome is confirmed by a comparison with objects that do not possess economic value. Dirt commands no economic value; it is plentiful everywhere and is not wanted. The same is true of mosquitoes and mice. Air ordinarily commands no economic value; it is useful for human wants but is present in inexhaustible supply. Snow ordinarily commands no economic value; it is scarce over most of the world most of the time, but is not much wanted. Such considerations as these seem clearly to support our conclusion.

To bring it to a still more decisive test, however, let us examine cases where an object that does not ordinarily possess economic value comes to command it. When a mountaineer ascends a high mountain, or an aviator tries for an altitude record, he takes along an oxygen tank; air for breathing has in his case an economic value. This is clearly because he is going to a place where it is scarce and where his need of it can only be supplied in special form requiring effort and specialized tools for its preparation. When a biologist wants mice for experimental purposes he usually has to pay for them, because he wants a particular kind and wants them in quantities which it requires labor and foresight to supply.

Moreover, there are cases where an object which ordinarily commands economic value loses it. When a farmer has apples rotting on the ground which he cannot profitably send to market, he will usually let anyone go in and help himself. The demand for apples is meager enough in relation to the plentiful supply so that apples

in that situation have no value. Not many years ago cotton stockings commanded a fair price; today they are practically without value. It still requires effort and special tools to make them, but they are no longer wanted.

These various comparisons all lead to the same conclusion, namely, that the essential causes of economic value are the two factors mentioned, and that no other cause need be invoked to explain it.

II. The best method brought to light in the author's experience for securing practice in the entire process of scientific discovery and formulation (outside of long and intensive laboratory work), beginning with the chaotic experiences which demand explanation and ending with the statement of an explanatory law in functional form, is found in the solution of cryptograms. In each of the following problems,

a. Solve the cryptogram and state its key in the form of a functional law, such as: "Substitute for each letter the third following letter in the English alphabet," "Read for each vowel the following vowel, for each consonant the letter equally distant from the letter m," etc. Some of the keys will be very simple, some rather complex.
b. State how the more general scientific criteria of empiricism, objectivity, and simplicity, and the principles of agreement, difference, and concomitant variation are revealed in your procedure.

The first seven cryptograms are separated into words, each of which represents an English word. In the last three the interpretative units themselves must be ferreted out by the student. In dealing with them he is thus in a situation fairly comparable to that of the archæologist attacking an inscription in an unknown tongue, or a Newton first disentangling the conception of mass, in terms of whose units he proposes to reduce the material world.

1. WLVH GSV NLIGZI SLOW GSV YIRXPH GLTVGSVI LI PVVK GSVN ZKZIG?
2. KE FKO QOYREQ DKIQ QOYREQ TUHK.
3. 5O *O *6E A9* *6OU %7U55A#3 2O9%I3E# 6E# ;A¢% A93 1E ;I%E ;6I26 6A"I95 9O O"E#%EE# O# #U7E# ?#O"-I3E*6 6E# 1#EA3 I9 *6E %U88E#.
4. NVUROKONVE UPHORHYK JVY CNNRUACJUOP OX LYXRYAJUHY JVOIWVJ JO JVY XIPZCQYPJCR PCJILY CPZ YKKYPJUCR KUWPUXUACPAY OX LYCRUJE UP WYPYLCR.

5. TFERQUISIP WI TUAJB RFER KIL QFUAJB BU ALRU WUA BU WI ISIL QU RU RFIK.
6. SDRAWKCAB DAER TSUJ.
7. 88O 1E O77 6O88 88O 1E 8833A88 I8 8833E 7UE888IO6.
8. LQWKHEHJLQQLQJJRGFUHDWHGWKHKHDYHQVD-QGWKHHDUWK.
9. RMDTITBMLTGEAHGRMDTITVTYEO.
10. YRNEANFGUBHTULBHJRERGBYVIRSBERIRE–YVIRNFGUBHTULBHJRERGBQVRGBZBEEBJ.

INDEX

This index aims to include all references which throw special light on the subjects treated, but not others.